A New Road to Massive Gravity?

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Why Higher-Derivative Gravity ?

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a\left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2 \ :$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no massless spin 2 !

$\Rightarrow$ "New Massive Gravity"

Hohm, Townsend + E.B. (2009)

- Can this be extended to four dimensions ?

Comparison to Massive Gravity

see talk by Deffayet

- Massive Gravity is an IR modification of Einstein gravity that describes a massive spin-2 particle via an explicit mass term

- modified gravitational force

$$V(r) \sim \frac{1}{r} \quad \rightarrow \quad V(r) \sim \frac{e^{-mr}}{r}$$

  - characteristic length scale $r = \frac{1}{m}$

- Cosmological Constant Problem

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Underlying Trick

- Higher-Derivative Gravity theories can be constructed starting from Second-Order Derivative FP equations and solving for differential subsidiary conditions

- This requires fields with zero massless degrees of freedom

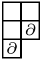
Massless Degrees of Freedom

cp. to Henneaux, Kleinschmidt and Nicolai (2011)

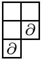
field $S \sim$ 

gauge parameters $\lambda_1 \sim$  $\lambda_2 \sim$ 

gauge transformation δ  $=$  $+$ 

curvature $R(S) \sim$ 

Zero Massless D.O.F.

"Einstein tensor" $\quad G(S) \sim \star\star$ 

Requirement : $G(S) \sim$  $\Rightarrow$ E.O.M. : $G(S) = 0$

two columns : $\quad p + q = D - 1$

Example : $\quad p = q = 1\,, D = 3\,, \qquad S \sim$ 

"Boosting Up the Derivatives"

Second-Order Derivative Generalized FP

Curtright (1980)

$$\left(\Box - m^2\right) S = 0\,, \qquad S^{\mathrm{tr}} = 0\,, \quad \partial\cdot S = 0$$

$$\partial\cdot S = 0 \quad \Rightarrow \quad S = G(T)$$

$$\left(\Box - m^2\right) G(T) = 0\,, \qquad G(T)^{\mathrm{tr}} = 0$$

Higher-Derivative Gauge Theory

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3D Einstein-Hilbert Gravity

Deser, Jackiw, 't Hooft (1984)

There are no massless gravitons : "trivial" gravity

Adding higher-derivative terms leads to "massive gravitons"

Free Fierz-Pauli

- $\left(\Box - m^2\right)\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\text{FP}} = \frac{1}{2}\tilde{h}^{\mu\nu} G^{\text{lin}}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Note : the numbers become 2 (4D) and 0 (3D) for $m = 0$

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \qquad \Rightarrow \qquad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \epsilon_\nu{}^{\gamma\delta} \partial_\alpha \partial_\gamma h_{\beta\delta} \equiv G_{\mu\nu}(h)$$

$$\left(\Box - m^2\right) \; G^{\text{lin}}_{\mu\nu}(h) = 0 \,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g} \left[-R - \frac{1}{2m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right) \right]$$

"New Massive Gravity" : unitary !

Mode Analysis

- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R

- lower number of derivatives from 4 to 2 by introducing an auxiliary symmetric tensor $f_{\mu\nu}$

- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient $\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$ and a massive spin 2 with mass $M^2 = -m^2\bar{\sigma}$

- special cases:
  - 3D NMG — Hohm, Townsend + E.B. (2009)
  - $D \geq 3$ "critical gravity" for special value of Λ

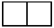
Li, Song, Strominger (2008); Lü and Pope (2011)

Outline

What did we learn?

- two theories can be equivalent at the linearized level (FP and boosted FP) but only one of them allows for a unique non-linear extension i.e. interactions !

- we need massive spin 2 whose massless limit describes 0 d.o.f.

Example :  in 3D

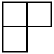
- what about 4D?

Generalized spin-2 FP

standard spin-2 :

$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 2 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

generalized spin-2 :



$$\text{describes} \begin{cases} 5 \quad \text{d.o.f.} \quad m \neq 0 \\ 0 \quad \text{d.o.f.} \quad m = 0 \end{cases}$$

Curtright (1980)

Connection-metric Duality

- Use first-order form with independent fields ${e_\mu}^a$ and ${\omega_\mu}^{ab}$

- linearize around Minkowski: ${e_\mu}^a = {\delta_\mu}^a + {h_\mu}^a$

  and add a FP mass term $-m^2(h^{\mu\nu}h_{\nu\mu} - h^2) \quad \longrightarrow$

$$\mathcal{L} \sim \text{“}\, h\,\partial\omega + \omega^2 \,\text{”} - m^2(h^{\mu\nu}h_{\nu\mu} - h^2)$$

- solve for ω $\longrightarrow$ spin-2 FP in terms of h and auxiliary $h_{[\mu\nu]}$

- solve for $h_{\mu\nu}$ and write ${\omega_\mu}^{ab} = \frac{1}{2}\epsilon^{abcd}\,\tilde{h}_{\mu cd}$ $\longrightarrow$ generalized

  spin-2 FP in terms of $\tilde{h}$ after elimination of auxiliary $\tilde{h}_{[\mu cd]}$

Boosting up the Derivatives

- start with generalized spin-2 FP in terms of

  and subsidiary conditions

$$\tilde{h}_{\mu\nu,\rho}\,\eta^{\nu\rho} = 0\,, \qquad\qquad \partial^\rho\,\tilde{h}_{\rho\mu,\nu} = 0$$

- solve for $\partial^\rho\,\tilde{h}_{\rho\mu,\nu} = 0 \rightarrow \tilde{h}_{\mu\nu,\rho} = G_{\mu\nu,\rho}(h) \;\rightarrow$ "NMG in 4D" :

$$\mathcal{L}_{\rm NMG} \;\sim\; -\tfrac{1}{2} h^{\mu\nu,\rho} G_{\mu\nu,\rho}(h) \;+\; \frac{1}{2m^2}\; \underbrace{h^{\mu\nu,\rho}\, C_{\mu\nu,\rho}(h)}_{\text{"conformal invariance"}}$$

- mode analysis $\;\rightarrow$

$$\mathcal{L}_{\rm NMG} \;\sim\; \text{massless spin 2 plus massive spin 2}$$

Interactions ?

cp. to Bekaert, Boulanger, Cnockaert (2005)

- compare to Eddington-Schrödinger theory

$$\mathcal{L}'_{\mathrm{ES}} = \sqrt{-\det g}\,[g^{\mu\nu} R_{\mu\nu}(\Gamma) - 2\Lambda] \quad \Leftrightarrow \quad \mathcal{L}_{\mathrm{ES}} = \sqrt{|\det R_{(\mu\nu)}(\Gamma)|}$$

$$g_{\mu\nu} = \frac{(D-2)}{2\Lambda} R_{(\mu\nu)}(\Gamma)$$

4D "Trivial" Gravity

avoids no-go theorem !

Example :  in 3D

- Chern-Simons formulation $\mathcal{L} \sim AdA + A^3$: $\left(e_\mu{}^a, \omega_\mu{}^a\right)$

  Achúcarro and Townsend (1986); Witten (1988)

first-order formulation of 4D "trivial" gravity :

- $\left(T_{\mu\nu}{}^a\,, \Omega_\mu{}^a\right)$ Zinoviev (2003); Alkalaev, Shaynkman and Vasiliev (2003)

  - interactions via CS formulation ?

A Common Origin

Both 3D NMG and 4D Massive Gravity stem from a

general class of bi-gravity models!

Bañados and Theisen (2009); Hassan and Rosen (2011); Paulos and Tolley (2012)

- 4D Massive Gravity: promote fixed reference metric to dynamical metric
- 3D NMG: exchange higher derivatives for auxiliary symmetric tensor

Suggestion

Can interactions be introduced by extending bi-gravity models to

- bi-metric models of different symmetry type?

Outline

Summary

- we discussed a general procedure for constructing Higher-Derivative Gravity Theories

- we investigated a new massive modification of 4D gravity

- Higher-Derivative gravity and Massive gravity have common origin as generalized bi-gravity models

Open Issues

- Interactions ?

- Extension to Higher Spins ?