# Multiple M-wave in 11D pp-wave background and BNM matrix model.

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- ***Introduction***
  - ***Superembedding approach to M0-brane (M-wave) and multiple M0-brane system (mM0 = multiple M-wave).***
  - ***Equations of motion for mM0 in generic 11D SUGRA background***
- ***Equations of motion for mM0 in supersymmetric 11D pp-wave background and Berenstein-Maldacena-Nastase (BMN) Matrix model.***
- ***Conclusion and outlook.***

# Introduction

- Matrix model (Matrix theory) was proposed by **Banks, Fischler, Shenker and Susskind** in 96- and remains an important tool for studying M-theory.
- Although the theory is **11 dimensional**, original BFSS Lagrangian is just a dimensional reduction of **D=10 SYM** down to d=1(low energy mD0). The symmetry enlargement to D=11 Lorentz symmetry was revealed by BFSS.
- However it was **not clear** how to write the action for **Matrix model in 11D supergravity background.**
- This is why matrix models are known (to be a bit provocative: they were guessed) for a few particular supergravity background, in particular
- for pp-wave background [***Berenstein-Maldacena-Nastase 2002***] =BMN Matrix model [Dasgupta, Sheikh-Jabbari and Van Raamsdonk- obtained it by reduction of light cone supermembrane in pp-wave background]
- for matrix Big Bang background [Craps, Sethi, Verlinde 2005]
- A natural way to resolve this problem was to obtain invariant action (or covariant eqs of motion) for multiple M0-brane system. But it was a problem to write it
- Purely bosonic mM0 action [***Janssen & Y. Losano*** 2002] – similar to Myers ‘dielectric’ ‘mDp-brane’ , neither susy nor Lorentz invariance.
- *Superembedding approach to **mM0** system* [***I.B.*** 2009-2010] (multiple M-waves or multiple massless superparticle in 11D) => **SUSY inv. Matrix model equations in an arbitrary 11D SUGRA.**
- ***This talk describes the first application of these equations.***
- ***It is natural to begin by specifying the general equations for pp-wave background and compear with equations of the BMN matrix model.***

# Matrix model eqs. in general 11D SUGRA background *[I.B. 2010]*

$$\ddot{\mathbf{X}}^i \quad = \frac{1}{16}\left[\mathbf{X}^j, \left[\mathbf{X}^j, \mathbf{X}^i\right]\right] + i\gamma^i_{qp}\left\{\Psi_q\,,\,\Psi_p\right\} + \frac{1}{4}\mathbf{X}^j \hat{R}_{\#j\,\#i} +$$

$$+\frac{1}{8}\hat{F}_{\#ijk}\left[\mathbf{X}^j,\,\mathbf{X}^k\right] - 2i\Psi_q \hat{T}_{\#i\,+q}\,.$$

$$\dot{\Psi}_q = -\frac{1}{4}\gamma^i_{qp}\left[\mathbf{X}^i\,,\,\Psi_p\right] + \frac{1}{24}\hat{F}_{\#ijk}\gamma^{ijk}_{qr}\Psi_r - \frac{1}{4}\mathbf{X}^i\hat{T}_{\#i\,+q}\,,$$

$$\left[\mathbf{X}^i\,,\,\dot{\mathbf{X}}^i\right] = 4i\left\{\Psi_q\,,\,\Psi_q\right\}$$

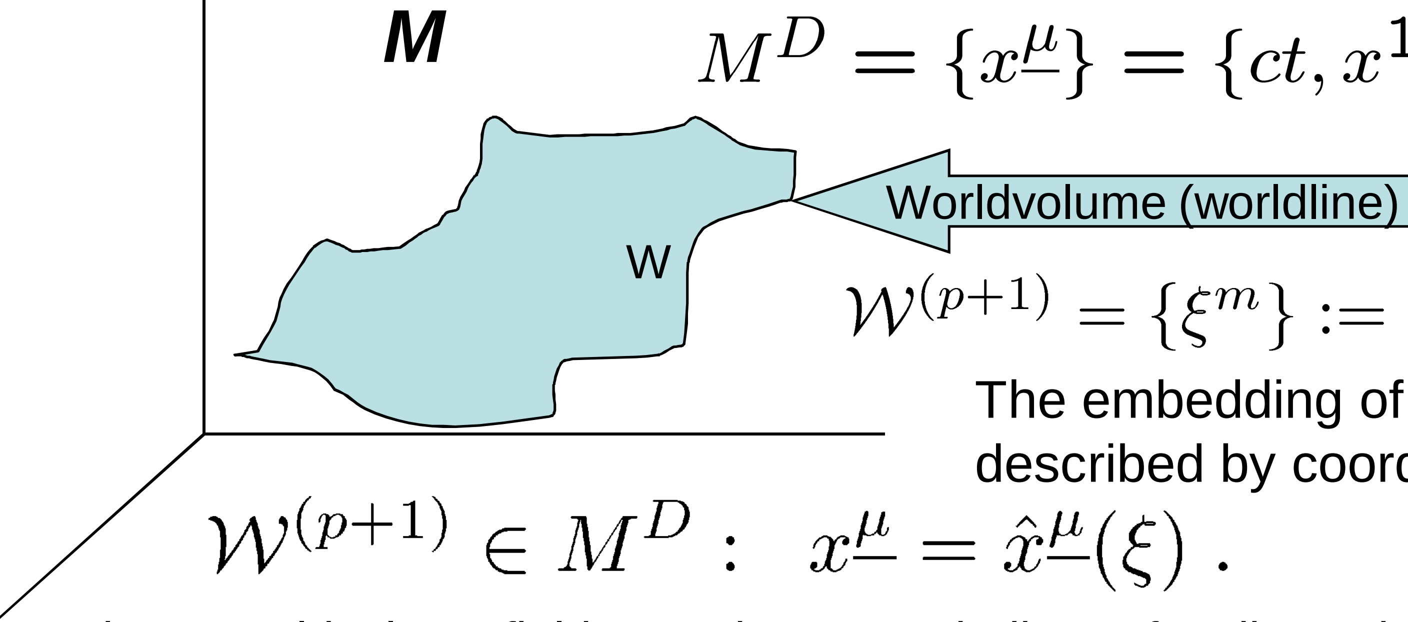
Obtained in the frame of superembedding approach

$$\dot{\mathbf{X}}^i := D_\#\mathbf{X}^i = \mathcal{E}^\tau_\#\partial_\tau\mathbf{X}^i + \mathcal{E}^{+\check{q}}_\#(\partial_{+\check{q}}\mathbf{X}^i)\big|_{\eta^{+\check{q}}=0} - \mathbf{X}^j A^{ji}_\# - 2\mathbf{X}^i A^{(0)}_\#$$

So we need to say what is the superembedding approach

# Bosonic particles and p-branes.

***Target space = spacetime***




$$M^D = \{x^{\underline{\mu}}\} = \{ct, x^1, ..., x^{(D-1)}\}$$

$$\mathcal{W}^1 = \{\tau\}$$

$$\mathcal{W}^{(p+1)} = \{\xi^m\} := (\tau, \sigma^1, ..., \sigma^p)$$

The embedding of ***W*** in ***Σ*** can be described by coordinate functions

$$\mathcal{W}^{(p+1)} \in M^D: \quad x^{\underline{\mu}} = \hat{x}^{\underline{\mu}}(\xi) \; .$$

These worldvolume fields carrying vector indices of D-dimensional spacetime coordinates are restricted by the ***p-brane equations of motion***

$$\partial_m(\sqrt{|g|}g^{mn}g_{\mu\nu}(\hat{x})\partial_n\hat{x}^\nu(\xi)) - \frac{1}{2}\sqrt{|g|}g^{mn}\partial_m\hat{x}^\nu\partial_n\hat{x}^\rho(\partial_\mu g_{\nu\rho})(\hat{x}) = 0$$

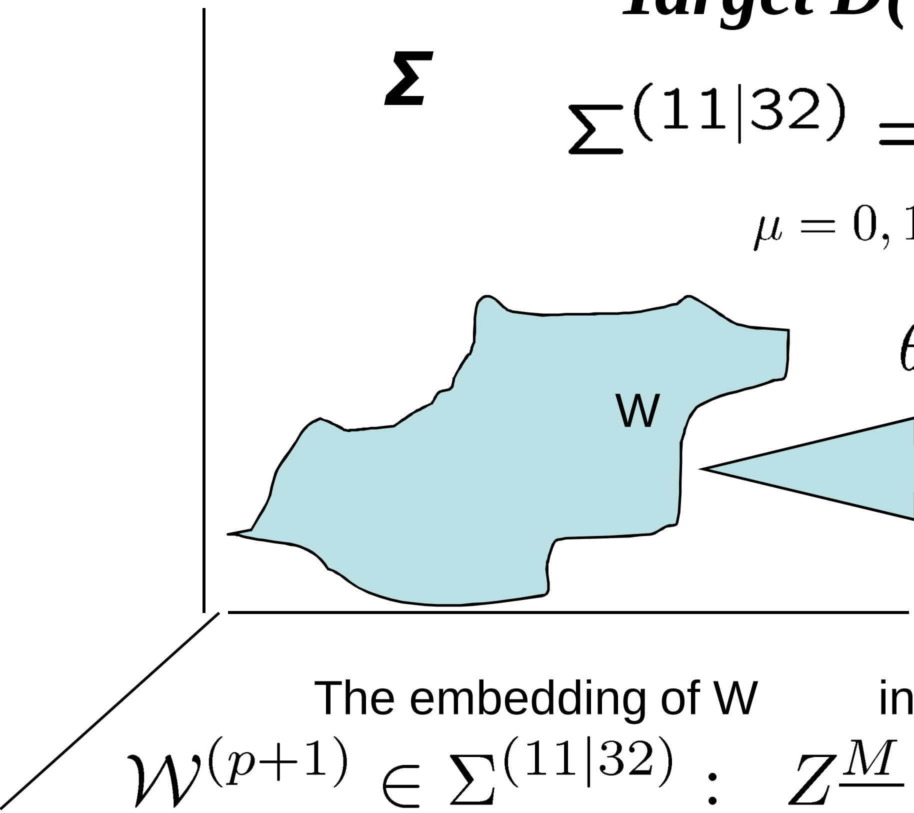
***minimal surface equation***
***(Geodesic eq. for p=0)***

$$\Leftarrow \; S_{DNG} = \int d^{p+1}\xi\sqrt{|g|}, \quad g_{mn} = \partial_m\hat{x}^\mu(\xi)\hat{x}^\nu(\xi)g_{\mu\nu}(\hat{x}(\xi))$$

or $\Leftarrow \; T^\mu{}_{\nu;\mu} = 0$

# Super-p-branes. Target superspaces and worldvolume

## ***Target D(=11) SUPERspace***

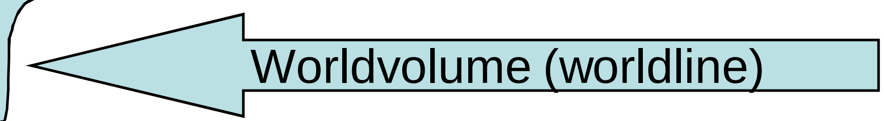



$$\Sigma^{(11|32)} = \{Z^{\underline{M}}\} := (x^{\mu}\, , \theta^{\check{\alpha}})$$

$$\mu = 0, 1, \ldots 9, 10, \quad \underline{\check{\alpha}} = 1, \ldots, 32$$

$$\theta^{\check{\alpha}}\theta^{\check{\beta}} = -\theta^{\check{\beta}}\theta^{\check{\alpha}}$$

***Grassmann or fermionic coordinates***




$$\mathcal{W}^{1} = \{\tau\}$$

$$\mathcal{W}^{(p+1)} = \{\xi^{m}\} := (\tau\, , \sigma^{1}, ..., \sigma^{p})$$

The embedding of W in Σ can be described by coordinate

$$\mathcal{W}^{(p+1)} \in \Sigma^{(11|32)}: \quad Z^{\underline{M}} = \hat{Z}^{\underline{M}}(\xi) \; = (\hat{x}^{m}(\xi)\, , \hat{\theta}^{\underline{\check{\alpha}}}(\xi)) \; .$$

These bosonic and fermionic worldvolume fields, carrying indices of 11D superspace coordinates, are restricted by the super-p-brane equations of motion

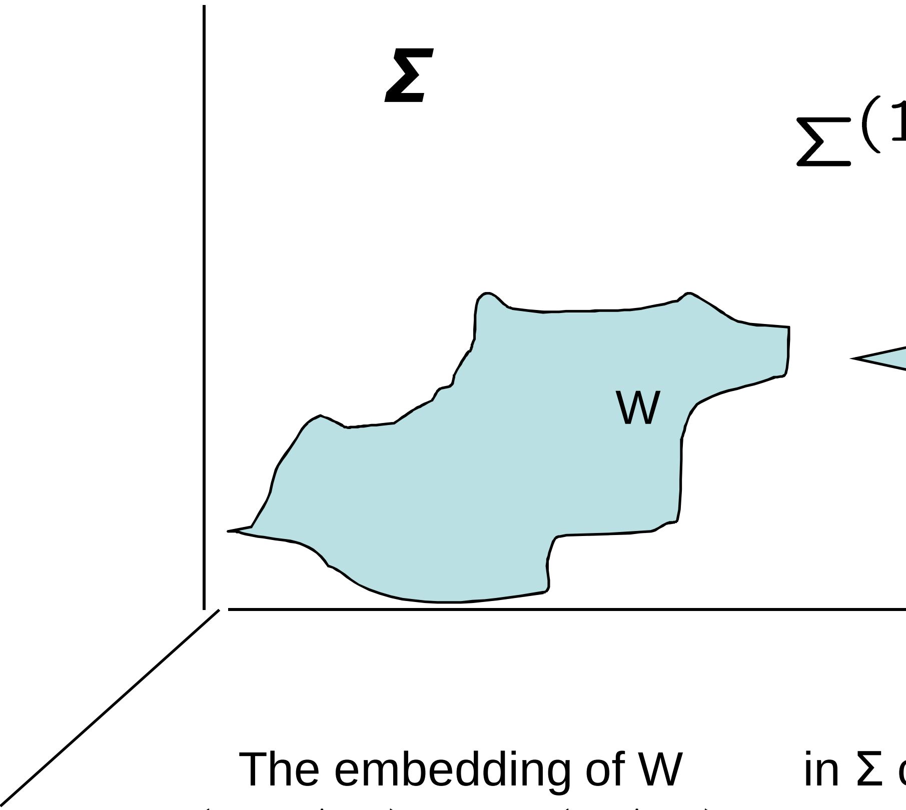
$$\Leftarrow \boxed{S = \int d^{p+1}\xi \sqrt{|g|} - \int_{\mathcal{W}^{p+1}} \hat{C}_{p+1}}\, , \; g_{mn} = \hat{E}_m{}^a \hat{E}_{na} \;\; , \;\; \hat{E}_m{}^a = \partial_m \hat{Z}^M(\xi) E_M^a(\hat{Z})$$

wave SSP and BMN

# Superembedding approach,

- [Bandos,Pasti.Sorokin, Tonin, Volkov 1995, Howe, Sezgin 1996],
- following the pioneer STV approach to superparticle and superstring [Sorokin, Tkach, Volkov MPLA 1989],
- provides a superfield description of the super-p-brane dynamics, in terms of embedding of superspaces,
- namely of embedding of worldvolume superspace $\mathcal{W}^{(p+1|n/2)}$ into a target superspace $\Sigma^{(D|n)}$.
- It is thus doubly supersymmetric, and the worldline (world-volume) susy repaces **[STV 89]** the enigmatic local fermionic kappa-symmetry [de Azcarraga+Lukierski 82, Siegel 93] of the standard superparticle and super-p-brane action.

# (M)p-branes in superembedding approach.
# I. Target and worldline superspaces.




***Target D=11 SUPERspace***

$$\Sigma^{(11|32)} = \{Z^{\underline{M}}\} := (x^{\mu}\, , \theta^{\underline{\check{\alpha}}})$$

$$\mu = 0, 1, \ldots 9, 10, \quad \underline{\check{\alpha}} = 1, \ldots, 32$$

Worldvolume (worldline) ***SUPERspace***

$$\mathcal{W}^{(p+1|16)} = \{\zeta^{\mathcal{M}}\} := (\xi^{m}\, , \eta^{\check{\alpha}})$$

$$\mathcal{W}^{(1|16)} = \{\zeta^{\mathcal{M}}\} := (\tau\, , \eta^{q})$$

$$\eta^q \eta^p = -\eta^p \eta^q .$$

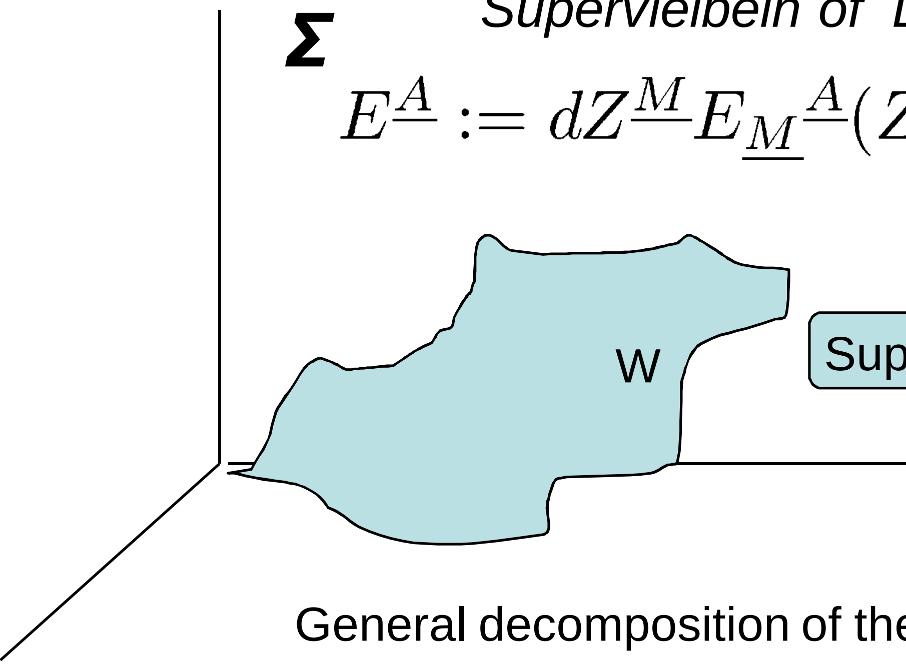
The embedding of W in Σ can be described by coordinate functions

$$\mathcal{W}^{(p+1|16)} \in \Sigma^{(11|32)} : \quad Z^{\underline{M}} = \hat{Z}^{\underline{M}}(\zeta) \; = (\hat{x}^{\underline{m}}(\zeta)\; , \hat{\theta}^{\underline{\check{\alpha}}}(\zeta)) \; .$$

These worldvolume superfields carrying indices of 11D superspace coordinates are restricted by the ***superembedding equation***.

# M0-brane (***M-wave***) in superembedding approach.
# I. Superembedding equation

*Supervielbein of D=11* **superspace** $\Sigma^{(11|32)}$




$$E^{\underline{A}} := dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z) = (E^{\underline{a}}, E^{\underline{\alpha}}) = E^{\underline{A}}(Z) \;,$$

$$\underline{a} = 0, 1, \ldots 9, 10, \qquad \underline{\alpha} = 1, \ldots, 32$$

Supervielbein of worldline ***superspace*** $\mathcal{W}^{(1|16)}$

$$\mathcal{E}^{\mathcal{A}} = d\zeta^{\mathcal{M}} \mathcal{E}_{\mathcal{M}}{}^{\mathcal{A}}(\zeta) = (\mathcal{E}^{\#} \;,\; \mathcal{E}^{+q}) \;,$$

$$q = 1, \ldots, 16$$

General decomposition of the pull-back of 11D supervielbein

$$\hat{E}^{\underline{A}} := E^{\underline{A}}(\hat{Z}) = d\hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(\hat{Z}) = \mathcal{E}^{\#} \hat{E}_{\#}{}^{\underline{A}} + \mathcal{E}^{+q} \hat{E}_{+q}{}^{\underline{A}} \;.$$

***Superembedding equation*** ***states that the pull-back of bosonic vielbein has vanishing fermionic projection:***.

$$\boxed{\hat{E}_{+q}{}^{\underline{a}} := D_{+q} \hat{Z}^{\underline{M}}\, E_{\underline{M}}{}^{\underline{a}}(\hat{Z}) = 0}$$

Superembedding equation

$$\boxed{\hat{E}_{+q}{}^{\underline{a}} := D_{+q}\hat{Z}^{\underline{M}}\, E_{\underline{M}}{}^{\underline{a}}(\hat{Z}) = 0}$$

Equations of motion of (single) M0-brane

(+ conv. constr.) geometry of worldline superspace $\mathcal{W}^{(1|16)}$

$$D\mathcal{E}^{\#} = -2i\mathcal{E}^{+q}\wedge\mathcal{E}^{+q}\;,$$ SO(1,1) curvature of $\mathcal{W}^{(1|16)}$ vanishes,

$$\mathcal{E}^{\mathcal{A}} = d\zeta^{\mathcal{M}}\mathcal{E}_{\mathcal{M}}{}^{\mathcal{A}}(\zeta) = (\mathcal{E}^{\#}\;,\;\mathcal{E}^{+q})\ \text{- induced supervilebein.}$$

$$D\mathcal{E}^{+q} = -\frac{1}{72}\mathcal{E}^{\#}\wedge\mathcal{E}^{+q}\hat{F}_{\#ijk}\gamma^{ijk}_{qp}\;,\qquad \hat{F}_{\#ijk} := F^{abcd}(\hat{Z})u_a{}^{=}u_b{}^{i}u_c{}^{j}u_d{}^{k}\;.$$

$$F_4 = dC_3$$

4-form flux of 11D SG= field strength of 3-form

Moving frame vectors

The 4-form flux superfield enters the solution of the 11D superspace SUGRA constraints [Cremmer & Ferrara 80, Brink & Howe 80] (which results in SUGRA eqs. of motion):

$$T^{\underline{a}} := DE^{\underline{a}} = -iE^{\alpha}\wedge E^{\beta}\Gamma^{\underline{a}}_{\alpha\beta}\;.$$

$$T^{\alpha} = -\frac{i}{18}E^{a}\wedge E^{\beta}\left(F_{ac_1c_2c_3}\Gamma^{c_1c_2c_3}{}_{\beta}{}^{\alpha} + \frac{1}{8}F^{c_1c_2c_3c_4}\Gamma_{ac_1c_2c_3c_4}{}_{\beta}{}^{\alpha}\right) + \frac{1}{2}E^{a}\wedge E^{b}T_{ba}{}^{\alpha}(Z)\;,$$

$$R^{ab} = E^{\alpha}\wedge E^{\beta}\left(-\frac{1}{3}F^{abc_1c_2}\Gamma_{c_1c_2} + \frac{i}{3\cdot 5!}(*F)^{abc_1\dots c_5}\Gamma_{c_1\dots c_5}\right)_{\alpha\beta} + E^{c}\wedge E^{\alpha}\left(-iT^{ab\beta}\Gamma_{c\beta\alpha} + 2iT_c{}^{[a\,\beta}\Gamma^{b]}{}_{\beta\alpha}\right) + \frac{1}{2}E^{d}\wedge E^{c}R_{cd}{}^{ab}(Z)\;.$$

wave SSP and BMN

# Moving frame and spinor moving frame variables

$$U_a^{(b)} = \left( \frac{1}{2}\left(u_a{}^{\#} + u_a{}^{=}\right), \, u_a{}^{i}, \, \frac{1}{2}\left(u_a{}^{\#} - u_a{}^{=}\right) \right) \in SO(1,10), \qquad V_{(\alpha)}{}^{\beta} =: \left( v_q{}^{-\beta}, \, v_q^{+\alpha} \right) \in Spin(1,10)$$

appear in the conventional constraints determining the induced supervielbein so that

$$\boxed{\hat{E}^a = \frac{1}{2}\, \mathcal{E}^{\#}\, u^{a\,=}} \quad , \qquad \hat{E}^{\alpha} = \mathcal{E}^{+q} v_q{}^{-\alpha} + \mathcal{E}^{\#} \chi_{\#}{}^{-q} v_q^{+\alpha}$$

Equivalent form of the superembedding eq.
+ conventional constraints.

$$\hat{E}_{+q}{}^{\underline{a}} = 0$$

on shell $\chi_{\#}^{-q} = 0$

$$v_q^{-} \Gamma_a v_p^{-} = u_a{}^{=} \delta_{qp} \, ,$$

$$2 v_q{}^{-\alpha} v_q{}^{-\beta} = \tilde{\Gamma}^{a\alpha\beta} u_a{}^{=}$$

## *M0 equations of motion*

$$D u_b{}^{=} = 0 \, , \qquad D v_q{}^{-\alpha} = 0,$$

Simplest representative (which we will use below):

$$u_a^{=} = \delta_a^{--} \equiv \delta_a^{0} - \delta_a^{10} \, , \qquad u_a^{\#} = \delta_a^{++} \equiv \delta_a^{0} + \delta_a^{10} \, , \qquad u_a^{i} = \delta_a^{i} \, .$$

$$v_q^{-\alpha} = \delta_{+q}^{\alpha} \qquad v_q^{+\alpha} = \delta_{-q}^{\alpha} \, ,$$

# Multiple M0 description by d=1, ☠=16 SU(N) SYM on $\mathcal{W}^{(1|16)}$.

- We describe the multiple M0 by 1d ☠=16 SYM on $\mathcal{W}^{(1|16)}$
- The embedding of $\mathcal{W}^{(1|16)}$ into the 11D SUGRA superspace is determined by the superembedding equation

$$\boxed{\hat{E}_{+q}{}^{\underline{a}} := D_{+q}\hat{Z}^{\underline{M}}\, E_{\underline{M}}{}^{\underline{a}}(\hat{Z}) = 0}$$

$\Rightarrow$ 'center of energy' motion of the mM0 system is defined by the single M0 eqs

$$\left.\begin{array}{r} Du_b{}^{=} = 0\ , \\ Dv_q^{-\alpha} = 0 \end{array}\right\} \quad , \quad \left\{ \begin{array}{l} D\mathcal{E}^{\#} = -2i\mathcal{E}^{+q} \wedge \mathcal{E}^{+q}\ , \\ D\mathcal{E}^{+q} = -\dfrac{1}{72}\mathcal{E}^{\#} \wedge \mathcal{E}^{+q}\hat{F}^{=ijk}\gamma^{ijk}_{qp}\ , etc. \end{array}\right.$$

# Multiple M0 description by d=1, ☠=16 SYM on $\mathcal{W}^{(1|16)}$

## Basic SYM constraints and superembedding-like equation.

$A = \mathcal{E}^{\#} A_{\#} + \mathcal{E}^{+q} A_{+q}$ is an su(N) valued 1-form potential on $\mathcal{W}^{(1|16)}$

with the field strength $G_2 = dA - A \wedge A = \frac{1}{2}\mathcal{E}^{+q} \wedge \mathcal{E}^{+p} G^{--}_{q\,p} + \mathcal{E}^{\#} \wedge \mathcal{E}^{+q} G_{+q\,\#}$

We impose constraint

$$\boxed{G^{--}_{q\,p} = i\gamma^i_{qp}\mathbf{X}^i\ .}$$

A clear candidate for the description of relative motion of the mM0-constituents!

Bianchi identities DG=0 $\Longrightarrow$ the *superembedding-like equation*

$$\boxed{D_{+q}\mathbf{X}^i = 4i\gamma^i_{qp}\Psi_q}$$

(in a gauge $E^a_{+q}u^i_a = D_{+q}X^i - 4i\gamma^i_{qp}\Theta^-_p + \ldots$ so that $E^a_{+q} = 0$ is equivalent to $D_{+q}X^i = 4i\gamma^i_{qp}\Theta^-_p + \ldots$)

Studying its selfconsistecy conditions, we find the dynamical equations describing the relative motion of mM0 constituents.

# Equations for the relative motion of multiple M0 in an arbitrary supergravity background follow from the constr.

## *1d Dirac equation*

$$D_{\#}\Psi_q = -\frac{1}{4}\gamma^i_{qp}\left[\mathbf{X}^i\,,\,\Psi_p\right] - \frac{1}{24}\hat{F}_{\#ijk}\gamma^{ijk}_{qr}\Psi_r - \frac{1}{4}\mathbf{X}^i\hat{T}_{\#i\,+q}\,.$$

$$\hat{F}_{\#ijk} := F^{abcd}(\hat{Z})u_a{}^{=}u_b{}^{i}u_c{}^{j}u_d{}^{k}\,, \qquad \hat{T}_{\#\,i-q} := \hat{T}_{ab}{}^{\beta}u^{a=}u^{bi}v_{\beta q}{}^{-}.$$

## *Gauss constraint*

$$\left[\mathbf{X}^i\,,\,D_{\#}\mathbf{X}^i\right] = 2i\left\{\Psi_q\,,\,\Psi_p\right\}$$

## *Bosonic equations of motion*

$$D_{\#}D_{\#}\mathbf{X}^i = \frac{1}{16}\left[\mathbf{X}^j\,,\,\left[\mathbf{X}^j\,,\,\mathbf{X}^i\right]\right] + i\gamma^i_{qp}\left\{\Psi_q\,,\,\Psi_p\right\}$$
$$+\frac{1}{4}\mathbf{X}^j\hat{R}_{\#j\;\#i} + \frac{1}{8}\hat{F}_{\#ijk}\left[\mathbf{X}^j\,,\,\mathbf{X}^k\right] - 2i\Psi_q\hat{T}_{\#i\,+q}\,.$$

Coupling to higher form characteristic for the Emparan-Myers dielectric brane effect

# mM0 in pp-wave background

Supersymmeric bosonic pp-wave solution of 11D SUGRA is very well known:

$$ds^2 \;=\; dx^{--}dx^{++} + \left[(\frac{\mu}{3})^2 x^I x^I + (\frac{\mu}{6})^2 x^{\tilde{J}} x^{\tilde{J}}\right] dx^{++}dx^{++} - dx^I dx^I - dx^{\tilde{J}} dx^{\tilde{J}} \;,$$

$$\begin{cases} I = 1,2,3, \\ \tilde{J} = 4,5,6,7,8,9, \end{cases}$$

constant 4 form flux $F_{++123} = \frac{\mu}{2}$:

$$F_{abcd} = 2\mu \delta_{[a}{}^{++} \delta_b^I \delta_c^J \delta_{d]}^K \epsilon_{IJK} \;, \qquad I, J, K = 1,2,3 \;.$$

# mM0 in pp-wave background

Constant 4 form flux

$$F_{abcd} = 2\mu\delta_{[a}{}^{++}\delta_b^I\delta_c^J\delta_{d]}^K\epsilon_{IJK} \ , \qquad I, J, K = 1, 2, 3 \ .$$

Vielbein one-forms

$$e^{++} = dx^{++} \ , \qquad e^I = dx^I \ , \qquad e^{\tilde{J}} = dx^{\tilde{J}} \ ,$$
$$e^{--} = dx^{--} + \left[\left(\frac{\mu}{3}\right)^2 x^I x^I + \left(\frac{\mu}{6}\right)^2 x^{\tilde{J}} x^{\tilde{J}}\right] dx^{++} \ ,$$

nonvanishing components of the spin connection:

$$\omega^{I--} = 2\left(\frac{\mu}{3}\right)^2 e^{++} x^I = 2\left(\frac{\mu}{3}\right)^2 dx^{++}\, x^I \ ,$$
$$\omega^{\tilde{J}--} = 2\left(\frac{\mu}{6}\right)^2 e^{++} x^{\tilde{J}} = 2\left(\frac{\mu}{6}\right)^2 dx^{++}\, x^{\tilde{J}} \ .$$

The nonvanishing components of the $SO(1,10)$ curvature are

$$R^{--I} = -2\left(\frac{\mu}{3}\right)^2 \ e^{++} \wedge e^I \ , \qquad R^{--\tilde{J}} = -2\left(\frac{\mu}{6}\right)^2 \ e^{++} \wedge e^{\tilde{J}} \ ,$$

- Thus it looks like we just have to substitute definite pure bosonic expressions into the general mM0 equations.
- However, this is not the case, because, for instance

$$D_{\#}\mathbf{X}^i = \mathcal{E}_{\#}^{\tau}\partial_\tau \mathbf{X}^i + \mathcal{E}_{\#}^{+\check{q}}\frac{\partial}{\partial\eta^{+\check{q}}}\mathbf{X}^i + \mathbf{X}^j A_{\#}^{ji} - 2\mathbf{X}^j A^{(0)}$$

- and also (in a more general case of, e.g., non-constant flux)

$$\hat{F}_{abcd} := F_{abcd}(\hat{Z}) = F_{abcd}(\hat{x}(\tau)) - 6\hat{\theta}^\alpha(\tau)\Gamma_{[ab|\alpha\beta}T_{|cd]}{}^{\beta}(\hat{x}(\tau)) + \propto \hat{\theta}^\alpha(\tau)\hat{\theta}^\beta(\tau)\,.$$

- Thus it is not sufficient to know pure bosonic supersymmetric solution
- To specify our mM0 eqs for some particular SUGRA background it is necessary to describe this background as a superspace $\Sigma^{(11|32)}_{pp-w}$
- and to find some details on the worldline SSP $\mathcal{W}^{(1|16)}$ embeddded in $\Sigma^{(11|32)}_{pp-w}$
  - as this allows to find $\mathcal{E}_{\#}^{+\check{q}}$, $\mathrm{so}(9)$ and $\mathrm{so}(1{,}1)$ connection $A_{\#}^{ji}$ and $A_{\#}^{(0)}$

- Fortunately the pp-wave superspace is a coset, so that
- one can write a definite expression for supervielbein etc. (but…)

$$E^{\underline{a}}(x,\Theta) = e^{\underline{a}}(x) - 2i\overset{0}{\mathcal{D}}\Theta^\beta \sum_{n=0}^{15} \tfrac{1}{(2n+2)!} \left((\Theta\Theta\mathcal{M})^n\Gamma^{\underline{a}}\Theta\right)_\beta \ ,$$

$$E^\alpha(x,\Theta) = \overset{0}{\mathcal{D}}\Theta^\alpha + \overset{0}{\mathcal{D}}\Theta^\beta \sum_{n=1}^{16} \tfrac{1}{(2n+1)!}\left((\Theta\Theta\mathcal{M})^n\right)_\beta{}^\alpha \ ,$$

$$\omega^{\underline{ab}}(x,\Theta) = \omega^{\underline{ab}}(x) + \overset{0}{\mathcal{D}}\Theta^\beta \sum_{n=0}^{15} \tfrac{1}{(2n+1)!} \left((\Theta\Theta\mathcal{M})^n\right)_\beta{}^\gamma R_{\gamma\alpha}{}^{\underline{ab}}\,\Theta^\alpha \ ,$$

$$\textit{where} \qquad (\Theta\Theta\mathcal{M})_\alpha{}^\beta := 2i\,(\Theta\Gamma^a)_\alpha\ \Theta^\gamma T_{a\,\gamma}{}^\beta - \tfrac{1}{4}\Theta^\gamma R_{\gamma\alpha}{}^{ab}\,(\Theta\Gamma_{ab})^\beta \ ,$$

$e^{\underline{a}}(x) = E^{\underline{a}}(x,0)$ is purely bosonic vielbein, $\omega^{\underline{ab}}(x) = \omega^{\underline{ab}}(x,0) = dx^\mu \omega_\mu^{\underline{ab}}(x)$ is specetime spin connection, $T_{a\,\beta}{}^\alpha = \frac{i}{18}\left(F_{a[3]}\Gamma^{[3]} + \frac{1}{8}F^{[4]}\Gamma_{a[4]}\right)_\beta{}^\alpha$ and

$$\overset{0}{\mathcal{D}}\Theta^\beta := \overset{0}{D}\Theta^\beta - e^{\underline{a}}(x)\,\Theta^\alpha T_{\underline{a}\,\alpha}{}^\beta = d\Theta^\beta - \frac{1}{4}\omega^{\underline{ab}}(x)\Theta^\alpha\Gamma_{\underline{ab}\,\alpha}{}^\beta - e^{\underline{a}}(x)\,\Theta^\alpha T_{\underline{a}\,\alpha}{}^\beta \ .$$

- The worldline (center of energy) superspace is embedded in this SSP.
- Its embedding is specified in terms of bosonic and fermionic coordinate superfields $\hat{Z}^{\underline{M}}(\tau,\eta^q) = (\hat{x}^\mu(\tau,\eta), \hat{\Theta}^\alpha(\tau,\eta))$
- Part of these are Goldstone (super)fields corresponding to (super)symmetries broken by brane/by center of energy of mM0
- and part can be identified with coordinates of $\mathcal{W}^{(1|16)}$
  - for instance, $\hat{\Theta}^\alpha(\tau,\eta) = \eta^{+q} S_q{}^p(\tau,\eta) v_q^{-\alpha} + \Theta^{-q}(\tau,\eta) v_q^{+\alpha}$ ,

Fermionic coordinate of W

Goldstone fermion superfield

- Let us begin by the simplest case when the Goldstone (super)fields describing the center of energy motion are =0, i.e. by describing a vacuum solution for center of energy SSP $\mathcal{W}_0^{(1|16)}$

- The embedding of the vacuum worldline superspace $\mathcal{W}_0^{(1|16)}$
- is characterized by that all Goldstone fields are zero or const.

$$\hat{x}^I(\tau,\eta) = 0 \, , \qquad \hat{x}^{\tilde{J}}(\tau,\eta) = 0 \, , \qquad \hat{x}^{++}(\tau,\eta) = \tau \, ,$$

$$\hat{\Theta}^\alpha(\tau,\eta) = \Theta^{+q} v_q^{-\alpha} \, , \qquad , \quad \hat{\Theta}^{+q} \propto \eta^{+q} \, , \qquad \hat{\Theta}^{-q} = 0 \, .$$

- more precisely: $\hat{\Theta}^{+q}(\tau,\eta) = \eta^{+p} \; \left( exp \left\{ -\frac{\mu}{6} \, \hat{x}^{++} \gamma^{123} \right\} \right)_{pq} .$
- by constant moving frame and spinor moving frame variables

$$u_a^{=} = \delta_a^{--} \equiv \delta_a^0 - \delta_a^{10} \, , \qquad u_a^{\#} = \delta_a^{++} \equiv \delta_a^0 + \delta_a^{10} \, , \qquad u_a^i = \delta_a^i \, .$$

$$v_q^{-\alpha} = \delta_{+q}^\alpha \qquad v_q^{+\alpha} = \delta_{-q}^\alpha \, ,$$

- With which the flux pull-back to *W* is: $\boxed{\widehat{F}_{\# ijk} = F_{++123} = \frac{\mu}{2}}$
- by $\qquad e^I(\hat{x}) = 0 \, , \quad e^I(\hat{x}) = 0 \, , \quad e^{++}(\hat{x}) = d\hat{x}^{++} \, , \qquad \omega^{ab}(\hat{x}) = 0 \, ,$
- One can check that equations of motion and superembedding equation are satisfied, $\hat{E}^I = 0, \, \hat{E}^{\tilde{J}} = 0, \, \hat{E}^{--} = 0 \text{ and } Du_a^{=} = 0.$

- But our main interest is in intrinsic geometry of $\mathcal{W}_0^{(1|16)}$ as the relative motion is described on this superspace.

$$\mathcal{E}^{\#} = d\hat{x}^{++} - 2id\eta^{+q}d\eta^{+q} - 4id\eta^{+p}(\eta\eta\mathcal{K})_{pq}\eta^{+q} \ ,$$
$$\mathcal{E}^{+q} = d\eta^{+p}S^{pq}(\tau,\eta) \ .$$

$$S^{pq}(\tau,\eta) := \left(\delta_{pp'} + \sum_{n=1}^{8} \frac{1}{(2n+1)!}((\eta\eta\mathcal{M})^{-+})^n)_{pp'}\right)\left(exp\left\{-\frac{\mu}{6}\,\hat{x}^{++}\gamma^{123}\right\}\right)_{p'q} \ ,$$
$$(\eta\eta\mathcal{K})_{pq} := \sum_{n=1}^{7} \frac{1}{(2n+2)!}((\eta\eta\mathcal{M})^{-+})^n)_{pq} \ .$$

$$(\eta\eta\mathcal{M})^{-+}_{pq} = -\frac{2i\mu}{3}\eta^{+p}(\eta^+\gamma^{123})_q - \frac{i\mu}{3}\epsilon^{IJK}(\eta^+\gamma^I)_p(\eta^+\gamma^{JK})_q - \frac{i\mu}{12}\frac{1}{4!}\epsilon_{\tilde{I}\tilde{J}\tilde{J}_1\ldots\tilde{J}_4}(\eta^+\gamma^{\tilde{J}_1\ldots\tilde{J}_4})_p(\eta^+\gamma^{\tilde{I}\tilde{J}})_q \ ,$$
$$(\eta\eta\mathcal{M})^{+-}_{pq} = -\frac{2i\mu}{3}(\eta^+\gamma^I)_p(\eta^+\gamma^I\gamma^{123})_q - \frac{2i\mu}{3}(\eta^+\gamma^I\gamma^{123})_p(\eta^+\gamma^I)_q - \frac{i\mu}{3}(\eta^+\gamma^{\tilde{J}})_p(\eta^+\gamma^{\tilde{J}}\gamma^{123})_q - \frac{i\mu}{6}\frac{1}{5!}\epsilon_{\tilde{J}\tilde{J}_1\ldots\tilde{J}_5}(\eta^+\gamma^{\tilde{J}_1\ldots\tilde{J}_5})_p(\eta^+\gamma^{\tilde{J}})_q$$

- What is really important:

$$\mathcal{E}^{+q}_{\tau}(\tau,\eta) \propto \mathcal{E}^{+q}_{++}(\tau,\eta) = 0$$
$$\mathcal{E}^{\#}_{\tau}(\tau,\eta) \propto \mathcal{E}^{\#}_{++}(\tau,\eta) = 1$$

$$\nabla_{\#} = \hat{\partial}_{++} \ , \qquad \nabla_{+q} = (L^{-1})^{qp}(D_{+p} + 4i(\eta\eta\mathcal{K})_{pp'}\eta^{+p'}\hat{\partial}_{++}) \ ,$$

- Furthermore, the induced SO(9) and SO(1,1) connection have only fermionic components, so that

$$\Omega^{(0)}_{\#} = 0, \ A^{ij}_{\#} = 0 \quad \text{and} \quad \boxed{D_{\#} = \nabla_{\#} = \hat{\partial}_{++}}$$

# Now we are ready to specify the Matrix model equations in general 11D SUGRA SSP

***for the case of completely SUSY pp-wave background***

***1d Dirac equation***

$$D_{\#}\Psi_q = -\frac{1}{4}\gamma^i_{qp}\left[\mathbf{X}^i\,,\,\Psi_p\right] - \frac{1}{24}\hat{F}_{\#ijk}\gamma^{ijk}_{qr}\Psi_r - \frac{1}{4}\mathbf{X}^i\hat{T}_{\#i\,+q}\,.$$

$$\hat{F}_{\#ijk} := F^{abcd}(\hat{Z})u_a{}^{=}u_b{}^{i}u_c{}^{j}u_d{}^{k}\,, \qquad \hat{T}_{\#\,i-q} := \hat{T}_{ab}{}^{\beta}u^{a=}u^{bi}v_{\beta q}{}^{-}.$$

***Gauss constraint***

$$\left[\mathbf{X}^i\,,\,D_{\#}\mathbf{X}^i\right] = 2i\left\{\Psi_q\,,\,\Psi_p\right\}$$

***Bosonic equations of motion***

$$D_{\#}D_{\#}\mathbf{X}^i = \frac{1}{16}\left[\mathbf{X}^j\,,\,\left[\mathbf{X}^j\,,\,\mathbf{X}^i\right]\right] + i\gamma^i_{qp}\left\{\Psi_q\,,\,\Psi_p\right\}$$
$$+\frac{1}{4}\mathbf{X}^j\hat{R}_{\#j\;\#i} + \frac{1}{8}\hat{F}_{\#ijk}\left[\mathbf{X}^j\,,\,\mathbf{X}^k\right] - 2i\Psi_q\hat{T}_{\#i\,+q}\,.$$

# mM0 eqs in pp-wave background

$$\begin{aligned}
\hat{\partial}_{++}\Psi_q &= -\frac{1}{4}\gamma^i_{qp}\left[\mathbf{X}^i\,,\,\Psi_p\right] - \frac{\mu}{4}\gamma^{123}_{qp}\Psi_p\,, \\
& \left[\mathbf{X}^I\,,\,\hat{\partial}_{++}\mathbf{X}^I\right] + \left[\mathbf{X}^{\tilde{J}}\,,\,\hat{\partial}_{++}\mathbf{X}^{\tilde{J}}\right] = 4i\left\{\Psi_q\,,\,\Psi_q\right\} \\
\hat{\partial}_{++}\hat{\partial}_{++}\mathbf{X}^I &= \frac{1}{16}\left[\mathbf{X}^j,\left[\mathbf{X}^j,\mathbf{X}^I\right]\right] + i\gamma^I_{qp}\left\{\Psi_q\,,\,\Psi_p\right\} - \left(\frac{\mu}{3}\right)^2\mathbf{X}^I - \\
& -\frac{\mu}{8}\epsilon^{IJK}\left[\mathbf{X}^J,\,\mathbf{X}^K\right]\,, \\
\hat{\partial}_{++}\hat{\partial}_{++}\mathbf{X}^{\tilde{J}} &= \frac{1}{16}\left[\mathbf{X}^i,\left[\mathbf{X}^i,\mathbf{X}^{\tilde{J}}\right]\right] + i\gamma^{\tilde{J}}_{qp}\left\{\Psi_q\,,\,\Psi_p\right\} - \left(\frac{\mu}{6}\right)^2\mathbf{X}^{\tilde{J}}\,.
\end{aligned}$$

**These eqs coincide with the ones which can be obtained by varying the BMN action up to the fact that they are formulated for traceless matrices.**

The trace part of the matrices should describe the center of energy motion.
In our approach it is described separately by the geometry of $\mathcal{W}^{(1|16)}$.

To find this, one should go beyond the ground state solution of the superembedding eq, which we have used above. This is under study now. But what one can see easily is…

**What one can see easily is** that the complete set of the BMN eqs is reproduced by the mM0 equations in the leading approximation on the center of energy Goldstone fermion $\hat{\theta}^{-q}(\tau)$.

To this end we can study the leading component $(\eta^{+q} = 0 \text{ limits})$ of the superembedding equation and its consequences with the ansatz

$$\hat{\Theta}^{\alpha}(\tau, \eta) = \theta^{-q}(\tau) v_q^{+\alpha}$$

$$u_a^{=} = \delta_a^{--} + \frac{1}{4} k^{--i} k^{--i} \delta_a^{++} - k^{--i} \delta_a^{i} ,$$

$$u_a^{\#} = \delta_a^{++} \equiv \delta_a^{0} + \delta_a^{10} , \qquad u_a^{i} = \delta_a^{i} - \frac{1}{2} k^{--i} \delta_a^{++} .$$

One finds $k^{--i} = 2\hat{\partial}_{++} \hat{x}^i$ and that the Goldstone fields have to obey

$$\hat{\partial}_{++}\hat{\partial}_{++}\hat{x}^{I}(\tau) + \left(\tfrac{\mu}{3}\right)^2 \hat{x}^{I}(\tau) = 0 , \qquad \hat{\partial}_{++}\hat{\partial}_{++}\hat{x}^{\tilde{J}}(\tau) + \left(\tfrac{\mu}{6}\right)^2 \hat{x}^{\tilde{J}}(\tau) = 0 ,$$

$$\hat{\partial}_{++}\theta^{-q}(\tau) = \tfrac{\mu}{6} \theta^{-p}(\tau) \gamma_{pq}^{123} .$$

**Bosonioc eqs coincide with the trace part of the BMN equations, BUT fermionic eqs….**

# Bosonioc eqs coincide with the trace part of the BMN equations, BUT fermionic equations

$$\hat{\partial}_{++}\theta^{-q}(\tau) = \frac{\mu}{6}\theta^{-p}(\tau)\gamma^{123}_{pq} \ .$$

BMN eqs describe relativistic fermion with mass $\mu/4$ rather than $\mu/6$.

However this is not a problem: in d=1 the fermionic mass is a matter of convenience. Indeed, using the field redefinition

$$\psi^{-q} := \theta^{-p} exp\left\{\left(\frac{\mu}{12}\hat{x}^{++}\gamma^{123}\right)\right\}_{pq}$$

We find that the redefined field obeys

$$\hat{\partial}_{++}\psi^{-q}(\tau) = \frac{\mu}{4}\psi^{-p}(\tau)\gamma^{123}_{pq}$$

Thus our superembedding approach to mM0 in pp-wave SSP does reproduce the complete set of the BMN equations in the leading order on center of energy Goldstone fields.

# Conclusions and outlook

- After reviewing of the superembedding approach to mM0 system and the generalization of Matrix model eqs. in an arbitrary 11D SUGRA background obtained from it
- we used them to obtain the mM0 equations of motion in the supersymmetric pp-wave background.
- The final answer is obtained for a particular susy solution of the center of energy equations of motion.
- The equations of the relative motion of mM0 constituents coincide with the BMN equations, but written for traceless matrices.
- To compare the complete set of equations, including the trace part of the matrices which describe relative motion of the mM0 constituents we need to find general solution of the superembedding approach equations to M0-brane.

- What we have shown is that our superembedding approach to mM0 in pp-wave SSP does reproduce the complete set of the BMN eqs in leading order on center of energy Goldstone fields.
- The search for the general solution of the superembedding approach equations to M0-brane in pp-wave background is on the way.

# Some other directions for future study

- **Matrix model equations in AdS(4)xS(7) and AdS(7)xS(4), and their application, in particular in the frame of AdS/CFT.**
- **Extension of the approach for higher p mDp- and mMp- systems (mM2-?, mM5-?). Is it consistent to use the same construction (SU(N) SYM on w/v superspace of a single brane)? And, if not, what is the critical value of p?**

*Thanks for your attention!*

*Thank you for your attention!*

# Appendix A: On BPS equations

for the supersymmetric pure bosonic solutions of mM0 equations

$$\epsilon^{+q}\mathbf{N}_{ipq}|_{\eta=0} = 0$$

SUSY preservation by center of energy motion

$$\epsilon^{+q}\mathbf{M}_{pq}|_{\eta=0} = 0$$

SUSY preservation by relative motion of mM0 constituents

$$D_{+p}\hat{T}_{\#\, i\, +q} = \frac{1}{2}\hat{R}_{\# ij\#}\gamma^{j}_{pq} + \frac{1}{3}D_{\#}\hat{F}_{\# ijk}\left(\delta^{i[j}\gamma^{kl]}_{pq} + \frac{1}{6}\gamma^{ijkl}_{pq}\right) + \hat{F}_{\# j_1 j_2 j_3}\hat{F}_{\# k_1 k_2 k_3}\Sigma^{i\,,\, j_1 j_2 j_3\,,\, k_1 k_2 k_3}_{pq} =: \mathbf{N}_{ipq}\ .$$

$$D_{+p}\Psi_q \ = \tfrac{1}{2}\gamma^{i}_{pq}D_{\#}\mathbf{X}^{i} + \tfrac{1}{16}\gamma^{ij}_{pq}\left[\mathbf{X}^{i}, \mathbf{X}^{j}\right] - \tfrac{1}{12}\mathbf{X}^{i}F_{\# jkl}\left(\delta^{i[j}\gamma^{kl]} + \tfrac{1}{6}\gamma^{ijkl}\right)_{pq} =: \mathbf{M}_{pq}\ .$$

½ BPS equation (16 susy's preserved)

$$\mathbf{N}_{ipq}|_{\eta=0} = 0$$
$$\mathbf{M}_{pq}|_{\eta=0} = 0$$

has fuzzy S² solution modeling M2 brane by mM0 configuration

$$\boxed{\mathbf{X}^{i} = f\delta^{i}_{I}T^{I}\ , \qquad [T^{I}, T^{J}] = \epsilon^{IJK}T^{K}}$$

$$\hat{F}_{\# ijk} = 3/4 f^{3}\delta^{i}_{I}\delta^{j}_{J}\delta^{k}_{K}\epsilon^{IJK}\ ,$$

1/4 BPS equation (8 susy's) with SO(3) symmetry and $\hat{F}_{\# ijk} = 0\ ,$

Nahm equation

$$D_{\#}\mathbf{X}^{I} + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^{J}, \mathbf{X}^{K}] = 0\ .$$

I. Bandos, mM0 in pp-wave SSP and BMN

The famous Nahm equation $$D_{\#}\mathbf{X}^I + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^J, \mathbf{X}^K] = 0\ ,$$

which has a(nother) fuzzy 2-sphere-related solution

$$\boxed{\mathbf{X}^i = f(\tau)\delta^i_I T^I\ , \qquad [T^I, T^J] = \epsilon^{IJK}T^K}$$

appears as an SO(3) inv

# ¼ BPS equation (8 susy's preserved)

with $\hat{F}_{\#ijk} = 0\ .$

½ BPS: $\epsilon^{+q}\mathbf{N}_{ipq}|_{\eta=0} = 0$ and $\epsilon^{+q}\mathbf{M}_{pq}|_{\eta=0} = 0$ with

$$\epsilon^{+q}(1-\bar{\gamma})_{pq} = 0 \qquad \bar{\gamma}_{qq} = 0,\ (\bar{\gamma}\bar{\gamma})_{qp} = \delta_{qp}$$

$(1+\bar{\gamma})_{pr}\mathbf{N}_{irq}|_{\eta=0} = 0$ is obeyed, in particular, for $\hat{F}_{\#ijk} = 0\ .$

$$\left.\begin{array}{l}(1+\bar{\gamma})_{pr}\mathbf{M}_{rq}|_{\eta=0} = 0 \\ \mathrm{SO}(3)\ \mathrm{symmetry} \leftrightarrow \bar{\gamma} = \gamma^1\gamma^2\gamma^3\end{array}\right\} \Rightarrow ((1+\bar{\gamma})\gamma^I)_{pq}\left(D_{\#}\mathbf{X}^I + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^J, \mathbf{X}^K]\right) = 0\ .$$

$$D_{\#}\mathbf{X}^I + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^J, \mathbf{X}^K] = 0\ ,$$

# Appendix B: Moving frame and spinor moving frame

(Auxiliary) moving frame superfields $u_a{}^{\#}(\zeta),\, u_a{}^{=}(\zeta)$ and $u_a{}^{i}(\zeta)\ (i=1,\ldots,9)$ are elements of the Lorentz group valued matrix.

$$U_a^{(b)} = \left( \frac{1}{2}(u_a{}^{\#} + u_a{}^{=}),\; u_a{}^{i}\;, \frac{1}{2}(u_a{}^{\#} - u_a{}^{=}) \right) \quad \in \quad SO(1,10)\;,$$

This is to say they obey

$$u_a^{=} u^{a\,=} = 0\;, \qquad u_a^{\#} u^{a\,\#} = 0\;, \qquad u_a^{\#} u^{a\,=} = 2\;,$$
$$u_a^{=} u^{a\,i} = 0\;, \qquad u_a^{\#} u^{a\,i} = 0\;, \qquad u_a^{i} u^{a\,j} = -\delta^{ij}\;.$$

Spinor moving frame superfields, entering $\hat{T}^{=\,i}{}_{q}^{-} := \hat{T}_{ab}{}^{\beta} u^{a=} u^{bi} v_{\beta q}{}^{-}\;,$ are elements of the Spin group valued matrix

$$V_{(\alpha)}{}^{\beta} =: \left(v_q{}^{-\beta}\,,\; v_q^{+\alpha}\right) \quad \in \quad Spin(1,10)$$

\`covering' the moving frame matrix $U_a^{(b)} = \left(u_a{}^{=}, u_a{}^{\#}, u_a{}^{i}\right)$ as an SO(1.10) group element

This is to say they are \`square roots' of the light-like moving frame variables, e.g.:

$$v_q^{-}\Gamma_a v_p^{-} = \; u_a{}^{=}\delta_{qp}\;, \qquad 2v_q^{-\alpha} v_q^{-\beta} = \tilde{\Gamma}^{a\alpha\beta} u_a{}^{=}$$

$$v_q^{+}\Gamma_a v_p^{+} = \; u_a{}^{\#}\delta_{qp}\;, \qquad 2v_q^{+\alpha} v_q^{+\beta} = \tilde{\Gamma}^{a\alpha\beta} u_a{}^{\#} \qquad v_q^{-}\Gamma_a v_p^{+} = \; u_a{}^{i}\gamma^{i}_{qp}\;.$$

One might wander *whether these spinor moving frame variables come from?*