Stochastic Acceleration of Particles and Problem of Plasma Overheating

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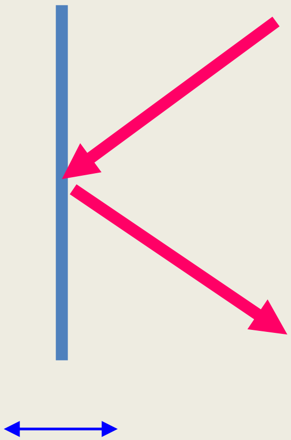
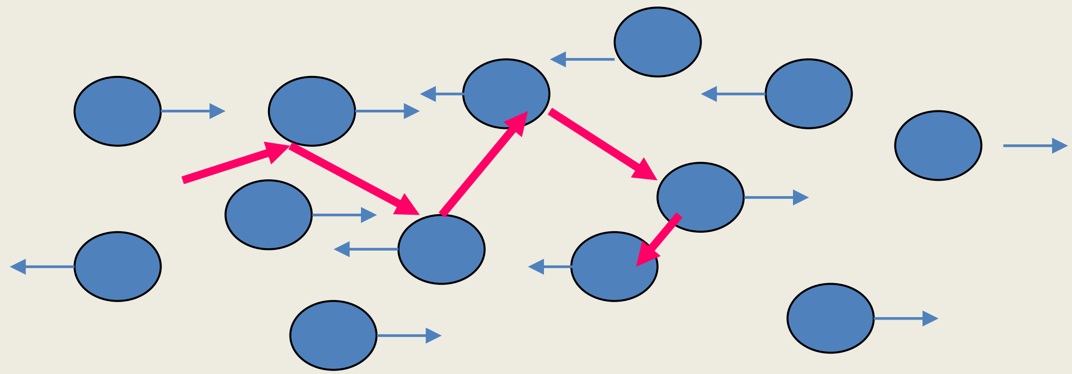
Stochastic acceleration





$$\nu \approx n_{cl}\sigma(v \pm u)$$

$$\Delta\nu \approx 2n_{cl}\sigma u$$

$$\Delta E = \pm\frac{u}{v} E\cos\theta \qquad \left(\frac{dE}{dt}\right)_F \approx \Delta E\,\Delta\nu = 2\sigma n_{cl} v\,\frac{u^2}{v^2}E$$

(E. Fermi, 1949, 1954)

Acceleration of the second order

(Fermi-II c.f. DSA <=> Fermi-I)

Stochastic acceleration

$$\frac{\partial f(\mathbf{u})}{\partial t} = \frac{\partial}{\partial u_\alpha}\left[\left(D_{\alpha\beta} + D^F_{\alpha\beta}\right)\frac{\partial f(\mathbf{u})}{\partial u_\beta} - F_\alpha f(\mathbf{u})\right]$$

$$D^F_{\alpha\beta} = a p^{\varsigma} \times \delta_{\alpha\beta}$$

$$f(p) \propto p^{-\varsigma-1}$$

- Power-law non-thermal tails
- Second order: less effective than DSA
- Solar flares, galaxy clusters, “Fermi bubbles” etc.

Acceleration from background plasma

- Start from Maxwell's distribution ($T << mc^2$)
- Supra-thermal: Coulomb collisions are essential

$$D_{\alpha\beta} = A \int \mathbf{Z}(\boldsymbol{u}, \boldsymbol{u}') f(\boldsymbol{u}')\, d^3u'$$

$$\boldsymbol{F} = -A \sum_{\beta} \int \left[\frac{\partial}{\partial u'_\beta} \mathbf{Z}(\boldsymbol{u}, \boldsymbol{u}') \right] f(\boldsymbol{u}')\, d^3u'$$

Nonlinear!

$$\mathbf{Z}(\boldsymbol{u}, \boldsymbol{u}') = \frac{r^2}{\gamma\gamma' w^3} \left[w^2 \delta_{\alpha\beta} - u_\alpha u_\beta - u'_\alpha u'_\beta + r(u_\alpha u'_\beta + u'_\alpha u_\beta) \right]$$

$$r = \gamma\gamma' - \boldsymbol{u} \cdot \boldsymbol{u}' / c^2 \quad w = c\sqrt{r^2 - 1}$$

(Wolfe & Melia, 2006)

Some simplifications

- Isotropy: spherically symmetric equation

$$\frac{\partial f}{\partial t} + \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[\left(\frac{dp}{dt}\right)_C f - \{D_C(p) + D_F(p)\}\frac{\partial f}{\partial p}\right] = 0$$

T(t) and N(t) are lost!

<u>Linearization</u>

$$||f - f_M|| \ll ||f_M|| \quad \Rightarrow$$
$$D_c \approx \int Z(p,p')f_M(p')dp'$$

$$\left(\frac{dp}{dt}\right)_c = A\left(1 + \frac{1}{p^2}\right), p \gg p_T$$

<u>Quasi-stationarity</u>

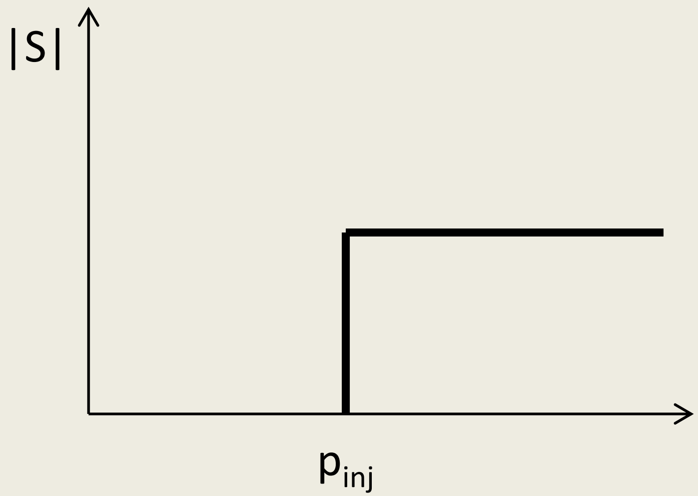
$$f(p,t) = f(p,T,N)$$

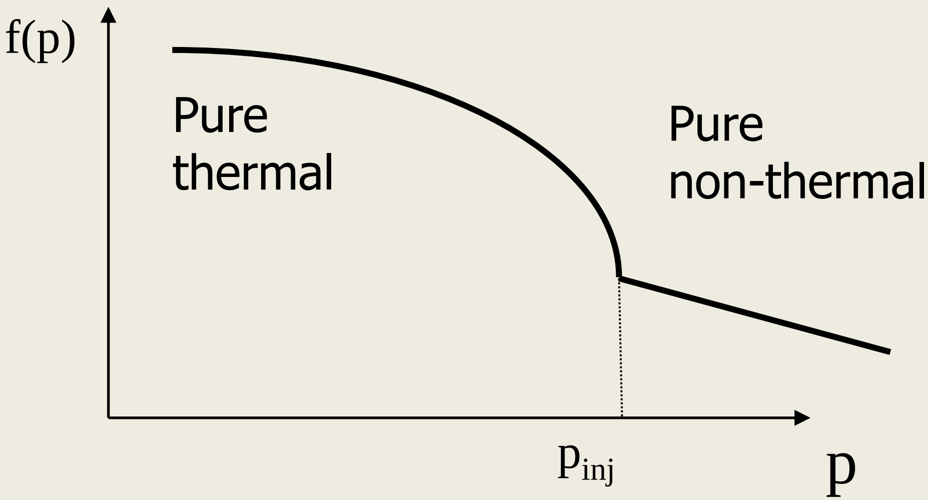
Quasi-stationary linear equation

- Acceleration start to dominate over losses at

$$p_{inj}\colon \left(\frac{dp}{dt}\right)_c = p \cdot D_F(p)$$

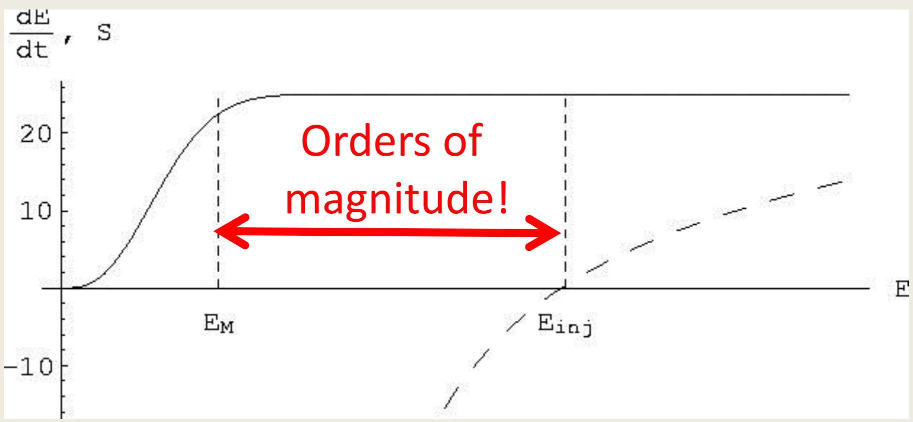
- How does the flux of accelerated particles look like?

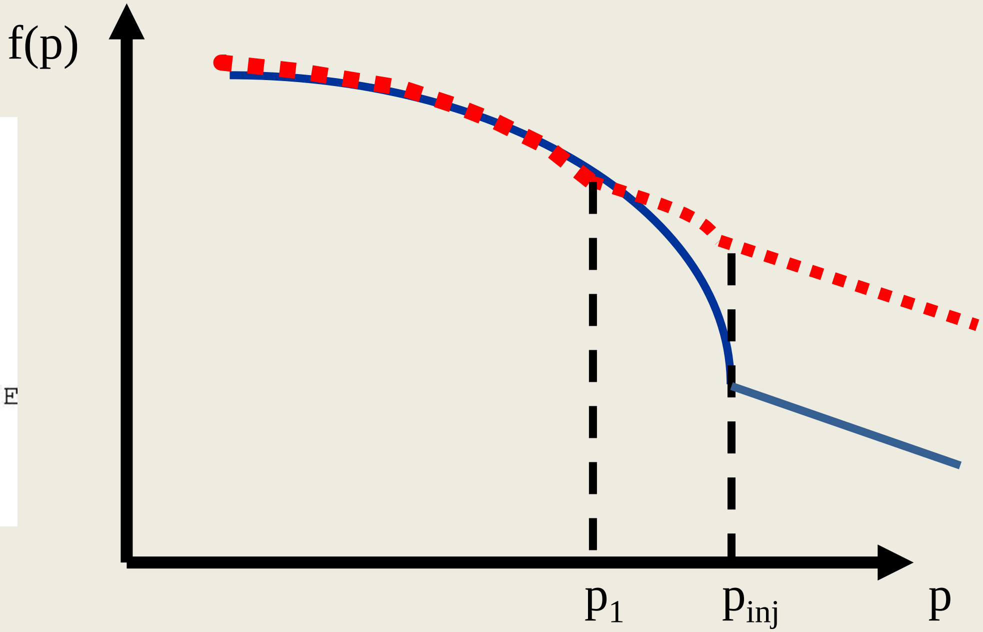




Gurevich, 1960: NO!

Quasi-stationary linear solution





- Good news: Fermi-II acceleration is powerful
- Bad news: particles in transition region are wasted
- Need to add equation for T(t) (since N(t) is obvious)

Is T(t) that important?

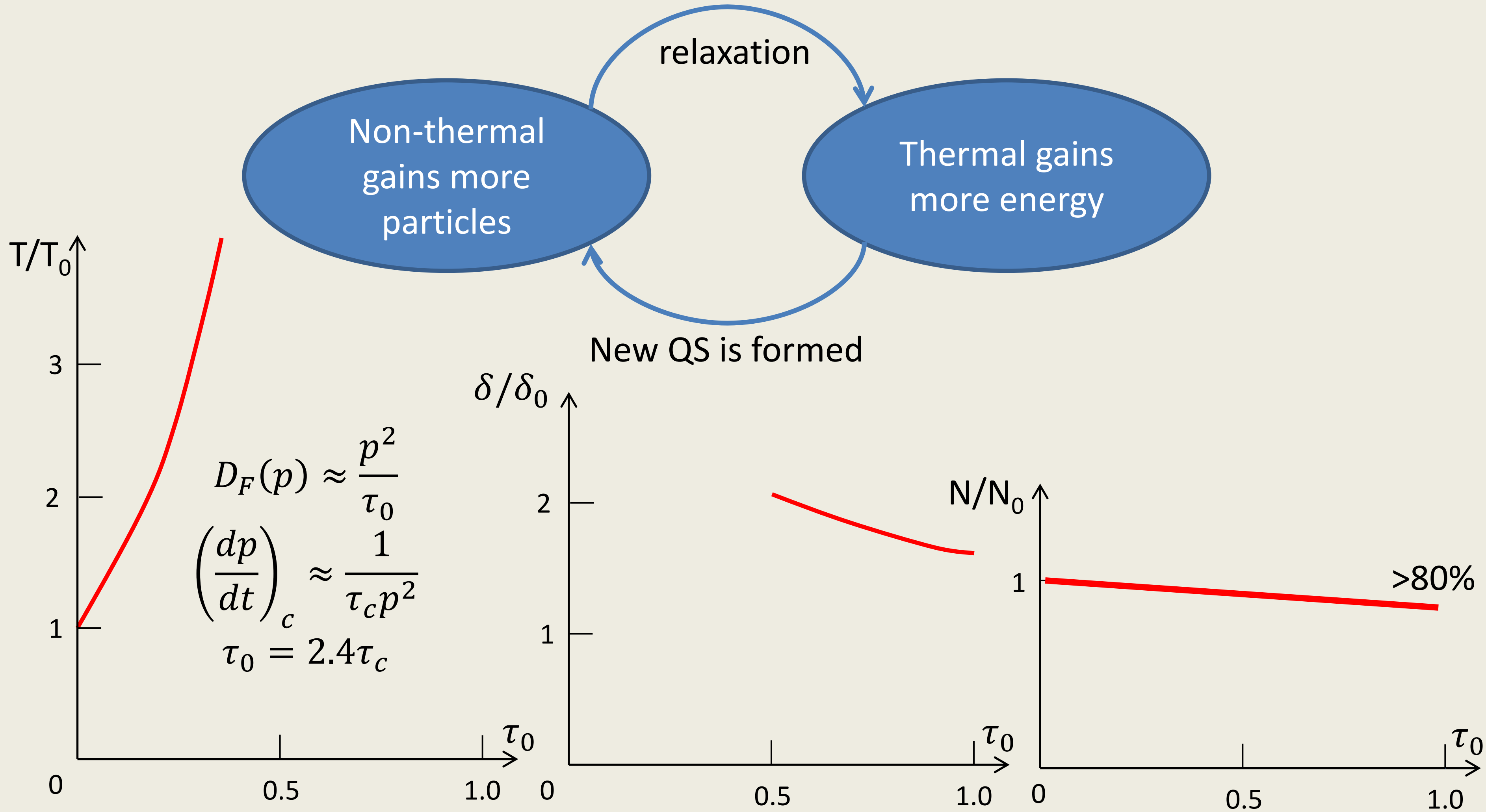
YES!

- Petrosian (2001): fast overheating of Coma cluster ($10^6 - 10^7$ yrs) [naïve linear]
- Dogiel et al. (2007): correct analysis of distribution required! [linear]

- Wolfe & Melia (2006) [fair non-linear]
- Petrosian & East (2008) [approx. non-linear]

No quasi-stationary distribution!

Vicious cycle of T(t)

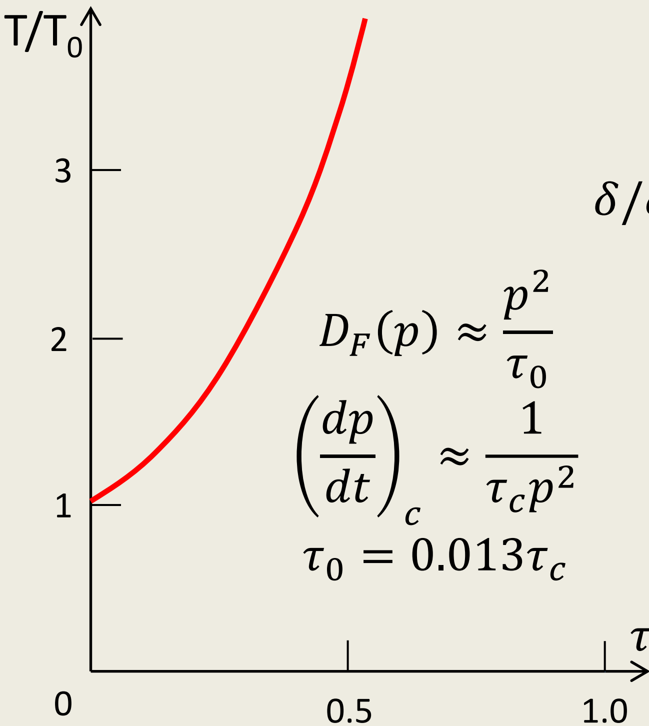


(Petrosian & East,2008)

How to break the cycle?

If you want to get somewhere else,
you must run at least twice as fast as that!

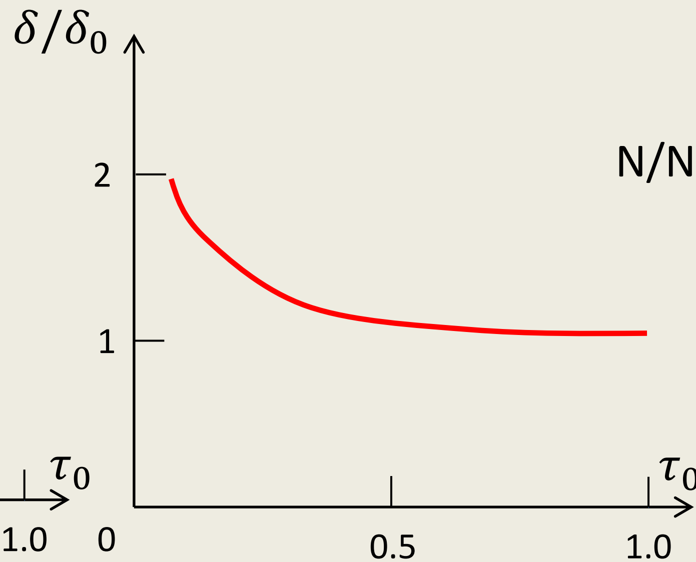
Lewis Carroll

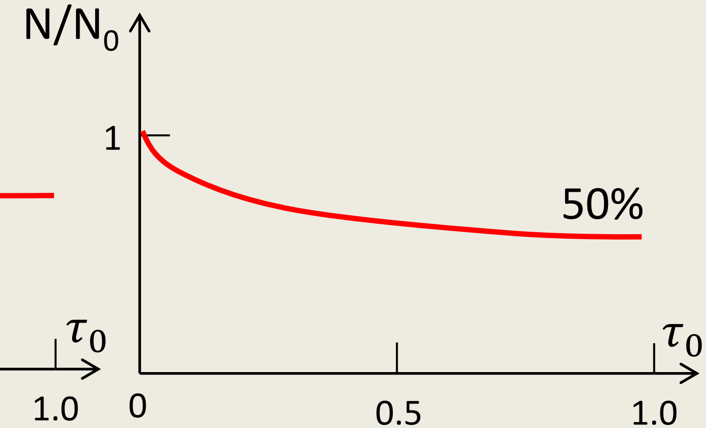


$$D_F(p) \approx \frac{p^2}{\tau_0}$$

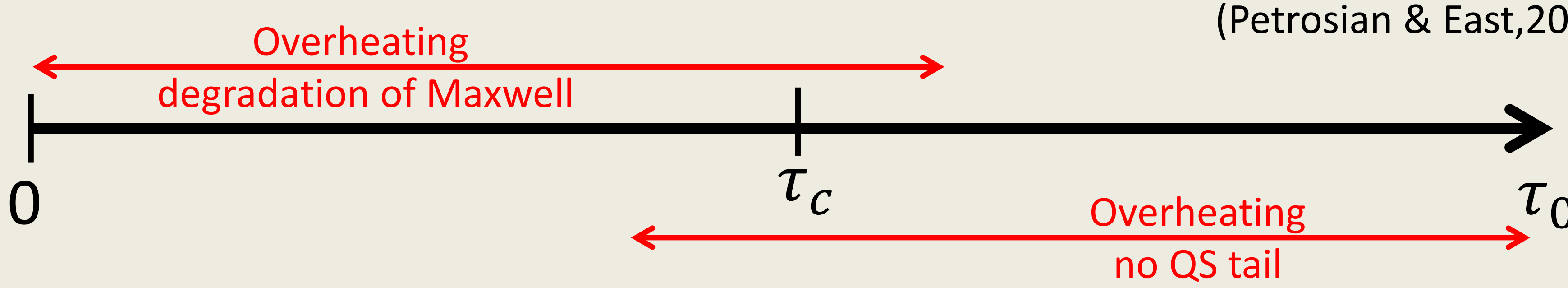
$$\left(\frac{dp}{dt}\right)_c \approx \frac{1}{\tau_c p^2}$$

$$\tau_0 = 0.013\tau_c$$



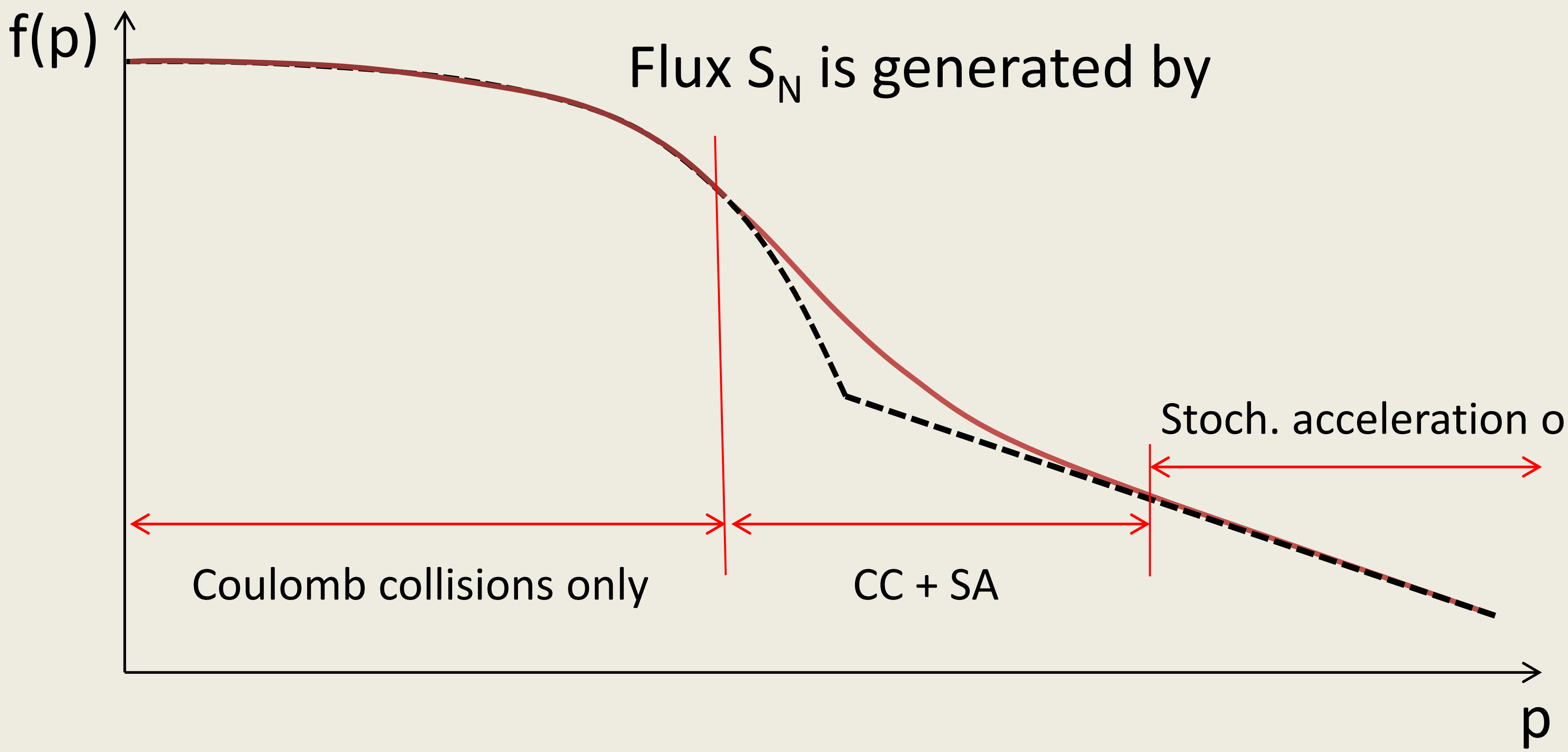


(Petrosian & East,2008)

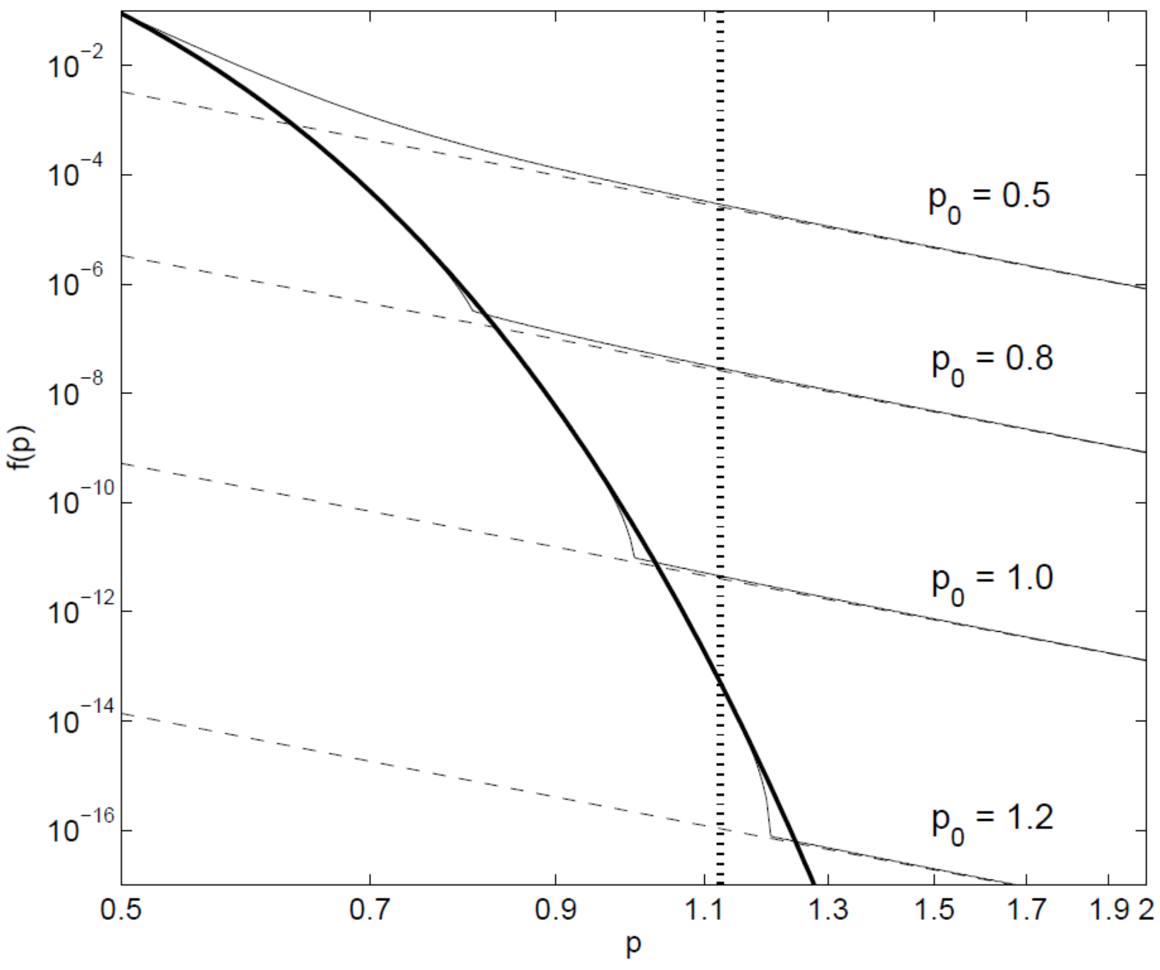


What causes overheating?

- Transitional region contains too much particles
- Unnecessary acceleration in thermal zone



Reduction of the transitional region



$$D_F(p) = \alpha p^{\varsigma} \theta(p - p_0)$$

$$p_0 \gg p_T$$

- For large p_0 there is no transitional region
- Acceleration is applied only where it needed

Plasma heating rate

$$\dot{W}_0 = \frac{\partial}{\partial t}\int\limits_0^{p_0} p^2 E f^I(p)dp = \int\limits_0^{p_0} E\frac{\partial}{\partial p}\left[p^2 D_c \frac{\partial f^I}{\partial p} - p^2\left(\frac{dp}{dt}\right)_c f^I\right]dp$$

$$\dot{W}_0 = -\int\limits_{p_0}^{\infty} E\frac{\partial}{\partial p}\left[p^2 D_c \frac{\partial f^{II}}{\partial p} - p^2\left(\frac{dp}{dt}\right)_c f^{II}\right]dp$$

$$\dot{W}_0 = E_0 S_N + \int\limits_{p_0}^{\infty} \frac{p^3}{\sqrt{p^2+1}}\left[D_c \frac{\partial f^{II}}{\partial p} - \left(\frac{dp}{dt}\right)_c f^{II}\right]dp$$

Negative
(«Maxwell's daemon»)

Positive
(relaxation of the tail)

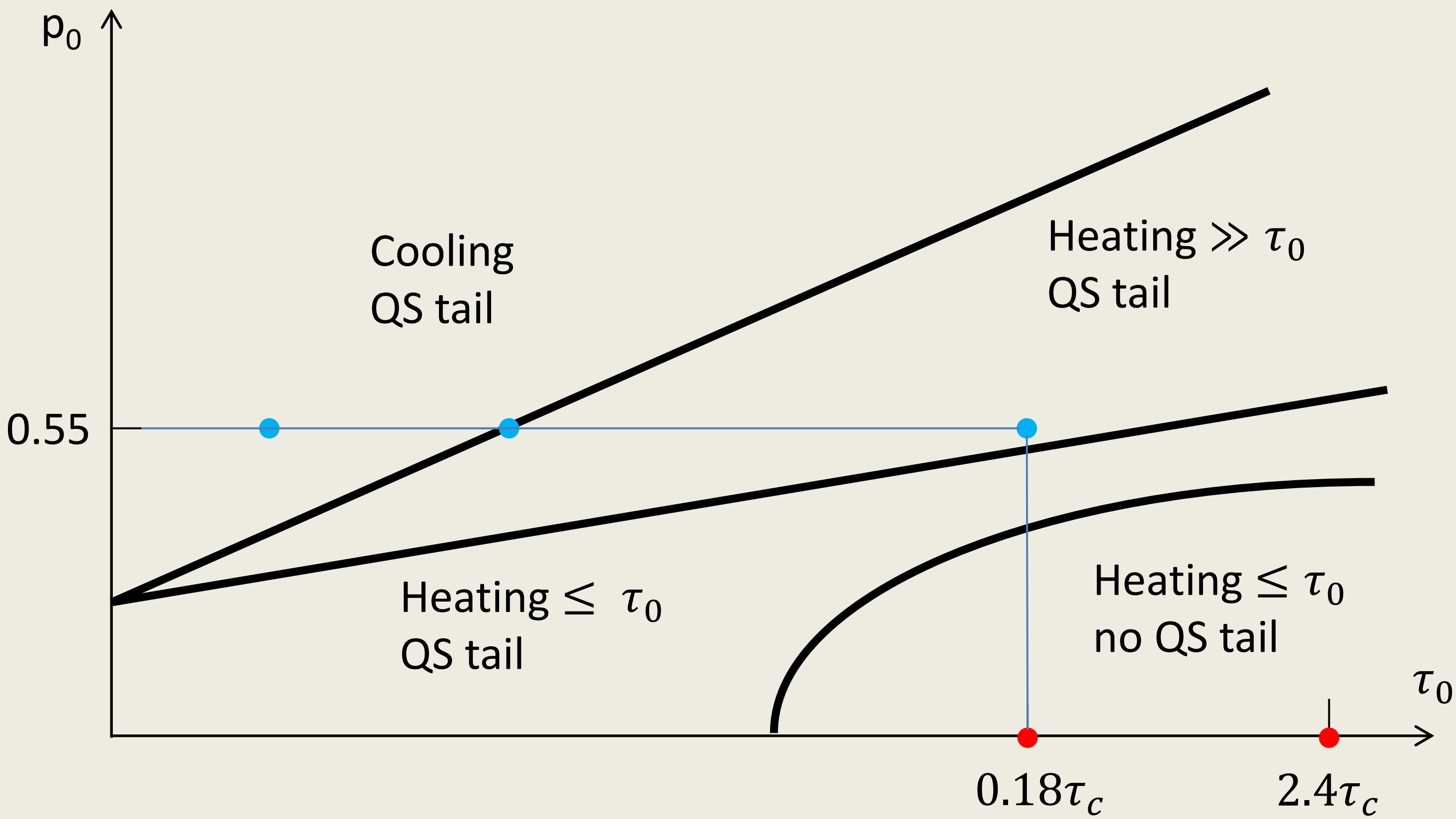
Special case $p_0 > p_{inj}$

$$\frac{dT}{dt} = \frac{2S_N}{3N}\left[\frac{AQ(p_0,\varsigma)}{\alpha(\varsigma+1)} - \mathcal{E}_0\right]$$

$$Q(p_0,\varsigma) = \int_{p_0}^{\infty} x^{-\varsigma}\sqrt{x^2+1}\,dx$$

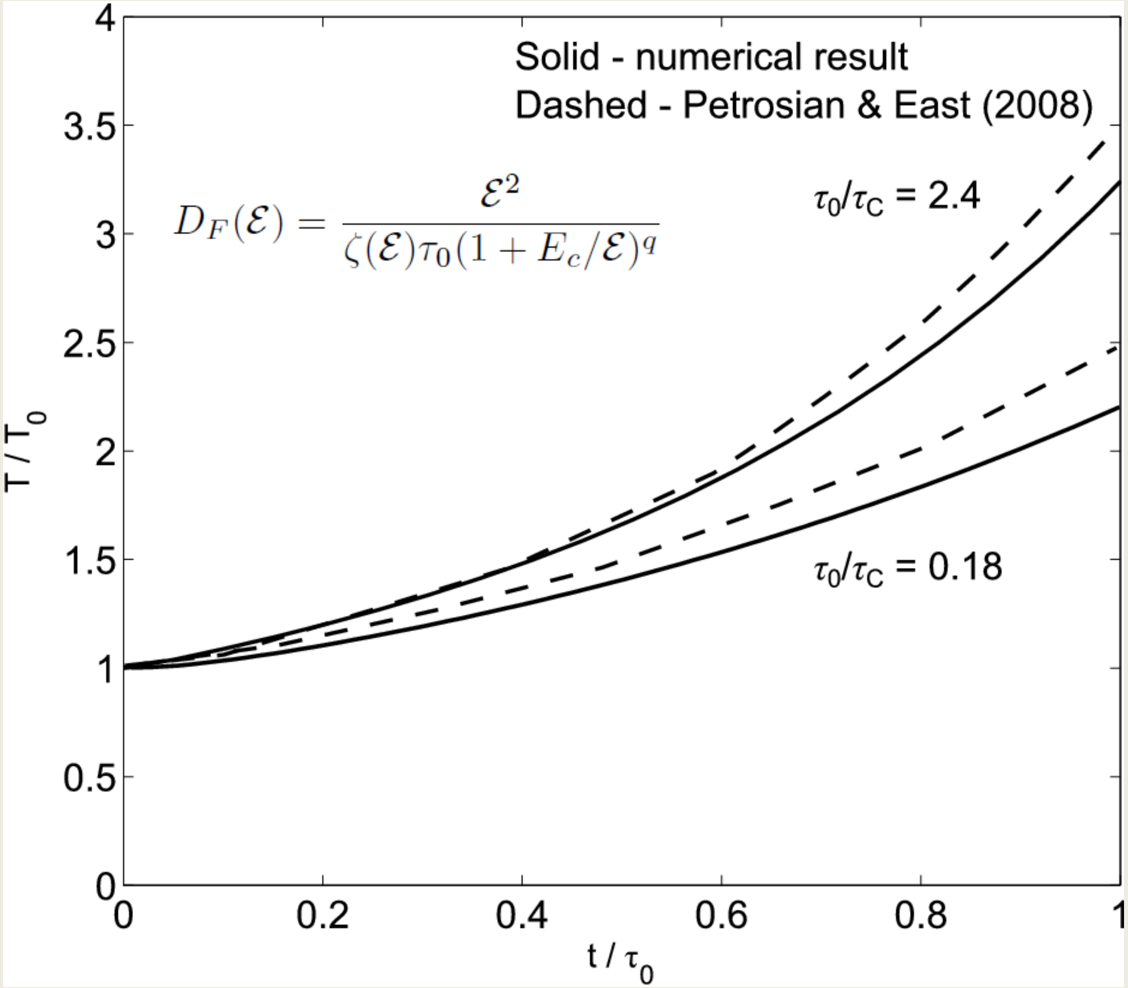
$$\left(\frac{dp}{dt}\right)_0 = -A\left(1+\frac{1}{p^2}\right) \quad A = 4\pi r_e^2 cN\ln\Lambda$$

$p_0 - \tau_0$ diagramm

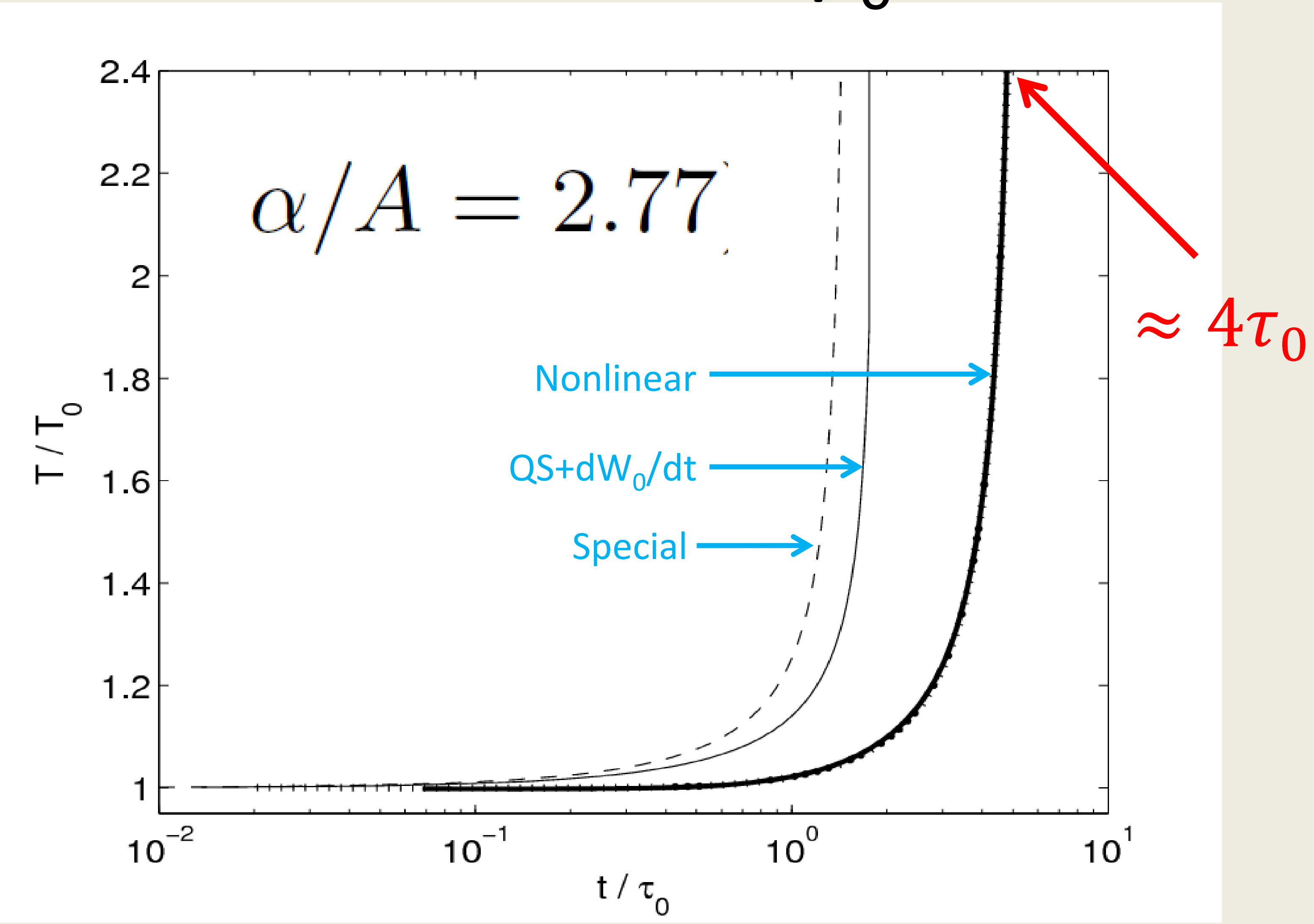


- Petrosian & East (2008)
- This work

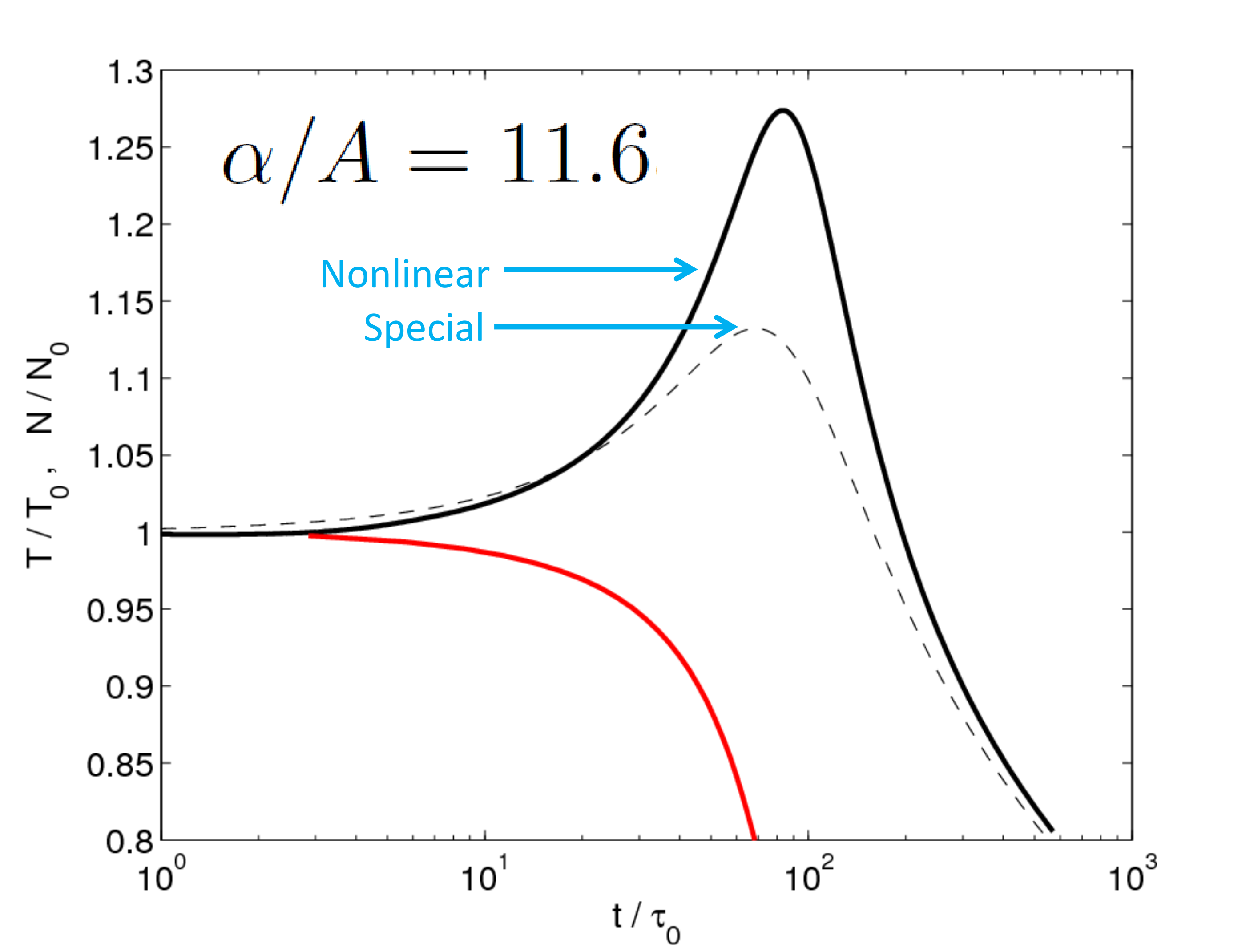
Solution of the nonlinear equation



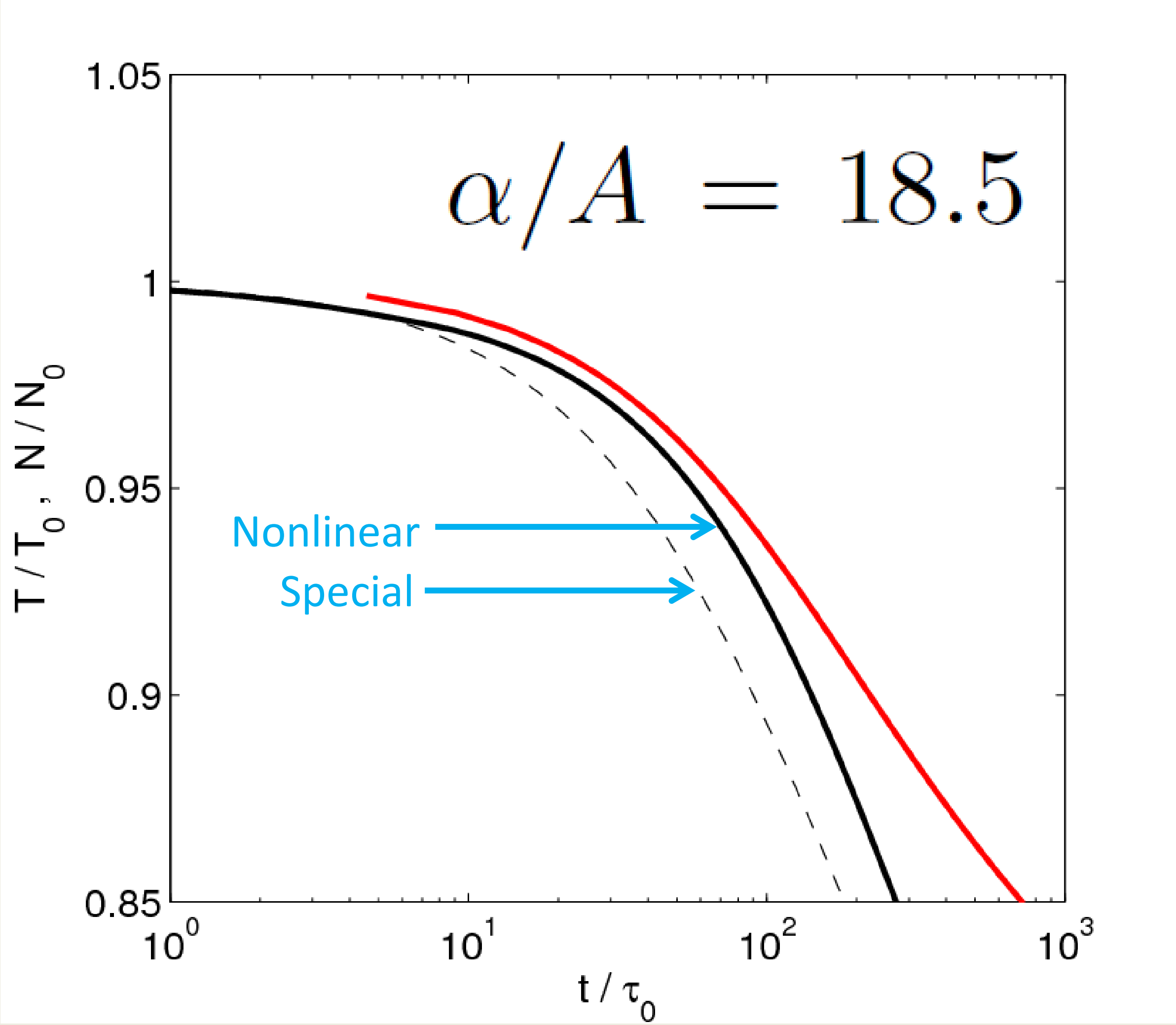
Slow acceleration, $p_0 = 0.55$



Moderate acceleration, $p_0 = 0.55$



Fast acceleration, $p_0 = 0.55$



Conclusions

- Under certain parameters stochastic acceleration is capable of acceleration particles and supply energy mainly to non-thermal tail

- Under certain parameters prominent non-thermal tail may be formed and the distribution can exist for a long time without overheating

- Behavior of the stochastic diffusion coefficient at suprathermal energies is essential

Additional slides

Quasi-stationary solution for $p_0 > 0$

$p < p_0$:

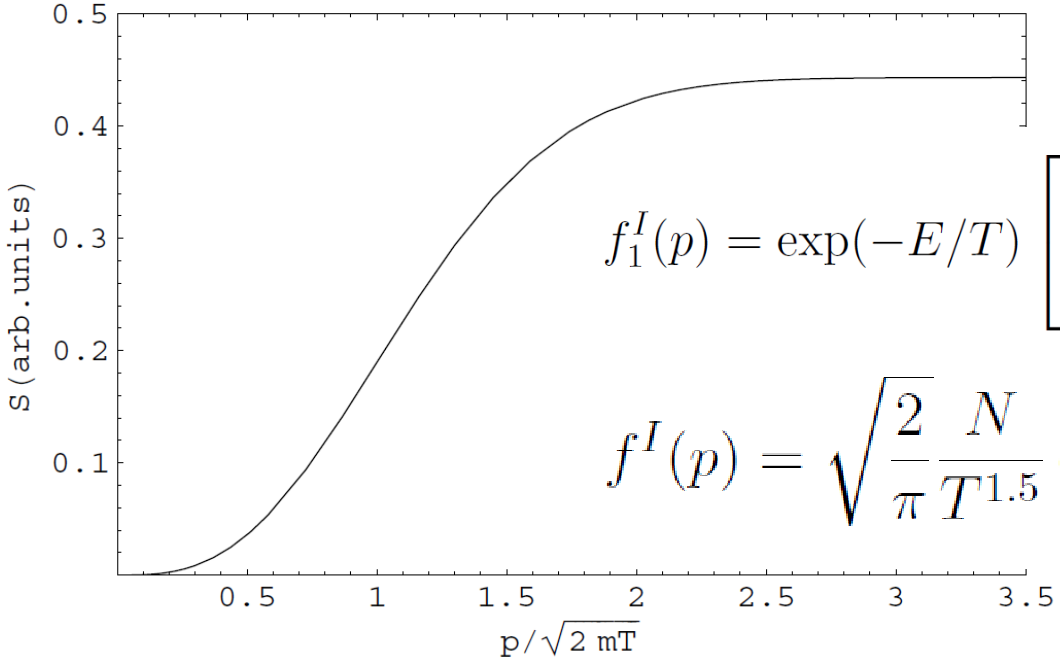
$$f_1^I(p) = \exp(-E/T)\left[C_1 + \int\limits_0^p \frac{S(u)}{u^2 D_c(u)} \exp(E(u)/T)du\right]$$

$p > p_0$:

$$f^{II}(p) = C^{II} \exp\left\{\int\limits_0^p \frac{(dp/dt)_c(u)du}{D_F(u) + D_c(u)}\right\} +$$

$$+S_N \exp\left\{\int\limits_0^p \frac{(dp/dt)_c(u)du}{D_F(u) + D_c(u)}\right\} \int\limits_0^p \frac{v^{-2}dv}{D_F(v) + D_c(v)} \exp\left\{-\int\limits_0^v \frac{(dp/dt)_c(u)du}{D_F(u) + D_c(u)}\right\}$$

Some properties



При p < p_0

$$f_1^I(p) = \exp(-E/T)\left[C_1 + \int_0^p \frac{S(u)}{u^2 D_c(u)} \exp(E(u)/T)du\right]$$

$$f^I(p) = \sqrt{\frac{2}{\pi}}\frac{N}{T^{1.5}}\exp\left(-\frac{E}{T}\right) + \frac{S_N}{A}$$

$S_N < 0$!

$$A = 4\pi r_e^2 cN \ln\Lambda$$

For p > p_{inj}

$$f^{II}(p) = f_0(p/p_0)^{-\varsigma-1} + f_\infty$$

If $p_0 > p_{inj}$ no transitional region

Why linear approach fails?

$$D_c = \int Z(p,p')f(p')dp' \qquad f(p) = f_0(p) + f_1(p)$$

$$\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[D_0\frac{\partial f_1}{\partial p} - \left(\frac{dp}{dt}\right)_0 f_1 + D_1\frac{\partial f_0}{\partial p} - \left(\frac{dp}{dt}\right)_1 f_0\right] = \frac{\partial f_0}{\partial t}$$

Tail to Maxwell

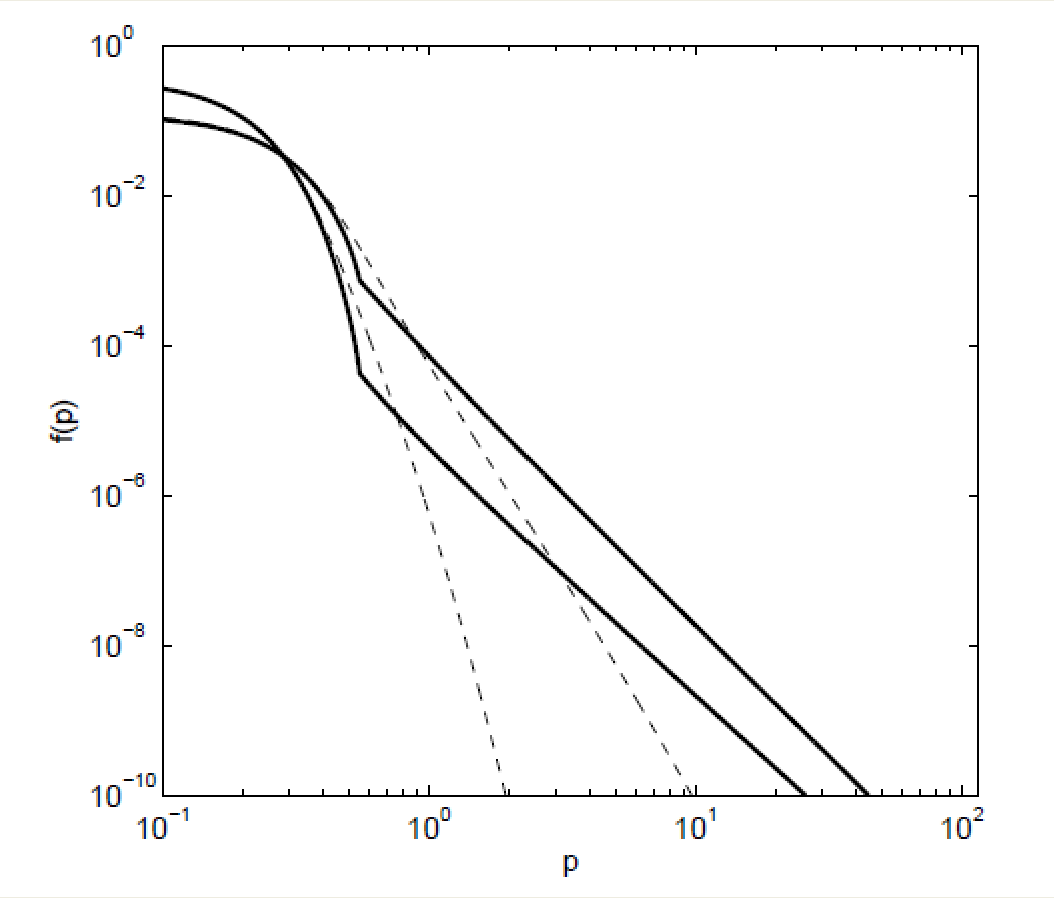
Maxwell to tail

$D_0 \gg D_1$ and $if\ p \gg p_T\ \ then\ \ f_1 \gg f_0$. Only linear terms

If $p \approx p_T$ the terms are of the same order

Conservation of energy!

Why 4 times different?



No transitional region!