Comments about Higher-Spin and Duality.

Part I: Spin-2 and E11

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Plan

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- Understand dual formulations of (linearised) gravity along lines of [N.B., S. Cnockaert, M. Henneaux (2003)]. In particular, to find a covariant action for Hull's double-dual graviton (2000) appearing in the dimensional reduction of exotic $\mathscr{N}_6 = (4,0)$ superconformal theory [J. Strathdree (1986)].

Dual graviton

- Hull (2000) : on-shell Hodge duality in linearised gravity $\rightarrow C_{[n-3,1]}$.
- West (2001) : E_{11} decomposes into an infinite set of highest-weight $SL(11,\mathbb{R})$ tensors. At low levels, $E_{11} \ni C_{[8,1]}$ s.t. $C_{[\mu_1\dots\mu_8,\nu]} \equiv 0$ which was identified with the dual graviton :
  $\hookrightarrow$ Einstein–Cartan action (first-order) re-written with $\omega_{\mathbf{1}}^{a[2]} \longrightarrow Y^a_{\mathbf{n-2}}$
  s.t. $S^{\mathrm{EC}}[e^a_{\mathbf{1}}, Y^a_{\mathbf{n-2}}] = \int_{M_n} e\ \left(\mathrm{d}e_a \wedge Y^a_{\mathbf{n-2}} + Y^2\right)$.
  On-shell and linearising, $Y^a_{\mathbf{n-2}} \rightsquigarrow \mathrm{d}C^a_{\mathbf{n-3}}$ the curl of dual graviton.
- In [N.B., S. Cnockaert, M. Henneaux (2003)], the off-shell Hodge dualisation of linearised gravity was done in $\mathbb{M}_n$. Using a parent action : Fierz–Pauli $\Longleftrightarrow$ action of [Curtright, Aulakh–Koh–Ouvry (1985-86)] for free $\mathfrak{gl}_n$-irreducible massless $C_{[n-3,1]}$ gauge field.

Double-dual graviton

- Hull (2000) conjectured a duality between an exotic $\mathscr{N}_6 = (4,0)$ superconformal theory and the strong coupling limit of $\mathscr{N}_5 = 8$ sugra. Upon dimensional reduction $6D \searrow 5D$ of the field content of the linearised theory, *not only* do the graviton $\ydiagram{2}$ and the dual graviton $\ydiagram{2,1}$ appear, *but also* the double-dual graviton $\ydiagram{2,2}$ $\rightsquigarrow$ "*triality*".
- The exotic interacting six-dimensional theory suggested is to maximally $\mathscr{N}_5 = 8$ sugra what superconformal $\mathscr{N}_6 = (2,0)$ theory is to maximally supersymmetric Yang–Mills theory in five dimensions.
  $\hookrightarrow$ Is there a corner of M-theory that contains the exotic $\mathscr{N}_6 = (4,0)$ theory ? [recent work `arXiv:1108.3085` by M. Chiodaroli, M. Gunaydin, and R. Roiban]

Goals

- Construct a $\mathfrak{gl}_n$-covariant action for the double-dual graviton $D_{\mu[n-3],\nu[n-3]}$ in $\mathbb{M}_n$;
- Consider *all possible further dualisations* of the graviton.

  In [N.B., S. Cnockaert, M. Henneaux (2003)], parent action that reproduces $S^{\mathrm{FP}}[h_{\mu,\nu}]$ upon elimination of some set of auxiliary fields, and reproduces $S[C_{[n-3,1]}]$ after elimination of other aux. fields.
- Relation between E_{11} and Hull's proposal ?

Some results in [1205.2277] with Paul P. Cook and Dmitry Ponomarev :

- A parent action that reproduces $S[C_{[n-3,1]}]$ on one hand and $S[D_{[n-3,n-3]}]$ on the other :

$$
\begin{array}{ccccc}
 & S[\Omega_{a[2],b}, Y_{a[3],b}] & & S[H_{a[n-3],b[2]}, D_{b[3],a[n-3]}] & \\
\swarrow & & \searrow \quad \swarrow & & \searrow \\
S_{\mathrm{FP}}(h_{\mu\nu}) & & S_{\mathrm{Curt.}}(C_{\mu[n-3],\nu}) & & S_{\mathrm{DD}}(D_{\mu[n-3],\nu[n-3]})
\end{array}
$$

- Three infinite *gravity towers* with fields $\tilde{h}_{\mu_1[n-2]\dots,\mu_k[n-2],\nu,\rho}$, $\tilde{C}_{\mu_1[n-2],\dots,\mu_k[n-2],\nu[n-3],\rho}$ and $\tilde{D}_{\mu_1[n-2],\dots,\mu_k[n-2],\nu[n-3],\rho[n-3]}$ $(k = 1, 2, \dots)$ referred to as the Fierz-Pauli tower, the dual graviton tower and the double-dual tower.
- By-product : places a conjecture of [Riccioni-West (2006)] (that $\tilde{C}_{\mu_1[n-2],\dots,\mu_k[n-2],\nu[n-3],\rho} \leftrightsquigarrow$ dual gravitons) on firm, off-shell footing.

- $$\tilde{h}^{(m)} \quad \sim \quad \begin{array}{cccccc} \boxed{\mathrm{n}} & \boxed{\mathrm{n}} & \dots & \boxed{\mathrm{n}} & \boxed{\mathrm{n}} & \boxed{\mathrm{n}} \\ \boxed{\mathrm{n\text{-}1}} & \boxed{\mathrm{n\text{-}1}} & \dots & \boxed{\mathrm{n\text{-}1}} & & \\ \vdots & \vdots & \dots & \vdots & & \\ \boxed{4} & \boxed{4} & \dots & \boxed{4} & & \\ \boxed{3} & \boxed{3} & \dots & \boxed{3} & & \end{array} \quad .$$

- $$\tilde{h}^{(m)} \quad \sim \quad \begin{array}{cccccc} \boxed{n} & \boxed{n} & \ldots & \boxed{n} & \boxed{n} & \boxed{n} \\ \boxed{n\text{-}1} & \boxed{n\text{-}1} & \ldots & \boxed{n\text{-}1} & & \\ \vdots & \vdots & \ldots & \vdots & & \\ \boxed{4} & \boxed{4} & \ldots & \boxed{4} & & \\ \boxed{3} & \boxed{3} & \ldots & \boxed{3} & & \end{array} \quad .$$

- $$\tilde{C}^{(m)} \quad \sim \quad \begin{array}{cccccc} \boxed{n} & \boxed{n} & \ldots & \boxed{n} & \boxed{n} & \boxed{n} \\ \boxed{n\text{-}1} & \boxed{n\text{-}1} & \ldots & \boxed{n\text{-}1} & \boxed{n\text{-}1} & \\ \vdots & \vdots & \ldots & \vdots & \vdots & \\ \boxed{4} & \boxed{4} & \ldots & \boxed{4} & \boxed{4} & \\ \boxed{3} & \boxed{3} & \ldots & \boxed{3} & & \end{array} \quad .$$

- $$\tilde{h}^{(m)} \sim \begin{array}{|c|c|c|c|c|c|}\hline n & n & \dots & n & n & n \\ \hline n\text{-}1 & n\text{-}1 & \dots & n\text{-}1 & & \\ \cline{1-4} \vdots & \vdots & \dots & \vdots & & \\ \cline{1-4} 4 & 4 & \dots & 4 & & \\ \cline{1-4} 3 & 3 & \dots & 3 & & \\ \cline{1-4} \end{array} .$$

- $$\tilde{C}^{(m)} \sim \begin{array}{|c|c|c|c|c|c|}\hline n & n & \dots & n & n & n \\ \hline n\text{-}1 & n\text{-}1 & \dots & n\text{-}1 & n\text{-}1 & \\ \cline{1-5} \vdots & \vdots & \dots & \vdots & \vdots & \\ \cline{1-5} 4 & 4 & \dots & 4 & 4 & \\ \cline{1-5} 3 & 3 & \dots & 3 & & \\ \cline{1-4} \end{array} .$$

- $$\tilde{D}^{(m)} \sim \begin{array}{|c|c|c|c|c|c|}\hline n & n & \dots & n & n & n \\ \hline n\text{-}1 & n\text{-}1 & \dots & n\text{-}1 & n\text{-}1 & n\text{-}1 \\ \hline \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ \hline 4 & 4 & \dots & 4 & 4 & 4 \\ \hline 3 & 3 & \dots & 3 & & \\ \cline{1-4} \end{array} .$$

Review of first dualisation

- Consider the quadratic parent action [N.B., S.C., M.H.]

$$S[\Omega, Y] = -\int d^n x \ \left(2\,\Omega_{ab,c}\partial_d Y^{dab,c} + \Omega^{ab,c}\Omega_{ab,c} + 2\,\Omega^{ab,c}\Omega_{ac,b} + 1 term\right)$$

  and vary w.r.t $Y_{abc,d} \equiv Y_{[abc],d} : \partial_{[d}\Omega_{ab],c} = 0 \implies \Omega_{ab,c} = \partial_{[a}h_{b],c}$ where $h_{[1,1]} \sim \Box \otimes \Box$.

- Eliminating Y that way, the action becomes the Fierz–Pauli action.
- On the other hand, $\Omega_{ab,c}$ is an auxiliary field.

  Eliminate it $\dashrightarrow$ resulting action equivalent to the Curtright action.

Second dualisation : 1

(i) Construct the action :

$$S[H^{a[n-3],}{}_{bc}, D^{bcd}{}_{,\,a[n-3]}] = \int d^n x \; \Big[H^{a[n-3],}{}_{bc}\, \partial_d D^{bcd}{}_{,\,a[n-3]} + \text{“}HH''\Big]$$

where “$H\,H$” must give the Curtright action via

$$H_{\mu[n-3],}{}^{\nu[2]} \;\longrightarrow\; 2\,\partial^{[\nu_1} C_{\mu[n-3],}{}^{\nu_2]}\,.$$

(ii) Eliminate $D^{b[3]}{}_{,\,a[n-3]}$ from the action, enforcing

$H_{\mu[n-3],}{}^{\nu[2]} = 2\,\partial^{[\nu_1} C_{\mu[n-3],}{}^{\nu_2]} \;\longrightarrow\; S^{\text{Curt.}}[H(C)]\,.$

Alternatively, extremise the action w.r.t. $H^{a[n-3],}{}_{b[2]}$ to get

$$S[D^{bcd}{}_{,\,a[n-3]}] = \int d^n x \; \Big[\partial^e D_{bce,}{}^{a[n-3]}\, \partial_d D^{bcd}{}_{,\,a[n-3]} + \cdots\Big]$$

which by construction is $\Longleftrightarrow$ to $S^{\text{Curt.}}[H(C)]$;

Towards second dualisation : 2

(iii) Decompose $D_{b[3]},{}^{a[n-3]}$ into $\mathfrak{gl}_n$-irreducible components :

$$D_{b[3]},{}^{a[n-3]} = X_{b[3]},{}^{a[n-3]} + Z_{b[3]},{}^{a[n-3]} \quad ,$$

$$Z_{b[3]},{}^{a[n-3]} = \delta^{[a_1}_{[b_1} Z^{(1)}{}_{b_2 b_3]},{}^{a_2 \dots a_{n-3}]} + \delta^{[a_1}_{[b_1} \delta^{a_2}_{b_2} Z^{(2)}{}_{b_3]},{}^{a_3 \dots a_{n-3}]} + Z^{(3)} \quad ,$$

$$X_{b_1 b_2 b_3},{}^{b_1 a[n-4]} \equiv 0 \equiv Z^{(1)}{}_{b_1 b_2},{}^{b_1\, a[n-5]} , \quad Z^{(2)}{}_{b},{}^{ba[n-6]} \equiv 0 ,$$

and exhibits the double-dual graviton

$D_{a[n-3],b[n-3]} := \frac{1}{(n-3)!}\, \epsilon_{c[3]a[n-3]}\, X^{c[3]},{}_{b[n-3]}$. The other components

$$E^{(1)}{}_{a[n-2],b[n-4]} := \frac{1}{(n-2)!}\, \epsilon_{c[2]a[n-2]}\, Z^{(1)\, c[2]},{}_{b[n-4]} ,$$

$$E^{(2)}{}_{a[n-1],b[n-5]} := \frac{1}{(n-1)!}\, \epsilon_{ca[n-1]}\, Z^{(2)\, c},{}_{b[n-5]} .$$

and $Z^{(3)}_{a[n-6]}$ are required off-shell.

Fierz–Pauli tower : 1

- Starting from the Fierz–Pauli action

$$S[h_{[1,1]}] = \int d^n x\, L^{\mathrm{FP}}(\partial_\alpha h_{\mu,\nu}) = \int d^n x\, \partial^\alpha h^{\mu,\nu}\, \partial_\alpha h_{\mu,\nu} + \ldots\,,$$

where $h_{[1,1]} \sim \square \otimes \square$, one introduces the independent field $G_1^{\alpha,\mu,\nu}$ which transforms in the representation $\square \otimes \square \otimes \square$ of $\mathfrak{gl}_n$ contrary to the curl $\Omega \sim \partial_{[\alpha} h_{\mu],\nu} \sim \begin{smallmatrix}\square\\\square\end{smallmatrix} \otimes \square$ from which one derives the Curtright action.

- Then writes the parent action

$$S^{(P1)}_{\mathrm{FP}}[G_1, F_1] = \int d^n x\, \left(G_{\alpha,\mu,\nu} \partial_\beta F^{\beta\alpha,\mu,\nu} - \tfrac{1}{2}\, L^{\mathrm{FP}}(G_1) \right) \quad ,$$

where $F_1 \sim \begin{smallmatrix}\square\\\square\end{smallmatrix} \otimes \square \otimes \square$.

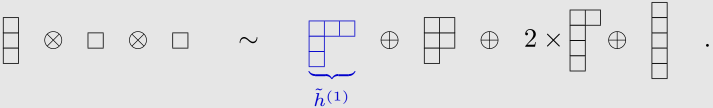
Fierz–Pauli tower : 2

- Repeating the procedure used previously, from $S_{\mathrm{FP}}^{(P1)}[G_1, F_1]$ one either reproduces the Fierz–Pauli action $S_{\mathrm{FP}}[h_{[1,1]}]$ upon extremising with respect to F_1 , or another equivalent action

$$S_{\mathrm{FP}}^{(1)}[h^{(1)}_{[n-2,1,1]}] = \int d^n x \, \left[\partial_{[\mu} h^{(1)}{}_{\mu[n-2]],\nu,\rho} \, \partial^{[\mu} h^{(1)\,\mu[n-2]],\nu,\rho} + \ldots\right] \quad ,$$

  expressed in terms of the field $h^{(1)}_{[n-2,1,1]}$ obtained by Hodge dualising F_1 on the first column.

- Take $n = 5$; $S_{\mathrm{FP}}^{(1)}$ features $h^{(1)}_{[3,1,1]}$:



Fierz–Pauli tower : 3

- Start from the resulting child action $S^{(1)}_{\rm FP}[h^{(1)}_{[n-2,1,1]}]$, integrating by parts in order to "undo" anti-symmetrisations appearing in the curls. Denote $\partial h^{(1)}_{[n-2,1,1]}$ by G_2 with symmetry type $[n-2]\otimes[1]\otimes[1]\otimes[1]$.
- Parent action $S^{(P2)}_{\rm FP}[G_2, F_2]$ featuring G_2 viewed as an independent field together with a new field $F_2 \sim [n-2]\otimes[2]\otimes[1]\otimes[1]$.
- Extremising the parent action w.r.t. G_2 and substituting the solution of the algebraic equation inside the action $\dashrightarrow$ $S^{(2)}_{\rm FP}[h^{(2)}_{[n-2,n-2,1,1]}]$ where $h^{(2)}_{[n-2,n-2,1,1]}$ obtained from F_2 by Hodge dualising the *second* column.
- *etc.* $\dashrightarrow$ $S^{(m)}_{\rm FP}[h^{(m)}_{[n-2,\dots,n-2,1,1]}]$.

Dual and double-graviton towers

- Exactly the same procedure can be done, starting from the Curtright action this time :

  $\hookrightarrow$ Example $(n = 5, m = 1)$: off-shell field $C^{(1)}_{[3,2,1]}$ $\mathfrak{gl}_5$ decomposition

$$\ydiagram{1,1,1} \otimes \ydiagram{1,1} \otimes \ydiagram{1} \;\sim\; \underbrace{\ydiagram{3,2,1}}_{\tilde{C}^{(1)}} \oplus \ydiagram{2,2,2} \oplus \ydiagram{3,1,1,1} \oplus 2 \times \ydiagram{2,2,1,1} \oplus 2 \times \ydiagram{1} \;.$$

- Again, the same off-shell dualisation procedure works starting from the double-dual graviton action.

Comments about E_{11}

- Work inspired by the argument [Hull, 2000] that the strong coupling limit of $\mathscr{N}_5 = 8$ sugra contains the double-dual graviton. We expected that E_{11} would know about that corner of M theory. However no such double-dual graviton contained within E_{11}. Instead : only the dual-graviton tower.
- The actions for the various gravitons in the 3 towers each retain a number of supplementary mixed-symmetry fields. These fields are seen on the E_{11} side : Given a real root $\leftrightsquigarrow$ a mixed-symmetry Young tableau (dual-graviton tower), one identifies a sequence of null and imaginary roots in the algebra whose generators have the symmetries of Young tableaux formed by repeatedly moving boxes to the left.
  $\hookrightarrow$ Reproduces the supplementary off-shell fields needed, associated with null and imaginary roots in the root system of E_{11}.