Voyage inside black hole

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Is there life beyond the horizon?

The answer is affirmative if black hole is rotating

Inhabited planets or Dyson spheres inside black holes

Canonical supermassive black holes

in the galactic centers

- Mass range of supermassive black holes
  $M \sim 10^6 - 10^{10} M_\odot, \quad \langle M \rangle \sim 10^8 M_\odot$
- Black holes without angular momentum are very exotic
- Canonical black holes rotate very fast
- Black holes are spinning up by disk accretion
  $a_* = 0.9982$ *K. S. Thorne, 1974*
- Observations

Microquasar GRS 1915+105	$a = 0.99$	*arXiv:1003.3887*
Cyg X-1 (disk inversion)	$a = 0.9$	*Shapiro and Lightman 1976*
GRO J1655-40 (QPO)	$a = 0.93 \div 0.95$	*astro-ph/9704072*
NGC 3783	$a \gtrsim 0.88$	*arXiv:1104.1172*
FRII	$a = 0.2 \div 1$	*arXiv:1103.0940*
Milky Way Center	$a \gtrsim 0.5$	

Typical observers infalling to black hole avoid the central singularity

Barriers toward the central singularity

- Angular momentum of black hole

  — relativistic centrifugal barrier

- Gravitational field of the black hole electric charge

  — negative radial pressure of the black hole electric charge (dark energy analog)

Eternal black hole: on the other side of horizon

Fate of observer in the other side world

- Tidal destruction
- Infall to central singularity
- Fly out to 'other' universe
- Permanent life inside black hole
- Infinite blue-sheet and mass inflation at the Cauchy horizon
  Hypothesis:
  Singularity at the Cauchy horizon is weak and traversable

It is supposed that Kerr-Newman metric is realized not only outside, but also inside the horizon

Classification of test particle orbits

Motion of planets and photons in the gravitational field of black hole

- Orbits of I kind: completely confined outside the BH event horizon
- Orbits of II kind: penetrate inside the black hole horizon

  S. Chandrasekhar, 'The mathematical theory of black holes' 1983

- Orbits of III kind: stable periodic orbits which are completely bound inside the black hole, not escaping outside, nor infalling into the central singularity

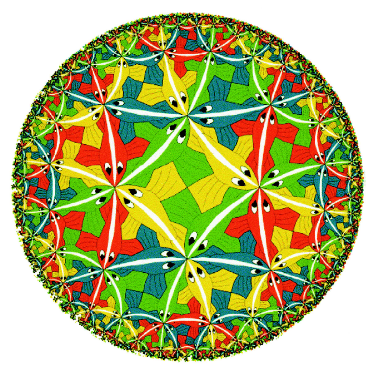
J. Bičák, Z. Stuchlík & V. Balek. Bull. Astron. Inst. Czechosl. **40**, 65, 1989; **40**, 135, 1989

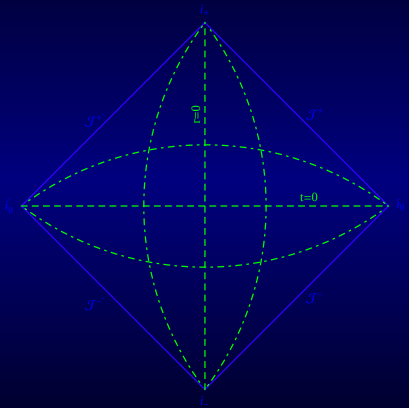
E. Hackmann, V. Kagramanova, J. Kunz & C. Lämmerzahl. Phys. Rev. D **81**, 044020, 2010

4D global geometry: Carter-Penrose diagrams

Compactification, light-cone conservation

Geodesic completeness $\sim$ maximal analytic extension





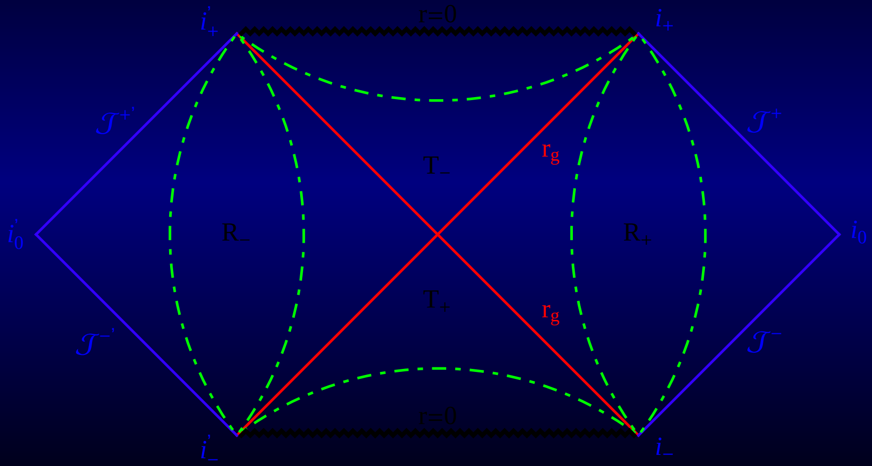
2D flat metric: M. Escher (Euclid)

4D flat metric: G. Minkowski

Global geometry: Schwarzschild black hole

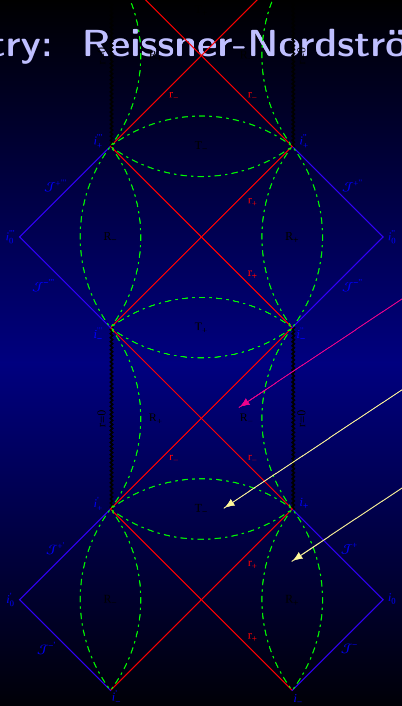
R-regions: $(t, r, \theta, \varphi) = (+, -, -, -)$ T-regions: $(-, +, -, -)$

I. Novikov 1962, Ya. Zeldovich & I. Novikov 1971



Intrinsic hyperbolicity of Einstein equations

Global geometry: Reissner-Nordström black hole

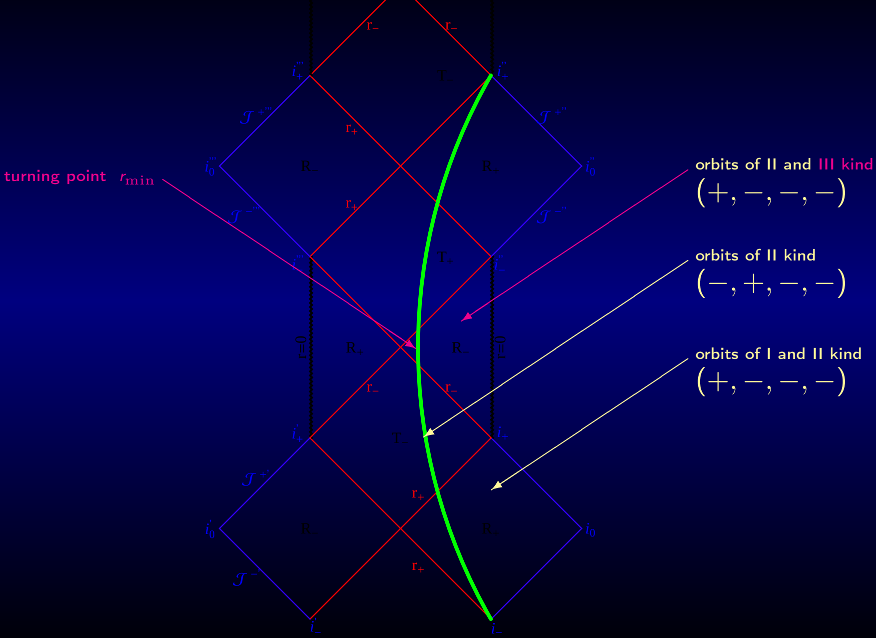


orbits of II and III kinds
$(+,-,-,-)$

orbits of II kind
$(-,+,-,-)$

orbits of I and II kinds
$(+,-,-,-)$

Reissner-Nordström black hole with a thin shell



Kerr-Newman metric

$$ds^2 = \frac{\rho^2\Delta}{\mathcal{A}}dt^2 - \frac{\mathcal{A}\sin^2\theta}{\rho^2}(d\phi - \omega dt)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2$$

$$A = e\rho^{-2}r(du - a\sin^2\theta d\phi), \quad u = t + r, \quad F = 2dA$$

$$\rho^2 = r^2+a^2\cos^2\theta, \quad \Delta = r^2-2r+a^2+e^2, \quad \mathcal{A} = (r^2+a^2)^2-a^2\Delta\sin^2\theta$$

Angular 'velocity' of metric

$$\omega = (2Mr - e^2)\frac{a}{\mathcal{A}}$$

Horizons $\Delta = 0, \;\; r_\pm = 1 \pm \sqrt{1 - a^2 - e^2}$

R-regions $\Delta > 0, \;\; r > r_+, \qquad 0 < r < r_-$

T-region $\Delta < 0, \;\; r_- < r < r_+$

Locally Non-Rotating Frame (LNRF):

$r = const, \;\; \theta = const, \;\; \varphi_0 = \omega t + const$

J. M. Bardeen 1970

Kerr-Newman metric: test particle motion

- M — black hole mass
- $J = \frac{GM^2}{c} a$ — black hole angular momentum
- e — black hole electric charge
- μ — particle mass
- ϵ — particle electric charge
- τ — particle proper time

Integrals of motion

- E — total particle energy
- L — azimuthal angular momentum
- Q — Carter constant

Motion in equatorial plane at $Q = 0$
Total angular momentum of particle $\mathcal{J} = \sqrt{Q + L^2}$ at $a = 0$

Equations of motion for test particles

B. Carter 1968

- $\rho^2 \frac{dr}{d\lambda} = \pm\sqrt{V_r}$ $\qquad \lambda = \frac{\tau}{\mu}, \quad \tau$ — particle proper time
- $\rho^2 \frac{d\theta}{d\lambda} = \pm\sqrt{V_\theta}$
- $\rho^2 \frac{d\varphi}{d\lambda} = L \sin^{-2}\theta + a(\Delta^{-1}P - E)$
- $\rho^2 \frac{dt}{d\lambda} = a(L - aE\sin^2\theta) + (r^2 + a^2)\Delta^{-1}P$

Effective radial potential

$$V_r = P^2 - \Delta[\mu^2 r^2 + (L - aE)^2 + Q]$$

Effective latitudinal potential (nutation)

$$V_\theta = Q - \cos^2\theta[a^2(\mu^2 - E^2) + L^2\sin^{-2}\theta]$$

$$P = E(r^2 + a^2) + \epsilon e r - aL, \qquad \rho^2 = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2r + a^2 + e^2$$

Horizons $\Delta = 0, \quad r = r_\pm$

$r_+ = 1 + \sqrt{1 - a^2 - e^2}$ — external (event) horizon

$r_- = 1 - \sqrt{1 - a^2 - e^2}$ — internal (Cauchy) horizon

Dimensionless variables and orbit parameters

$$r \Rightarrow \frac{r}{M}, \quad a \Rightarrow \frac{a}{M}, \quad e \Rightarrow \frac{e}{M}$$

$$\mu \Rightarrow \frac{\mu}{\mu} = 1, \quad \epsilon \Rightarrow \frac{\epsilon}{\mu}, \quad E \Rightarrow \frac{E}{\mu} = \Gamma, \quad L \Rightarrow \frac{L}{M\mu}, \quad Q \Rightarrow \frac{Q}{M^2\mu^2}$$

Impact parmeters:

$$b = \frac{L}{E}, \quad q = \frac{Q}{E^2}$$

Particular case of circular (spherical) orbits with $r = const$:

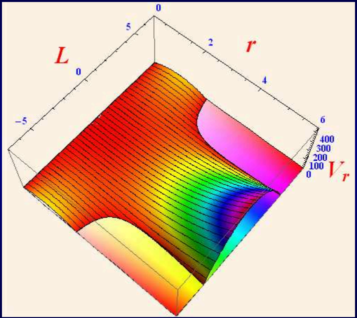
$$V_r(r, a, e, \epsilon, E, L, Q) = 0, \quad V_r'(r) \equiv \frac{dV_r}{dr} = 0$$

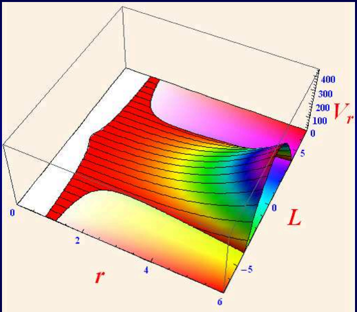
Orbits are stable at $V_r''(r) < 0$

Charged black hole: $V_r = V_r(r, e, E, L, \epsilon)$

Massive neutral particles do not reach singularity at $r = 0$

All turning points $V_r = 0$ are at $r > 0$ due to negative radial pressure of electric field ('dark energy' analogue)





At left: 'T-shirt' — Schwarzschild

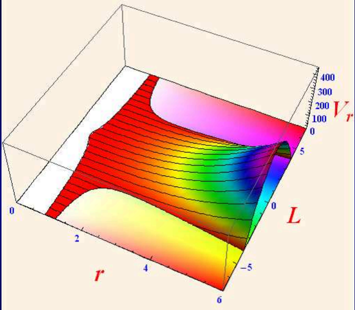
$a = 0, \quad e = 0, \quad E = 1, \quad r_- = 0, \quad r_+ = 2$

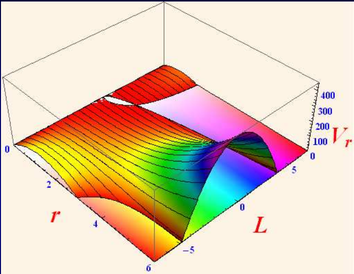
At right: 'Oriental shirt (кафтан)' — extreme Reissner-Nordström

$a = 0, \quad e = 1, \quad E = 1, \quad r_- = r_+ = 1$

Rotating black hole: $V_r(r, a, e, \epsilon, E, L, Q)$

Not all particles reach the central singularity at $r = 0$





At left: 'Oriental shirt (кафтан)' — extreme Reissner-Nordström

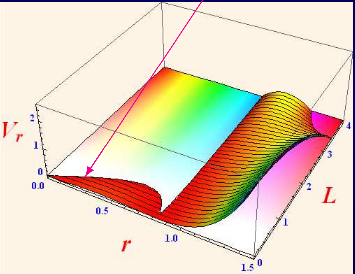
$a = 0, \quad e = 1, \quad E = 1, \quad r_- = r_+ = 1$

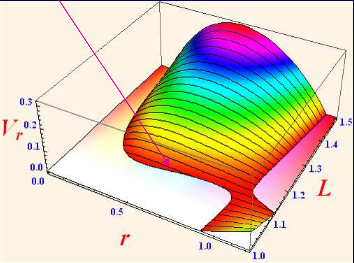
At right: 'Russian shirt (косоворотка)' — extreme Kerr

$a = 1, \quad e = 0, \quad \epsilon = 0, \quad E = 1, \quad Q = 0, \quad r_- = r_+ = 1$

Orbits of III kind: inside Cauchy horizon

Finite motion inside black hole in R-region, $0 < r < r_-$, at $V_r \geq 0$ with two turning points at $V_r = 0$





At left: near extreme Reissner-Nordström

$a = 0$, $e = 0.99$, $E = 1.9$, $\epsilon = -1.69$, $r_- = 0.859$, $r_+ = 1.141$

At right: near extreme Kerr-Newman

$a = 0.9982$, $e = 0.05$, $E = 0.57$, $Q = 0.13$, $r_- = 0.997$, $r_+ = 1.033$

Statements

- Schwarzschild $(a = 0,\ e = 0)$
  There are no orbits of III kind
- Reissner-Nordström $(e \neq 0,\ a = 0)$
  Orbits of III kind only for charged planets
- Kerr $(a \neq 0,\ e = 0)$
  Only non-equatorial orbits of III kind
- Kerr-Newman $(a \neq 0,\ e \neq 0)$
  Equatorial and non-equatorial orbits of III kind
  for planets and photons

Charged black hole: circular orbits $r = \mathrm{const}$

Total energy E and angular momentum L of planet with charge ϵ on the circular orbit: $V_r = V_r' = 0$, $V_r'' < 0$

$$E_{1,2} = \frac{\pm\Delta D_1 - e\epsilon(r^2 - 4r + 3e^2)}{2r(r^2 - 3r + 2e^2)}$$

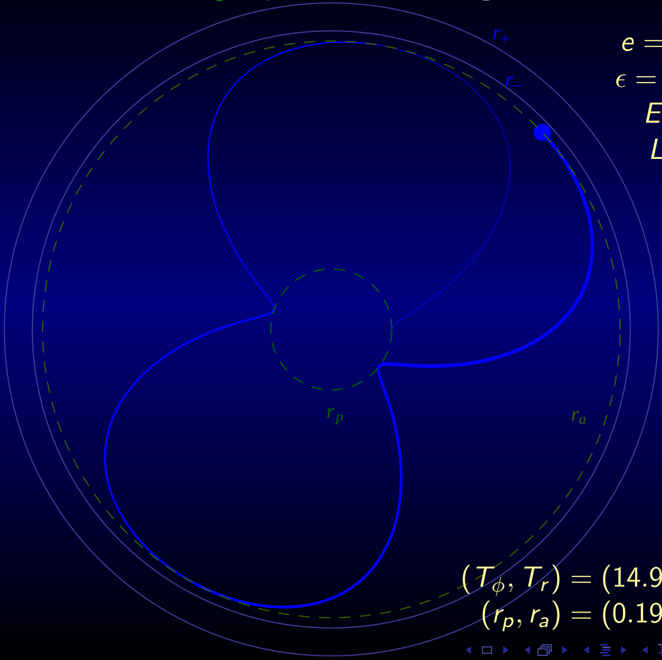
$$L_{1,2}^2 = \frac{r^2}{r^2 - 3r + 2e^2}\left[r - e^2 + \frac{e\epsilon\Delta(e\epsilon \pm D_1)}{2(r^2 - 3r + 2e^2)}\right]$$

where

$$D_1^2 = e^2(\epsilon^2 + 8) + 4r(r - 3)$$

Stable circular orbits $r = const$ exist for particles with charge $|\epsilon| > \mu$ inside the inner horizon of charged black hole at $0 < r < r_-$

Stable periodic orbit of charged planet inside charged black hole



$e = 0.999$
$\epsilon = -1.45$
$E = 1.5$
$L = 0.2$

$(T_\phi, T_r) = (14.9, 7.17)$
$(r_p, r_a) = (0.19, 0.92)$

Spherical orbits $r = \mathrm{const}$:

equatorial at $Q = 0$ and nonequatorial with nutation at $Q \neq 0$

Total energy E and angular momentum L for neutral planet, $\epsilon = 0$, inside Kerr-Newman black hole $(a, e) \neq 0$

$$E_{1,2}^2 = \frac{\mp 2D_2 + \beta_1 r^2 + a^2[2(r-e^2)\Delta - r^2(r-1)^2]Q}{r^4[(r^2-3r+2e^2)^2 - 4a^2(r-e^2)]}$$

$$b_{1,2} = \frac{L_{1,2}}{E_{1,2}} = \frac{\pm D_2 r - a^2(r-e^2)\{\beta_2 r + [a^2 - r(r-e^2)]Q\}}{a(r-e^2)\{r[(\Delta - a^2)^2 - a^2(r-e^2)] + a^2(1-r)Q\}}$$

where

$$\beta_1 = (r^2 - 3r + 2e^2)(r^2 - 2r + e^2)^2 - a^2(r-e^2)[r(3r-5) + 2e^2]$$

$$\beta_2 = e^4 - a^2(r-e^2) + 2e^2 r(r-2) - r^2(3r-4)$$

$$D_2^2 = [a(r-e^2)\Delta]^2[(r-e^2)r^4 - r^2(r^2-3r+2e^2)Q + a^2Q^2]$$

At $\epsilon \neq 0$ formulas for E и L for spherical orbits are rather cumbersome

Spherical photon orbits

Ultra-relativistic limit $E = \Gamma \to \infty$

$$b_1 = \frac{L}{E} = \frac{a^2(1+r)+r(r^2-3r+2e^2)]}{a(1-r)}$$

$$q_1 = \frac{Q}{E^2} = \frac{r^2[4a^2(r-e^2)-(r^2-3r+2e^2)^2]}{a^2(1-r)^2}$$

Stability condition $V_r'' \leq 0$ for spherical photon orbits

$$a^2 + e^2 - r(r^2 - 3r + 3) \leq 0$$

$$q_1 \leq \left(\frac{1-\delta^{1/3}}{a}\right)^2 [3 - 4e^2 - 2(3-2e^2)\delta^{1/3} + 3\delta^{2/3}]$$

where $\delta = 1 - a^2 - e^2$

$$q_{1,\mathrm{max}} = 4 - a^{-2} \leq 3$$

Maximal nutation is reached at extreme case
$a = \sqrt{1-e^2} \leq 1/2,\ e \leq \sqrt{3}/2$

Circular photon orbits: $r = const, \quad Q = 0$

Ultra-relativistic limit $E = \Gamma \to \infty$ **for circular planet orbits**

Relation for circular photon orbits

$$4a^2(r - e^2) = (r^2 - 3r + 2e^2)^2$$

Impact parameter for circular photon orbit

$$b_{1,2} = \frac{a\beta_2 \pm r^2\sqrt{(r - e^2)\Delta^2}}{(r^2 - 2r + e^2)^2 - a^2(r - e^2)}$$

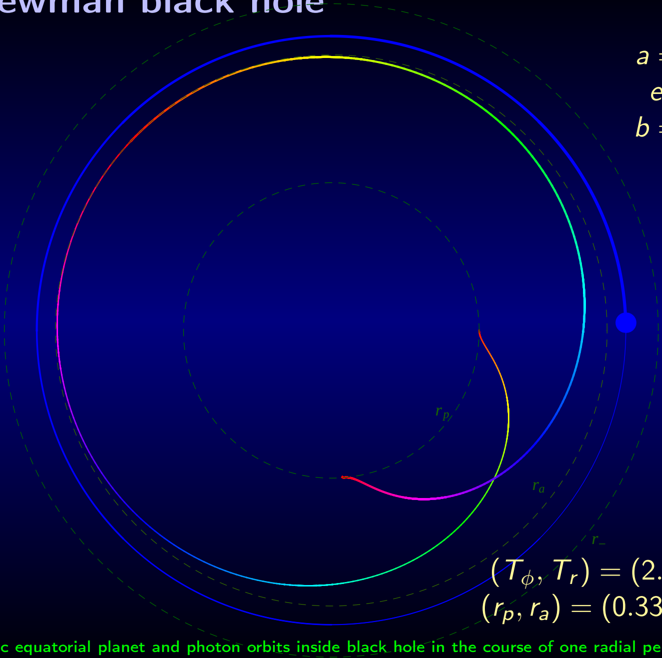
Silution with sign 'minus' is unstable

Circular photon orbits exist at

$$e^2 \le r \le \frac{4}{3}e^2, \quad a \ne 0, \quad 0 < e \le \frac{\sqrt{3}}{2}, \quad 0 < b < \frac{5}{2}$$

Kerr-Newman black hole

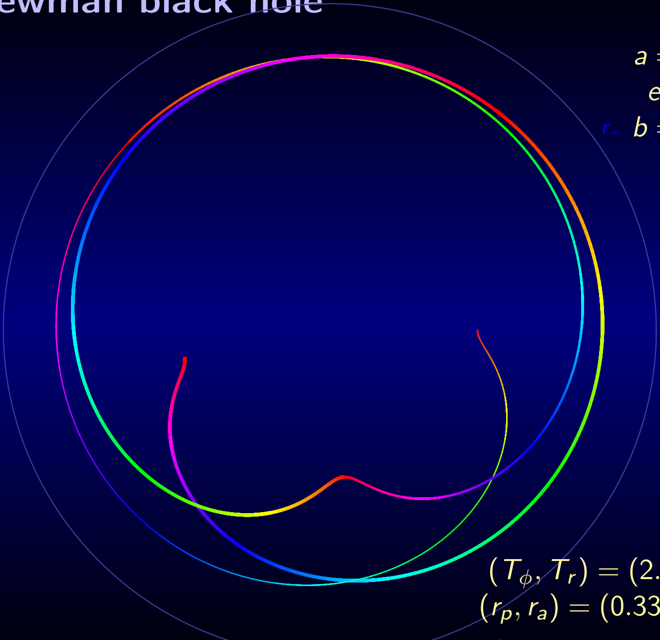
$a = 0.75$
$e = 0.6$
$b = 1.53$



$(T_\phi, T_r) = (2.1, 2.7)$
$(r_p, r_a) = (0.33, 0.61)$

Periodic equatorial planet and photon orbits inside black hole in the course of one radial period T_r

Kerr-Newman black hole



$a = 0.75$

$e = 0.6$

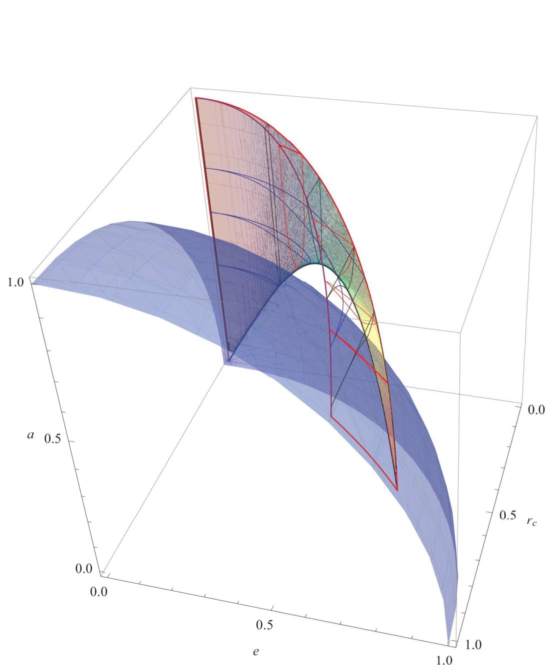
$b = 1.53$

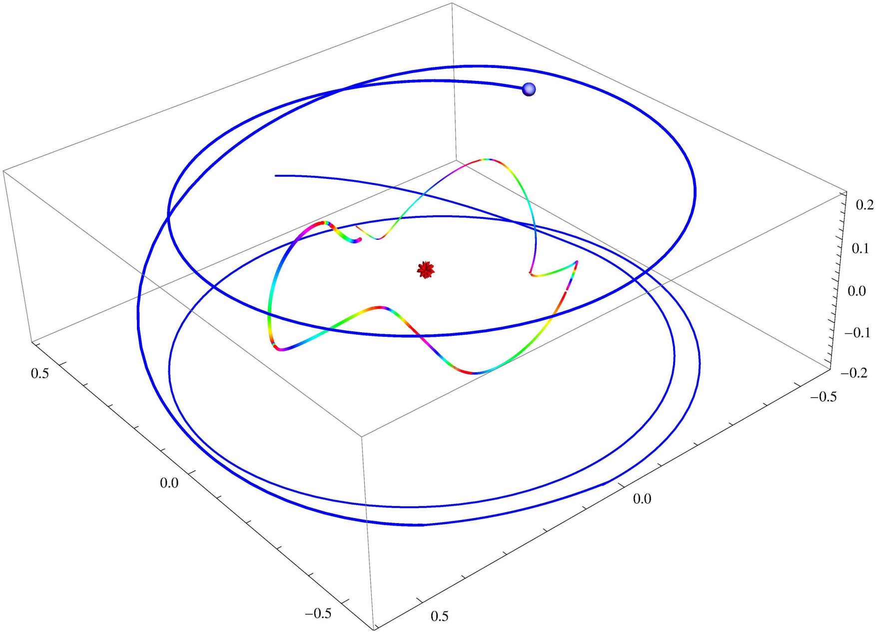
$(T_\phi, T_r) = (2.1, 2.7)$

$(r_p, r_a) = (0.33, 0.61)$

Periodic equatorial photon orbit inside black hole in the course of two radial periods T_r

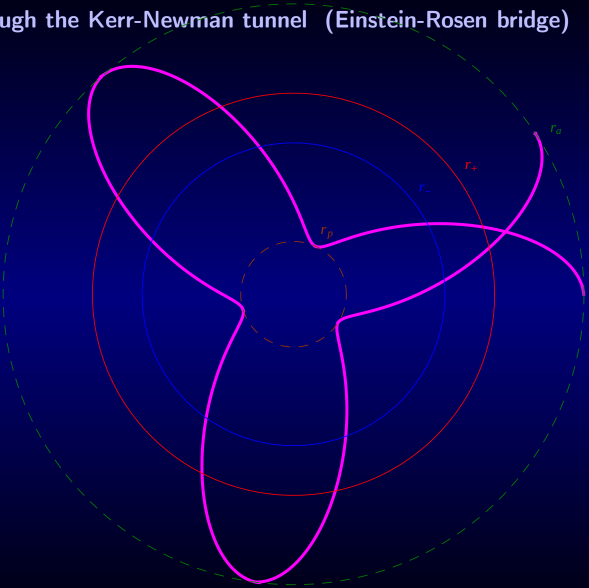
Stability region for circular orbits of neutral particles inside black hole




0.2
0.1
0.0
−0.1
−0.2
0.5
0.0
−0.5
−0.5
0.0
0.5

3D voyage to other universes

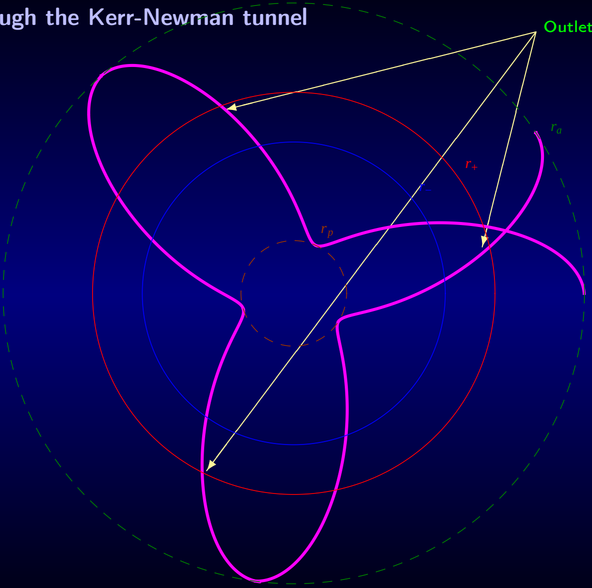
Evacuation through the Kerr-Newman tunnel (Einstein-Rosen bridge)



$a = 0,\ e = 0.99,\ \epsilon = -1.5,\ E = L = 0.5,\ r_p = 0.29 < r_-,\ r_a = 1.65 > r_+$

3D voyage to other universes

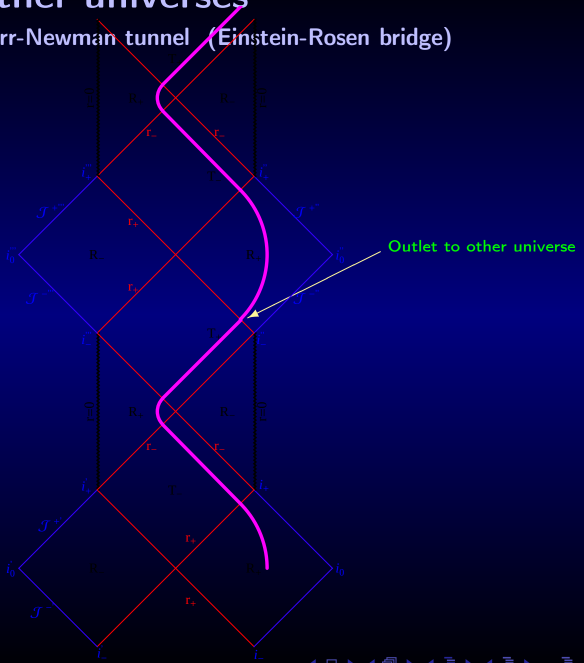
Evacuation through the Kerr-Newman tunnel



$a = 0,\ e = 0.99,\ \epsilon = -1.5,\ E = L = 0.5,\ r_p = 0.29 < r_-,\ r_a = 1.65 > r_+$

4D voyage to other universes

Evacuation through the Kerr-Newman tunnel (Einstein-Rosen bridge)



Interior of quantum black hole?

Black atom — black hole as atomic nucleus

- Stationary states of electrons and photons inside black hole?
- Quantum transitions (tunneling) of internal electrons and photons beyond the horizon?
- Modification of Hawking radiation due to internal states?
- Quantum mining of black hole interiors?

Conclusion: Life is possible in the other side world

- There are stable periodic orbits of planets and photons (orbits of III kind) inside rotating black holes

Hypotheses

- Civilizations of III type (on the N. Kardashev scale) dwell the interiors of supermassive black holes in the galactic nuclei
- Resolving the E. Fermi paradox (problem of the Great Silence of Cosmos): inhabitants of black holes observe the all outside Universe, being invisible from outside
- Existence of III kind orbits may be verified or falsified by observations of white holes

Bonuses for living inside black holes

Central singularity — source of energy
Highlighting at nights by orbiting photons
Recycling the accreting matter
Energy accumulation near the inner surface of Cauchy horizon
Possibility of evacuation through Einstein-Rosen bridge