Enhancement of the critical temperature of superconductors by Anderson localization

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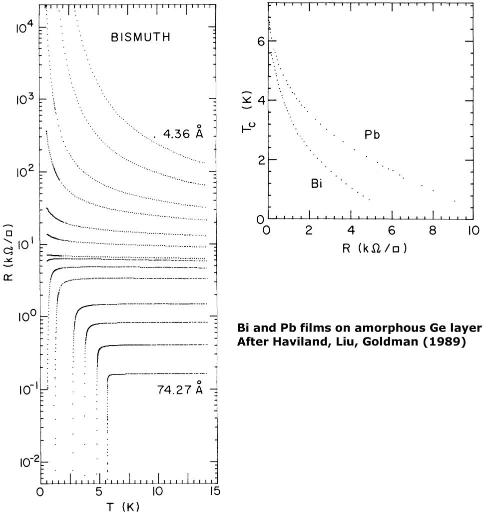
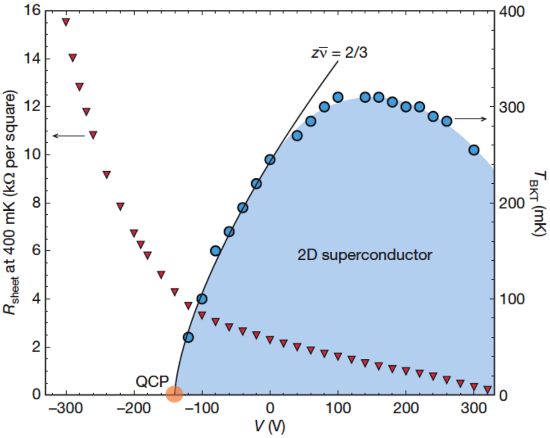
- Superconductor-insulator transition in homogeneously disordered materials
  - amorphous Mo-Ge films (thickness $b = 15 - 1000$ Å) Graybeal, Beasley (1984)
  - Bi and Pb layers on amorphous Ge ($b = 4 - 75$ Å) Strongin, Thompson, Kammerer, Crow (1971); Haviland, Liu, Goldman (1989)
  - ultrathin Be films ($b = 4 - 15$ Å) Bielejec, Ruan, Wu (2001)
  - amorphous thick In-O films ($b = 100 - 2000$ Å) Shahar, Ovadyahu (1992); Gantmakher (1998); Gantmakher, Golubkov, Dolgopolov,Tsydynzhapov, Shashkin (1998),(2000); Sambandamurthy, Engel, Johansson, Shahar (2004); Sacépé, Dubouchet, Chapelier, Sanquer, Ovadia, Shahar, Feigel'man, Ioffe (2011)
  - thin TiN films Baturina, Mironov, Vinokur, Baklanov, Strunk (2007)
  - Li_xZrNCl powders Kasahara, Kishiume, Takano, Kobayashi, Matsuoka, Onodera, Kuroki, Taguchi, Iwasa (2009)
  - $LaAlO_3/SrTiO_3$ interface Caviglia, Gariglio, Reyren, Jaccard, Schneider, Gabay, Thiel, Hammerl, Mannhart, Triscone (2008)

for recent review, see Gantmakher, Dolgopolov, (2010)



Bi and Pb films on amorphous Ge layer
After Haviland, Liu, Goldman (1989)

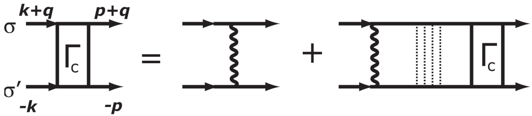
Phase diagram of the $LaAlO_3/SrTiO_3$ interface after Caviglia et al. (2008)



Giant background dielectric constant: Coulomb interaction strongly screened

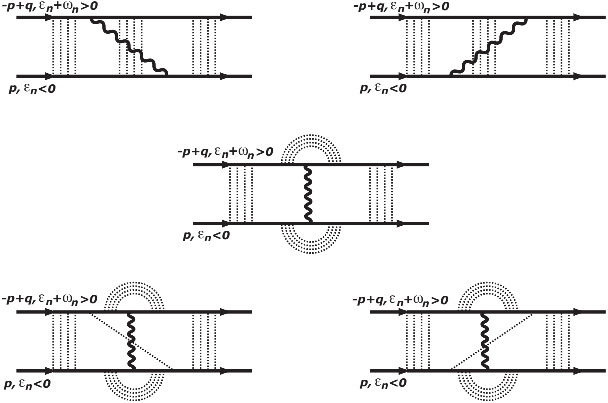
- Nonmagnetic impurities do not affect s-wave superconductors

  Cooper-instability is the same for diffusive electrons:



Mean free path l does not enter expression for T_c

- Disorder, Coulomb (long-ranged) repulsion, (short-ranged) attraction in the Cooper channel



Diagrams for renormalization of attraction in the Cooper channel

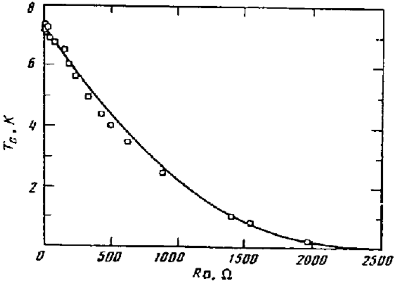
- Suppression of T_c in a film as compared with BCS result

$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\square \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^3 < 0$$

Ovchinnikov (1973) (wrong sign); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982); Finkelstein (1987)

- RG theory for disorder and interactions

Finkelstein (1983); Castellani, Di Castro, Lee, Ma (1984)



after Finkelstein (1994). Experiments on Mo-Ge films

- T_c vanishes at the sheet resistance

$$R_\square \sim \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^{-2}$$

Finkelstein (1987)

- BCS model in the basis of exact electron states ϕ_ε for a given disorder (No Coulomb repulsion)

  Bulaevskii, Sadovskii (1984); Ma, Lee (1985); Kapitulnik, Kotliar (1985)

  superconductivity survive as long as

$$T_c^{BCS} \gtrsim \delta_\xi \propto \xi^{-d}$$

  where ξ – localization length, d – dimensionality

- Enhancement of T_c as compared with BCS results ($T_c^{BCS} \propto \exp(-2/\lambda)$)

$$T_c \propto \lambda^{d/|\Delta_2|}$$

  Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007); Feigelman, Ioffe, Kravtsov, Cuevas (2010)

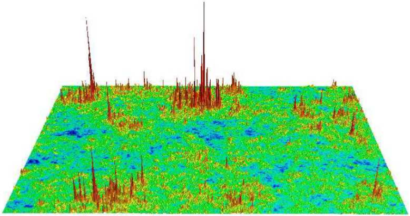
  where $\Delta_2 < 0$ – multifractal exponent for inverse participation ratio

- Multifractality near Anderson transition (No e-e interactions)

Wegner (1980); Kravtsov, Lerner (1985); Pruisken(1985); Castellani, Peliti (1986); Wegner (1987)

$$\left\langle \int d^d \boldsymbol{r} \, |\phi_\varepsilon(\boldsymbol{r})|^{2q} \right\rangle \sim L^{-\tau_q}$$

perfect metal: $\tau_q = d(q-1)$
criticality: $\tau_q = d(q-1) + \Delta_q$ with Δ_q being non-trivial function of q
perfect Anderson insulator $\tau_q = 0$



from Evers, Mildenberger and Mirlin

if $f(\alpha)$ is Legendre transform of τ_q: $f(\alpha) = q\alpha - \tau_q, \quad \alpha = d\tau_q/dq$
then $L^{f(\alpha)}$ measures a set of points where $|\phi_\varepsilon|^2 \sim L^{-\alpha}$

Can suppression of T_c due to Coulomb repulsion and enhancement of T_c due to multifractality be described in a unified way?

Does weak multifractality enhances T_c in 2D systems ?

Does the enhancement of T_c hold if one takes into account short-ranged repulsion in particle-hole channels ?

- Free electrons:

$$H_0 = \int d^d\boldsymbol{r}\, \bar{\psi}_\sigma(\boldsymbol{r}) \left[-\frac{\nabla^2}{2m} \right] \psi_\sigma(\boldsymbol{r})$$

where $\sigma = \pm 1$ is spin projection

- Scattering off white-noise random potential :

$$H_{\text{dis}} = \int d^d\boldsymbol{r}\, \bar{\psi}_\sigma(\boldsymbol{r}) V(\boldsymbol{r}) \psi_\sigma(\boldsymbol{r})$$

Gaussian distribution: $\langle V(\boldsymbol{R})\rangle = 0, \quad \langle V(\boldsymbol{R}_1) V(\boldsymbol{R}_2)\rangle = \dfrac{1}{2\pi\nu_0\tau}\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$

where ν_0 denotes the thermodynamic density of states

- Electron-electron interaction:

$$H_{\text{int}} = \frac{1}{2}\int d^d\boldsymbol{r_1}\, d^d\boldsymbol{r_2}\, \bar{\psi}_\sigma(\boldsymbol{r_1})\psi_\sigma(\boldsymbol{r_1})\, U(\boldsymbol{r_1}-\boldsymbol{r_2})\, \bar{\psi}_{\sigma'}(\boldsymbol{r_2})\psi_{\sigma'}(\boldsymbol{r_2})$$

$$U(\boldsymbol{R}) = u_0\left[1+\frac{R^2}{a^2}\right]^{-\alpha/2} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R}), \qquad \alpha > d, \qquad u_0 > 0$$

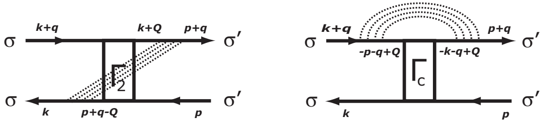
  - In the case of short-ranged repulsion with BCS-type attraction ($\lambda > 0$)

$$U(\boldsymbol{R}) = \frac{e^2}{\epsilon R} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R})$$

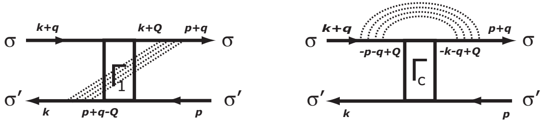
  - In the case of Coulomb repulsion with BCS-type attraction ($\lambda > 0$)

Renormalization of interaction parameters / First order in interaction

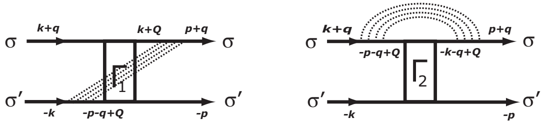
$\delta\Gamma_1$



$\delta\Gamma_2$



$\delta\Gamma_c$



Maekawa, Fukuyama (1982); Takagi, Kuroda (1982); Castellani, Di Castro, Lee, Ma (1984); Finkelstein (1987)

Here $\gamma_1 = (\gamma_t - \gamma_s)/2$ and $\gamma_2 = \gamma_t$

$$\frac{dt}{dy} = t^2\Big[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c\Big]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}\big[1+\gamma_s\big]\big[\gamma_s + 3\gamma_t + 2\gamma_c\big]$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}\big[1+\gamma_t\big]\big[\gamma_s - \gamma_t - 2\gamma_c(1+2\gamma_t)\big]$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\big[\gamma_s - 3\gamma_t + \gamma_c(\gamma_s + 3\gamma_t)\big] - 2\gamma_c^2$$

Finkelstein (1984); Castellani, Di Castro, Lee, Ma (1984)

Castellani, DiCastro, Forgacs, Sorella (1984); Ma, Fradkin (1986); Finkelstein (1984)

where $y = \ln L/l$ and $f(x) = 1 - (1 + x^{-1})\ln(1+x)$

- lowest order in disorder, $t = 2/\pi g$, g is conductivity in units e^2/h
- exact in γ_s (singlet p-h channel)and γ_t (triplet p-h channel)
- lowest order in γ_c (cooper channel)

- Coulomb (long-ranged) interaction: $\gamma_s = -1$

$$\frac{dt}{dy} = t^2\Big[1 + 1 + 3f(\gamma_t) - \gamma_c\Big]$$

$$\frac{d\gamma_t}{dy} = \frac{t}{2}\big[1 + \gamma_t\big]\big[1 + \gamma_t + 2\gamma_c(1 + 2\gamma_t)\big]$$

$$\frac{d\gamma_c}{dy} = \frac{t}{2}\big[1 + 3\gamma_t\big] - 2\gamma_c^2 \qquad \Longrightarrow \qquad \gamma_c^2 \sim t(1 + 3\gamma_t) > 0$$

Destruction of superconductivity by disorder and Coulomb interaction!

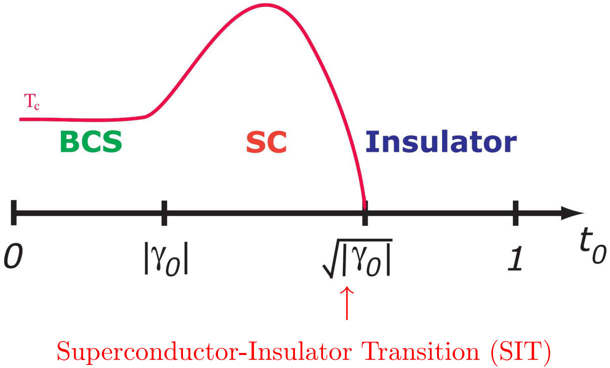
Finkelstein (1987)

$$\frac{dt}{dy} = t^2\Big(1 - \big[\gamma_s + 3\gamma_t + 2\gamma_c\big]/2\Big)$$

$$\frac{d}{dy}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} = -\frac{t}{2}\begin{pmatrix}1 & 3 & 2\\ 1 & -1 & -2\\ 1 & -3 & 0\end{pmatrix}\begin{pmatrix}\gamma_s\\ \gamma_t\\ \gamma_c\end{pmatrix} - \begin{pmatrix}0\\ 0\\ 2\gamma_c^2\end{pmatrix}$$

- Weak interaction, $|\gamma_s|, |\gamma_t|, |\gamma_c| \ll 1$
- Weak disorder, $t \ll 1$.
- Initial values $\gamma_s(0) = \gamma_{s0} < 0$, $\gamma_t(0) = \gamma_{t0} > 0$, $\gamma_c(0) = \gamma_{c0} < 0$, $t(0) = t_0$

- Sketch of phase diagram



- Enhancement of T_c due to weak multifractality: $T_c \sim E_0 e^{-2/t_0} \gg T_c^{BCS}$

- RG equations near free electron fixed point $t = t_c$, $\gamma = 0$:

$$\frac{dt}{dy} = \frac{1}{\nu}(t - t_c) + \eta\gamma, \qquad \frac{d\gamma}{dy} = -\Delta_2\gamma - a\gamma^2 \,, \quad a \sim 1$$

Initial values: $t(0) = t_0$ and $\gamma(0) = \gamma_0 < 0$

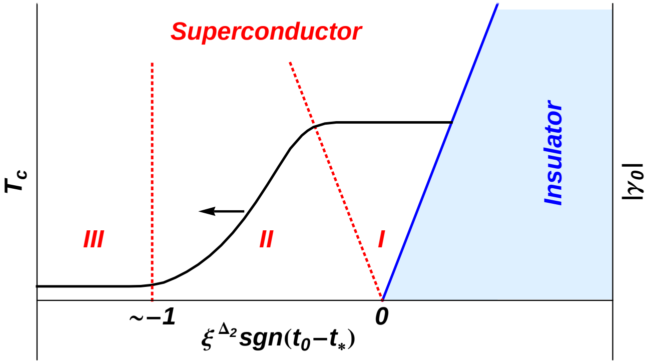
- Correlation length:

$$\xi = \left|\tilde{t}_0 - t_c\right|^{-\nu}$$

where $\tilde{t} = t - \frac{\eta\nu\gamma}{|\Delta_2|\nu - 1}$ and $\tilde{t}_0 = \tilde{t}(0)$

- Transform from t to $\tilde{t}$ removes $\eta\gamma$ from the first equation
- 3D Anderson transition (orth. sym. class): $\nu = 1.57 \pm 0.02$ and $\Delta_2 = -1.7 \pm 0.05$

Schematic phase diagram in the interaction–disorder plane and T_c



III: $T_c = T_c^{BCS}$ II: $T_c = \xi^{-d} E_0 \exp\left(-\frac{d}{a|\gamma_0|\xi^{|\Delta_2|}}\right)$ I: $T_c = E_0 |\gamma_0|^{d/|\Delta_2|}$

T_c for region I coincides with Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007)

Conclusions

- Strong enhancement of T_c for 2D electron system with short-ranged interactions in intermediate range of disorder, $|\gamma_{c0}| \ll t_0 \ll \sqrt{|\gamma_{c0}|} \ll 1$
- Strong enhancement of T_c near (free electron) Anderson transition in a system with short-ranged interactions
- Strong enhancement of T_c occurs due to multifractality of electron wave functions in the absence of interactions
- Future works: the effect of Coulomb interaction with $\varkappa l \ll 1$, the effect of magnetic field, the effect of localization inside SC