Quantum simulation of thermodynamic and transport properties of quark – gluon plasma

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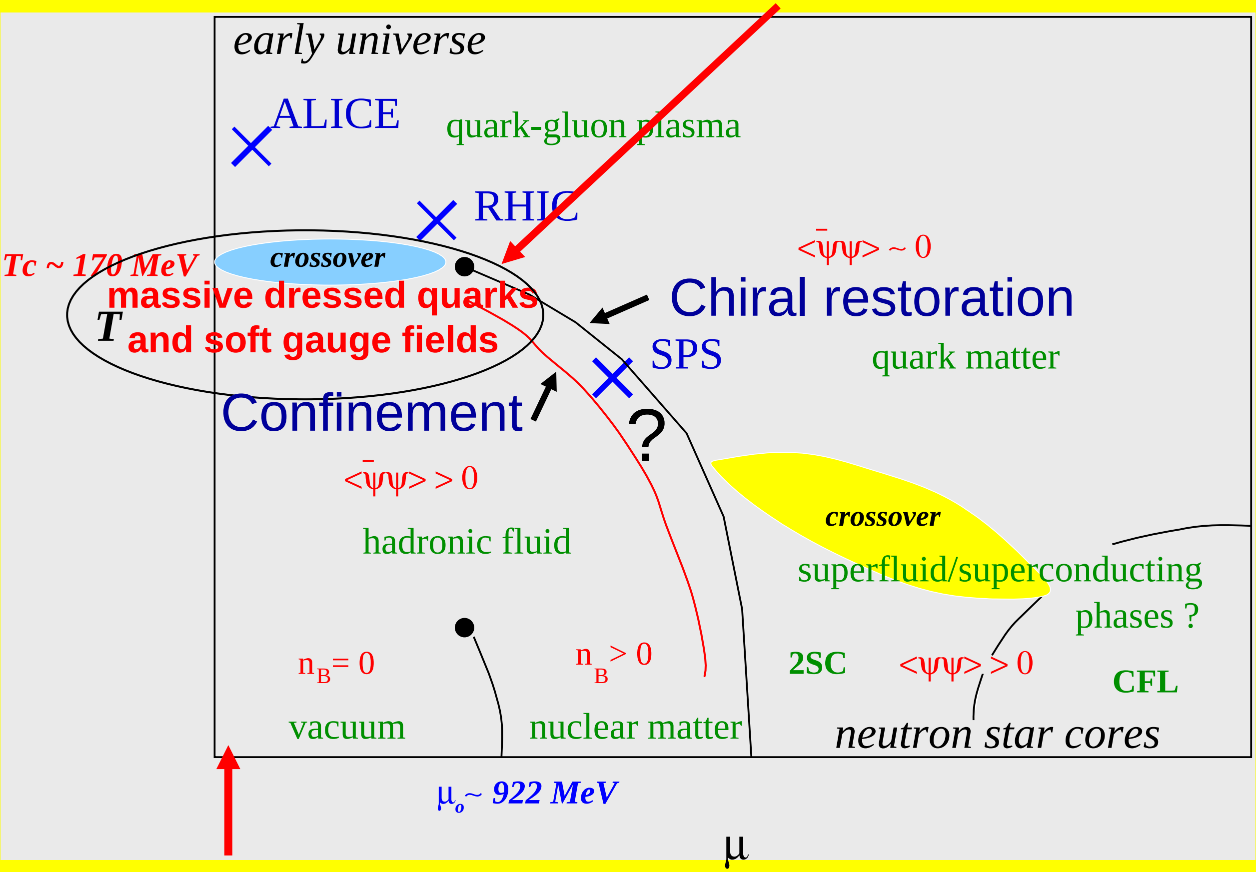
Outlook

- Path integral approach to quark-gluon plasma
- Quantum effects in particle interactions and Kelbg potentials
- Thermodynamic quantities and pair distribution functions
- Wigner formulations of quantum mechanics
- Integral form of the color Wigner – Liouville equation
- Quantum dynamics and kinetic properties

Quasiparticle model of QGP

In restricted part of phase diagram results of resummation technique and lattice simulations allow for consideration of quark-gluon plasma as system of dressed quarks, antiquarks and gluons which can be presented by massive color Coulomb quasiparticles with T-dependent dispersion curves and width (at least at $\mu=0$ at $T \sim T_d$ or above T_d and below T_c if $T_d < T_c$)

Feinberg, Litim, Manuel, Stoecker,Bleicher,, Richardson, Bonasera,Maruyama, Hatsuda, Shuryak,….



Phase diagram

(F.Karsch)

Basic asumptions of quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.

(Shuryak , Phys.Lett.B478,161(2000), Phys. Rev. C, **74**, 044909, (2006))

- We consider relativistic color quasiparticles representing gluons and the most stable quarks of three flavors (up, down and strange).
- Up, down and strange quasiparticles have the same masses
- Interparticle interaction is domonated by a color Coulomb potential with distance dependent coupling constant.
- The color operators are substituted by their average values
  – classical color vectors in SU(3) (8D vectors with 2 Casimirs conditions.).

The model input requires :

- The temperature dependence of the quasiparticle masses.
- The temperature dependence of the coupling constant.

All input quantities should be deduced

from lattice QCD calculations or experimental data

and substitued in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C =$$

$$= \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

Grand canonical partition function

$$\Omega(\mu, \mu_g = 0, V, \beta) = \sum_{N_u, N_d, N_s, N_{\bar{u}}, N_{\bar{d}}, N_{\bar{s}}, N_g} \exp(\beta\mu(N_q - N_{\underline{q}})) \times$$

$$\times Z(N_q, N_{\bar{q}}, N_g, \beta) / N_u! N_{\underline{d}}! N_s! N_{\bar{u}}! N_{\bar{d}}! N_{\bar{s}}! N_g!$$

$$N_q = N_u + N_{\underline{d}} + N_s; N_{\bar{q}} = N_{\bar{u}} + N_{\bar{d}} + N_{\bar{s}}$$

SU(3) Haar measure with two Casimirs !!!!

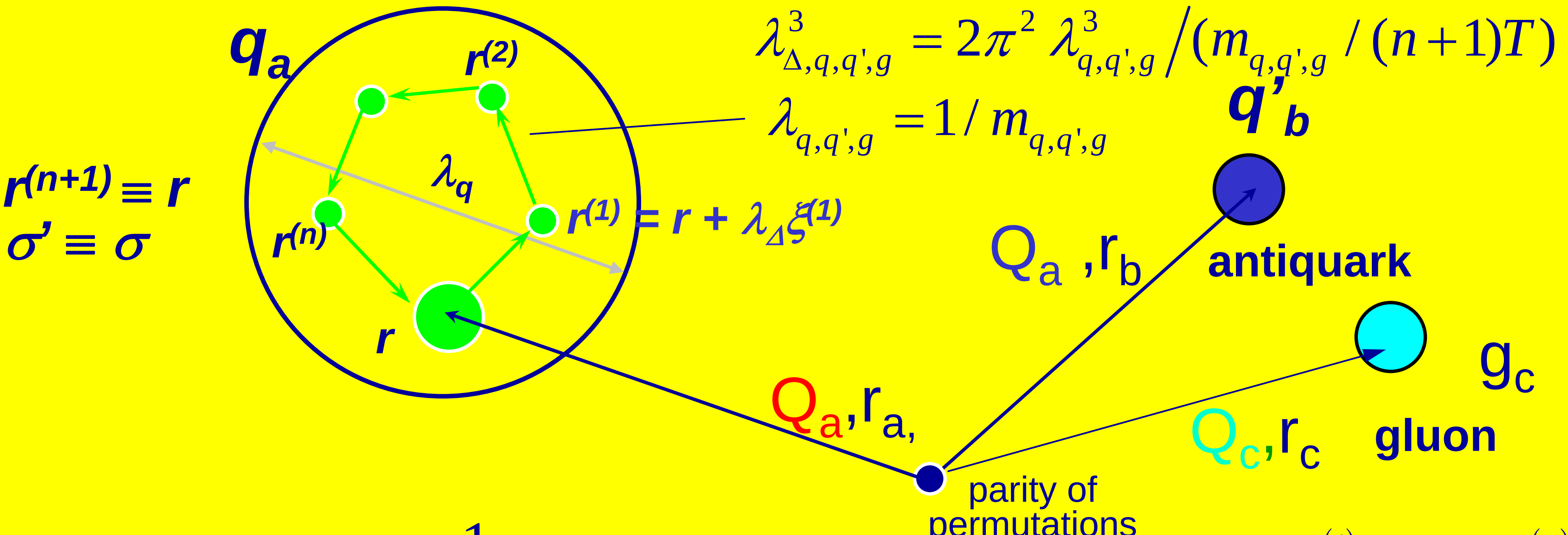
$$Z(N_q, N_{\underline{q}}, N_g, \beta) = \sum_\sigma \int_V dr\, d\mu \vec{Q}\, \rho(r, \vec{Q}, \sigma; \beta)$$

$$\rho = \exp(-\beta H(\beta)) = \underbrace{\exp(-\Delta\beta H(\beta)) \times \ldots \times \exp(-\Delta\beta H(\beta))}_{n+1}$$

$$\beta = 1/kT$$

$$\Delta\beta = \beta/(n+1)$$

PATH INTEGRAL
MONTE-CARLO METHOD



$$\rho\left(r,\vec{Q},\sigma;\beta\right) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta \bar{q}}^{3N_{\bar{q}}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{\bar{q}},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)} \ldots dr^{(n)} d\mu\vec{Q}^{(1)} \ldots d\mu\vec{Q}^{(n)} \times$$

$$\rho\left(r,\vec{Q};r^{(1)},\vec{Q}^{(2)};\Delta\beta\right) \ldots \rho\left(r^{(n)},\vec{Q}^{(n)};\hat{P}r^{(n+1)},\hat{P}\vec{Q}^{(n+1)};\Delta\beta\right) S\left(\sigma,\hat{P}\sigma'\right)$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)} - Q^{(l+1)})\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l)})$$

Density matrix

$$\sum_{\sigma} \rho\left(r,\vec{Q},\sigma;\beta\right) = \frac{1}{\lambda_{\Delta}^{3N_q}\lambda_{\Delta}^{3N_{\bar{q}}}\lambda_{\Delta}^{3N_g}} \sum_{\sigma} \rho\left(\left[r\vec{Q}\right],\beta\right)$$

$$\rho\left([rQ],\beta\right) = \exp\left\{-\beta U\left([rQ],\beta\right)\right\} \times$$

$$\times \prod_{l=1}^{n} \prod_{p=1}^{N_q} \varphi_{pp}^{l} \det\left|\psi_{ab}^{n,1}\right|_{N_q} \prod_{p=1}^{N_{\bar{q}}} \tilde{\varphi}_{pp}^{l} \det\left|\tilde{\psi}_{ab}^{n,1}\right|_{N_{\bar{q}}} \prod_{p=1}^{N_g} \tilde{\tilde{\varphi}}_{pp}^{l}\, per\left|\tilde{\tilde{\psi}}_{ab}^{n,1}\right|_{N_g}$$

Relativistic measure instead of Gaussian one

$$U\left([rQ],\beta\right) = \sum_{l=0}^{n} \frac{U_l\left(\left[r^{(l)}Q\right],\beta\right)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix

$$\left\|\psi_{ab}^{n,1}\right\|_s \equiv \left\| \delta_{\sigma_a,\sigma_b} \delta_{f_a,f_b} K_2\left\{\sqrt{(m_a/((n+1)T))^2 + \left|\left(r_a^{(0)} - r_a^{(n)}\right)\right|^2 \Big/ \Delta\lambda_a^2}\right\}\right\|$$

Color Kelbg potential

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Levai, Kalman (r=0 ?)

$$x_{ab} = |\mathbf{r}_{ab}| / \tilde{\lambda}_{ab} \qquad \tilde{\lambda}_{ab} = \hbar^2 \Delta\beta / 2\mu_{ab}$$

$$\Phi^{ab}(x_{ab}, \Delta\beta) = \frac{<\vec{Q}_a | \vec{Q}_b > g^2}{4\pi\tilde{\lambda}_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi} x_{ab} \left[1 - erf(x_{ab}) \right] \right\}$$

$$|\mathbf{r}_{ab}| \to 0$$

$$\sim \frac{< Q_a | Q_b > g^2 \sqrt{\pi}}{4\pi\tilde{\lambda}_{ab}}$$

$$|\mathbf{r}_{ab}| >> \tilde{\lambda}_{ab}$$

$$\frac{< Q_a | Q_b > g^2}{4\pi\tilde{\lambda}_{ab} |x_{ab}|}$$

Objects Q are color coordinates of quarks and gluons

There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev,

$T_c < T$, $m_a \sim k_B T_c / c^2$,

$L_o \sim hc / k_B T_c$, $r_s = \langle r \rangle / L_o \sim 0.3$,

$L_o \sim 1.2 \; 10^{-15}$ m

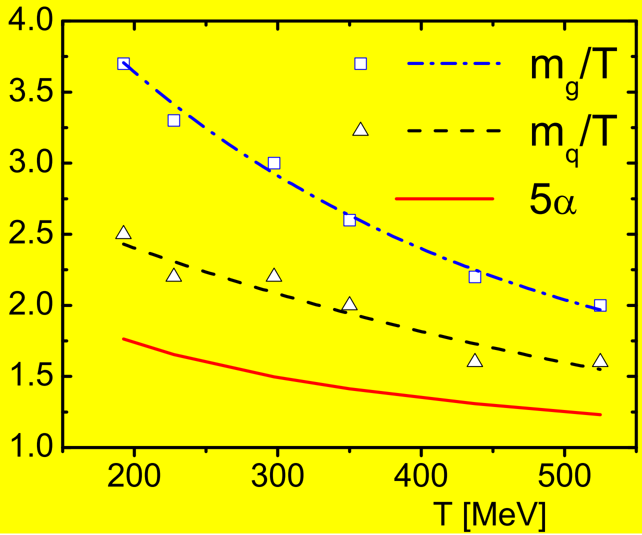
Input quantities

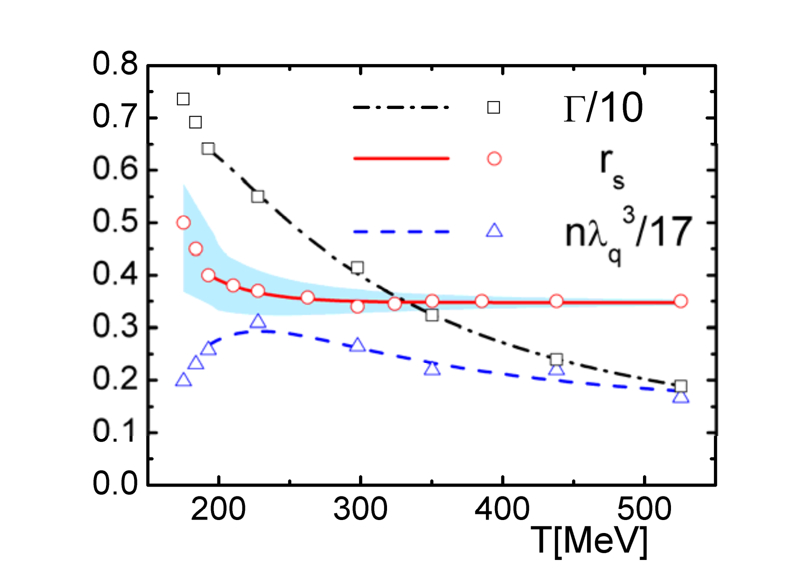
1) Coupling constant

$$\alpha(T) = g^2(T)/4\pi < 1$$

$$\mu_B = 0$$

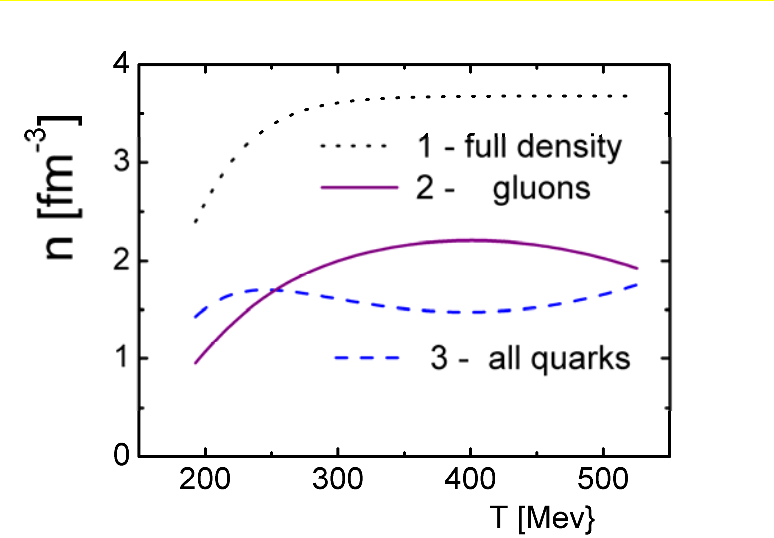
2) Quasiparticle masses: m_q, m_q, m_g





Ratio of potential to kinetic energy per quasiparticle

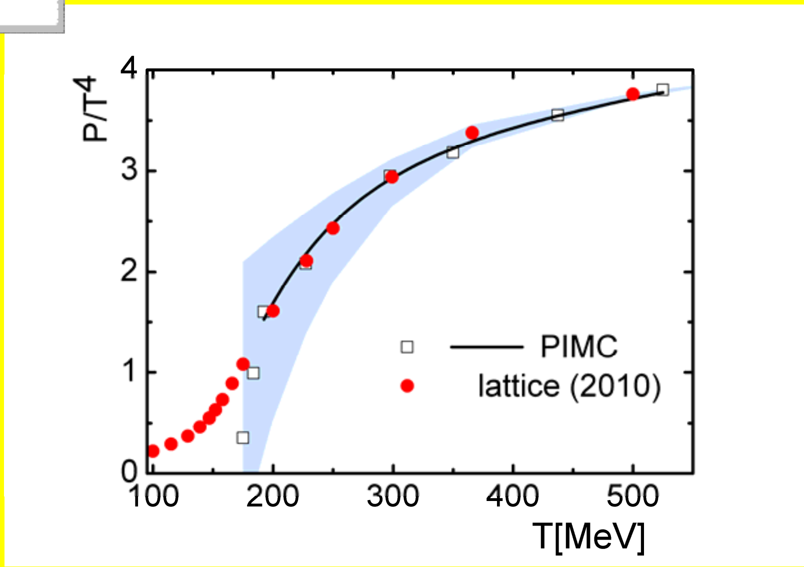
$$\Gamma(T) \sim U/K \sim 5$$

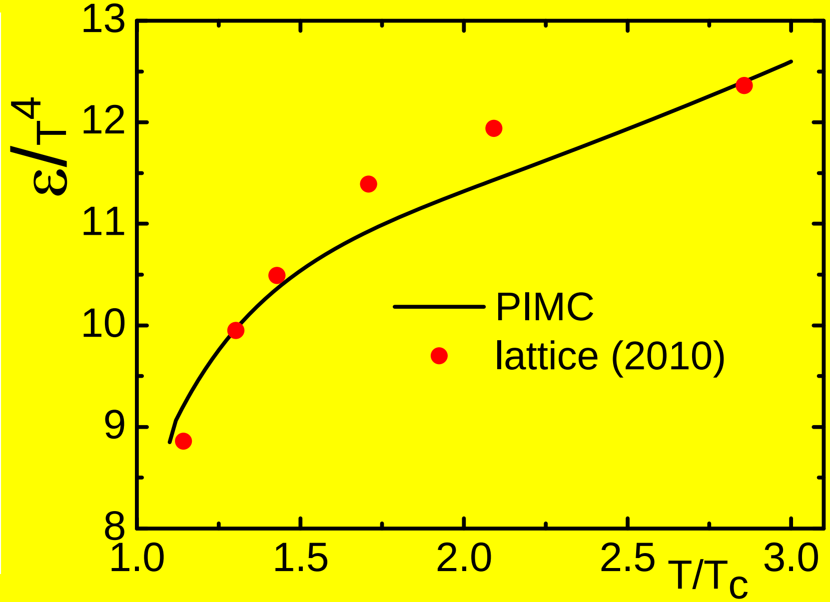


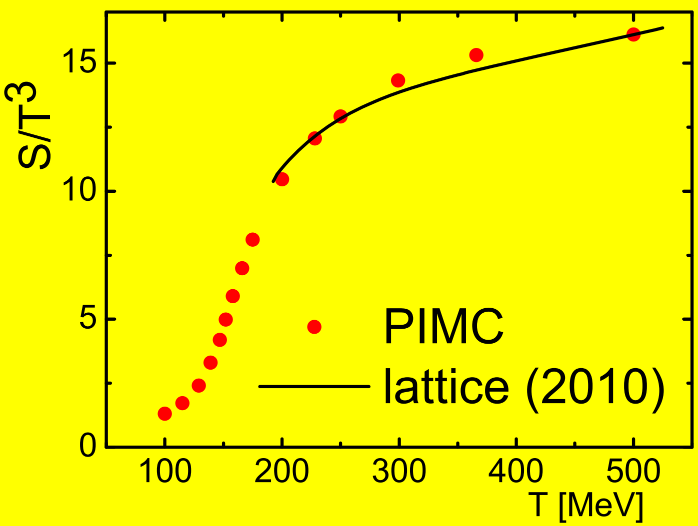
Density from grand canonical ensemble

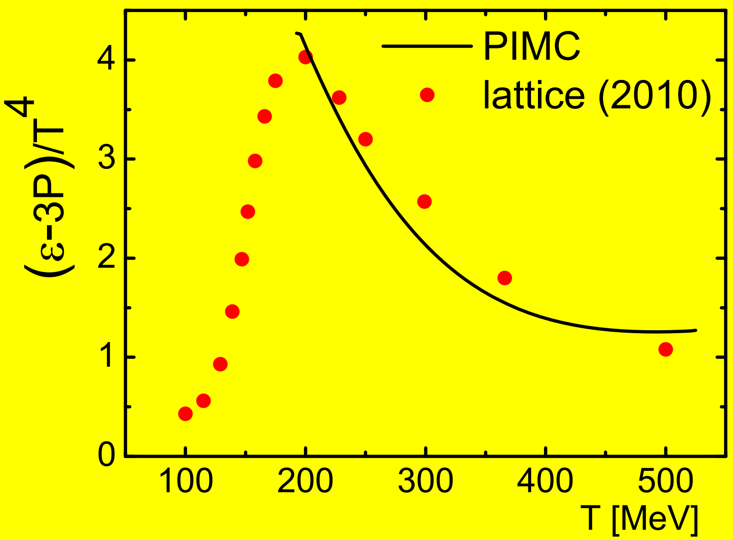
r_s - Wigner-Seitz radis

Equation of State. The entropy density. The trace anomaly.
Comparison path integral results with lattice (2+1) QCD









The QCD equation of state with dynamical quarks **Szabolcs Borsanyi, Gergely Endrodi, Zoltan Fodor, Antal Jakovac, Sandor D. Katz, Stefan Krieg, Claudia Ratti, Kalman K. Szabo, JHEP 11 (2010) 077**

Pair distribution functions in canonical emsemble

$$H_{\beta} = \sum_{a} \sqrt{m_a(\beta)^2 + p_{\alpha}{}^2} + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

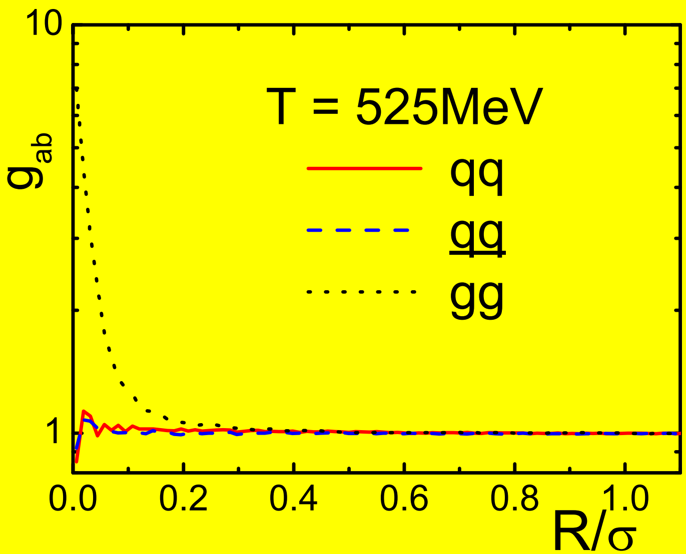
$$g_{ab}(|R_1 - R_2|) = g_{ab}(R_1, R_2) = \frac{1}{Z(N_q, N_{\bar{q}}, N_g)} \times$$

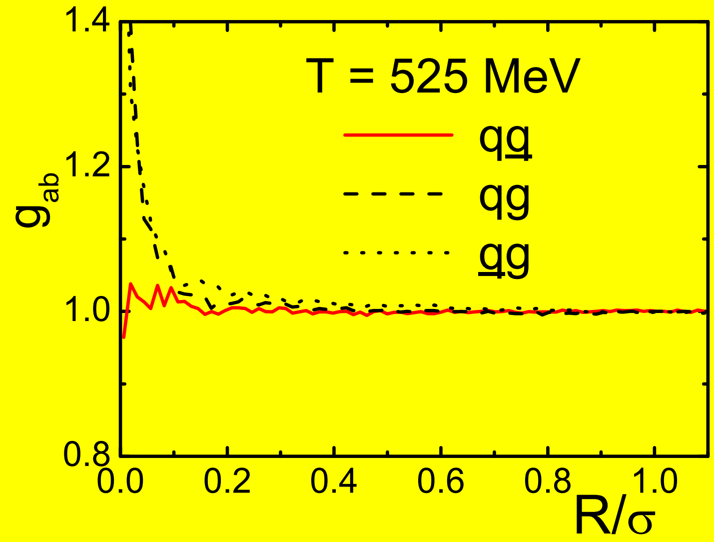
$$\sum_{\sigma} \int_V dr\, dQ\, \delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) \rho(r, Q, \sigma; \beta),$$

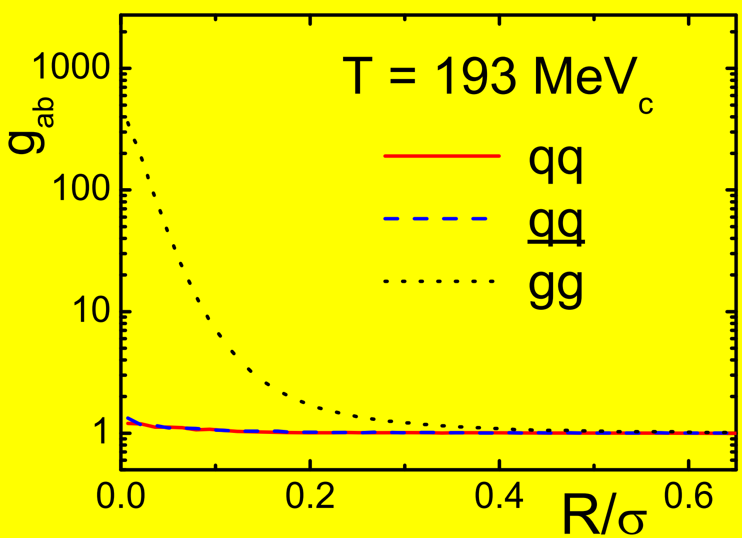
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

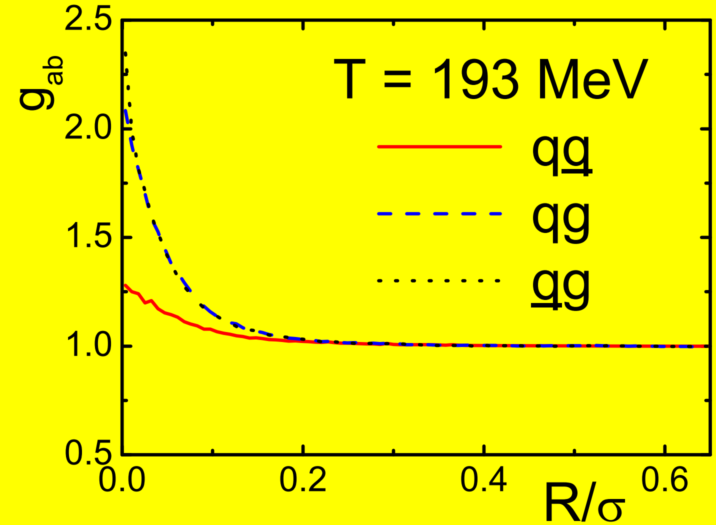
Similar quasiparticles

Different quasiparticles









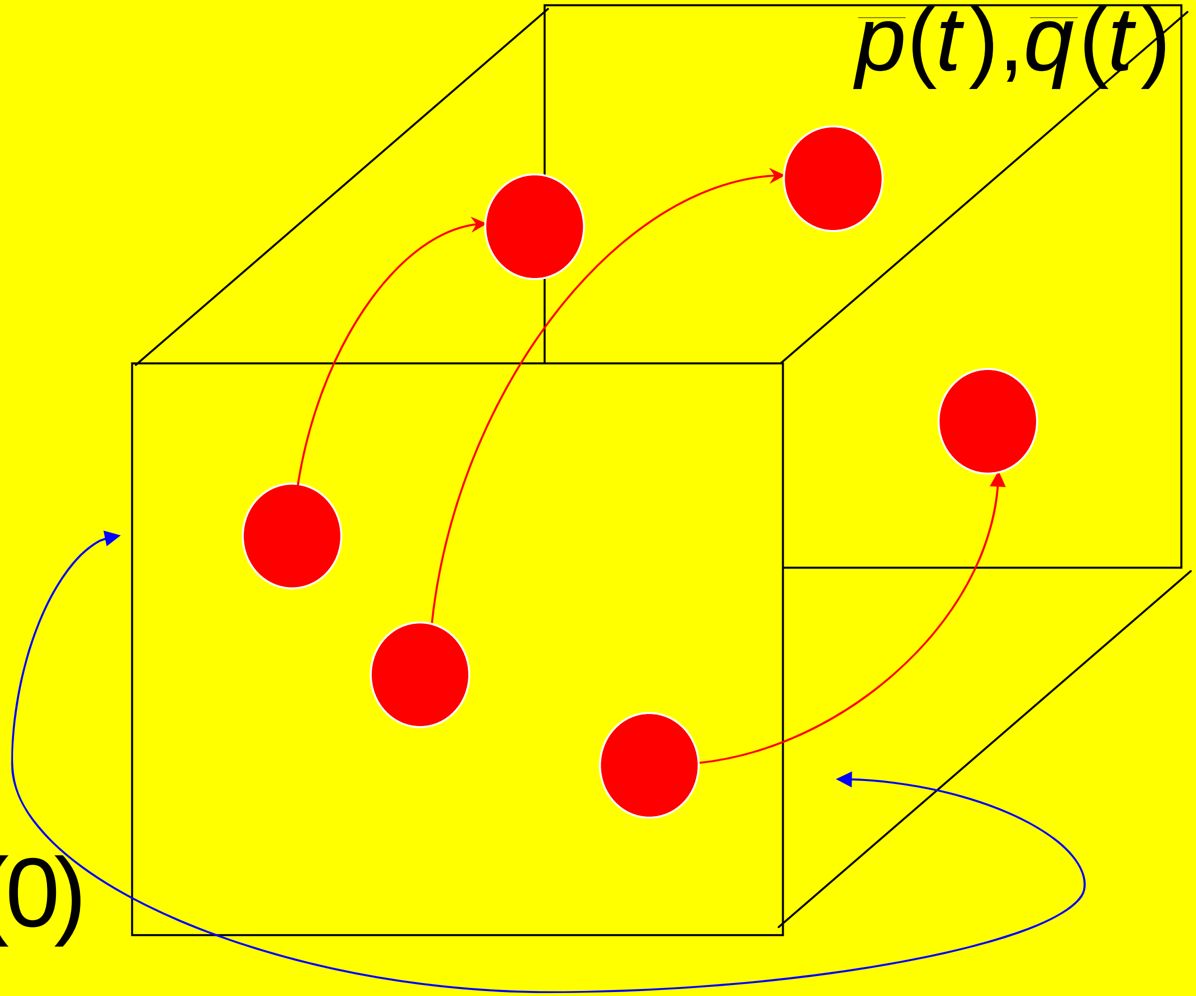
Classical dynamics in phase space

$$W(\bar{p}(0),\bar{q}(0)) \sim \exp(-\beta H(\bar{p}(0),\bar{q}(0)))$$

$$\frac{d\bar{p}}{dt} = \bar{F}(q(t))$$

$$\frac{d\bar{q}}{dt} = \frac{\bar{p}(t)}{m}$$

$$<\bar{p}(t)\bar{p}(0)>$$



$\bar{p}(0),\bar{q}(0)$

QUANTUM DYNAMICS IN WIGNER REPRESENTATION

Quasi-distribution function in phase space for the quantum case

Density matrix: $\rho(q',q'')=\psi^*(q')\psi(q'')$ $\quad \psi \in C \qquad i\frac{\partial \rho}{\partial t}=[\hat{H},\rho]$

↓

Wigner function: $W^L(q,p)=\frac{1}{(2\pi)^{Nd}}\int \rho\left(q+\frac{\xi}{2},q-\frac{\xi}{2}\right)e^{-ip\xi}d\xi \qquad W^L \in R$

$$\rho(q',q'')=\int W^L\left(\frac{q'+q''}{2},p\right)e^{i(q'-q'')p}dp$$

Evolution equation: $\frac{\partial W^L}{\partial t}+\left\langle \frac{p}{m}\middle|\frac{\partial W^L}{\partial q}\right\rangle=\int ds W^L(p-s,q,t)\omega(s,q)ds$

$$\omega(s,q)=\frac{2}{(2\pi)^{Nd}}\int dq' U(q-q')\sin\left[\frac{2sq'}{\hbar}\right]$$

Classical limit $\hbar \to 0$:

$$\frac{\partial W^L}{\partial t}+\left\langle \frac{p}{m}\middle|\frac{\partial W^L}{\partial q}\right\rangle-\left\langle \frac{\partial U}{\partial q}\middle|\frac{\partial W^L}{\partial p}\right\rangle=0$$

Characteristics (Hamilton equations):

$$\langle \dot{q}|=\left\langle \frac{p}{m}\right| \qquad \langle \dot{p}|=-\left\langle \frac{\partial U}{\partial q}\right|$$

SOLUTION OF THE WIGNER EQUATION IN INTEGRAL FORM

$$W^L(p,q,t) = \int \Pi^W(p,q,t;p_0,q_0,0) \times W_0(p_0,q_0)\,dp_0\,dq_0 +$$

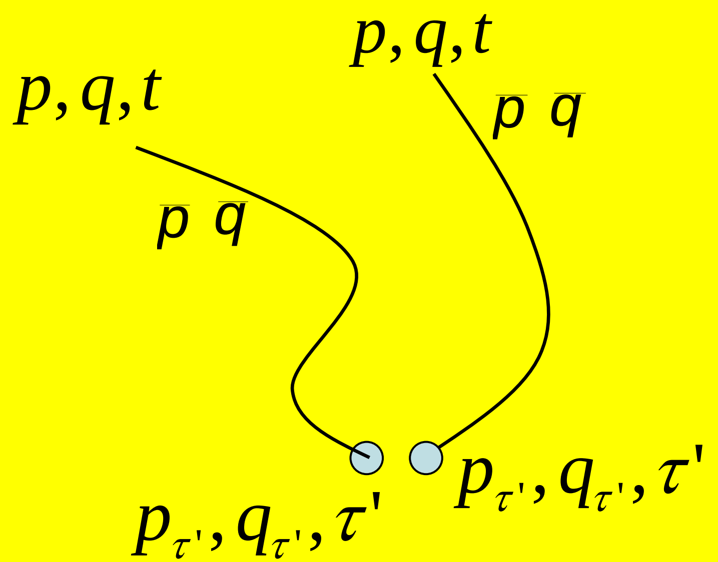
$$\int_0^t d\tau' \int\int dp_{\tau'}\,dq_{\tau'}\,\Pi^W(p,q,t;p_{\tau'},q_{\tau'},\tau') \int_{-\infty}^{\infty} ds\, W^L(p_{\tau'}-s,q_{\tau'},\tau')\,\omega(s,q_{\tau'})$$

Dynamical trajectories:

p,q,t

$$\frac{d\bar{p}}{dt} = F(\bar{q}(t)),\ \bar{q}_t(t|_{t=\tau'};p_{\tau'},q_{\tau'},\tau') = q_{\tau'}$$

$$\frac{d\bar{q}}{dt} = \bar{p}(t)/m,\ p_t(t|_{t=\tau'};p_{\tau'},q_{\tau'},\tau') = p_{\tau'}$$



Propagator:

$$\Pi^W(p,q,t;p_{\tau'},q_{\tau'},\tau') = \delta(p-\bar{p}_t(t;p_{\tau'},q_{\tau'},\tau'))\,\delta(q-\bar{q}_t(t;p_{\tau'},q_{\tau'},\tau'))$$

Quantum dynamics in phase space

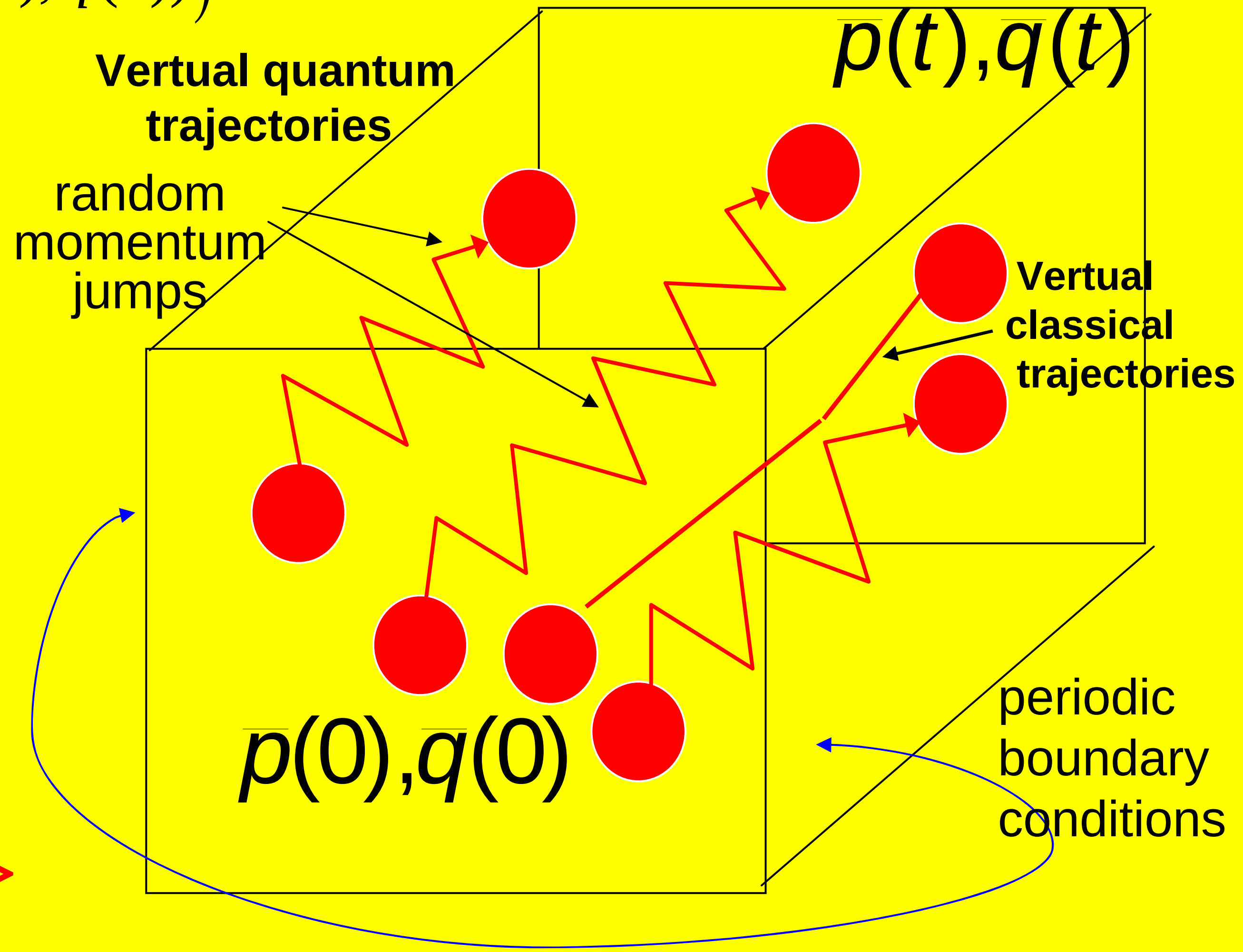
$$P \sim \left| W^{L}(\overline{p}(0), \overline{q}(0)) \right|$$

W^L - Wigner-Louville function

$$weight = \mathrm{sign}\left(W^{L}(\overline{p}(0), \overline{q}(0)) \right)$$

$$\frac{d\overline{p}}{dt} = \overline{F}(q(t))$$

$$\frac{d\overline{q}}{dt} = \frac{\overline{p}(t)}{m}$$



Averaged Weil symbols of operators

$$< \overline{p}(t)\overline{p}(0) >$$

Kinetic properties of quark – gluon plasma in canonical ensemble

$$G_{FA}(t) = Z^{-1}Tr\{\exp(-\beta H)F\exp(i\frac{Ht}{h})A\exp(-i\frac{Ht}{h})\}$$

$$C_{FA}(t) = Z^{-1}Tr\{F\exp(i\frac{Ht_c^*}{h})A\exp(-i\frac{Ht_c}{h})\};C_{FA}(\omega) = \exp(-\frac{\beta h\omega}{2})G_{FA}(\omega)$$

$$H = K + V(qQ), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$

$$Z = Tr\{\exp(-\beta H)\}$$

$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}}\iint d\mu Q_1 dp_1 dq_1 d\mu Q_2 dp_2 dq_2 F(p_1, q_1, Q_1)A(p_2, q_2, Q_2)\times$$

$$W(p_1, q_1, Q_1; p_2, q_2, Q_2; t; i\beta h),$$

$$A(p, q, Q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2}|A|q + \frac{\xi}{2} >$$

$$W(p_1, q_1, Q_1; p_2, q_2, Q_2; t; i\beta h) = Z^{-1}\iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h})\exp(i\frac{p_2\xi_2}{h})\times$$

$$< q_1 + \frac{\xi_1}{2}|\exp(i\frac{Ht_c^*}{h})|q_2 - \frac{\xi_2}{2} >< q_2 + \frac{\xi_2}{2}|\exp(-i\frac{Ht_c}{h})|q_1 - \frac{\xi_1}{2} >$$

Integral color Wigner – Liouville equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_2;t;i\beta h)=\overline{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)\delta(Q_1^0-Q_2^0)+$$

$$+\int_0^t d\tau\iint ds\iint d\eta W(p_1^\tau-s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq'V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\bullet\frac{d\delta(s)}{ds}$$

Positive time direction

$$\frac{dq_1^t}{dt}=\frac{1}{2}\frac{p_1^t}{\sqrt{m^2+(p_1^t)^2}},\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

$$\frac{dq_2^t}{dt}=-\frac{1}{2}\frac{p_2^t}{\sqrt{m^2+(p_2^t)^2}},\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_2)=Q_2$$

Color Wong dynamics in SU(3)

Initial conditions

Hamiltonian eqations

Negative time direction

Initial conditions

$$\bar{W}(p_1,q_1,Q_1;p_2,q_2,Q_2;0;i\beta h) = Z^{-1}\iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h})\exp(i\frac{p_2\xi_2}{h})\times$$

$$< q_1+\frac{\xi_1}{2}|\exp(-\beta\frac{H}{2})|q_2-\frac{\xi_2}{2}>< q_2+\frac{\xi_2}{2}|\exp(-\beta\frac{H}{2})|q_1-\frac{\xi_1}{2}>\delta(Q_1-Q_2)$$

$$\exp(-\frac{\beta}{2}H)=\exp(-\varepsilon H)\exp(-\varepsilon H)...\exp(-\varepsilon H), \varepsilon=\beta/2M, t=0$$

$$\exp(-\varepsilon H)=\exp(-\varepsilon K)\exp(-\varepsilon V)\exp(-\varepsilon^2[K,V]/2)...,$$

$$\bar{W}(p_1,q_1,Q_1;p_2,q_2,Q_2;0;i\beta h)\approx\iint d\bar{q}_1 d\bar{q}_2...d\bar{q}_M d\tilde{q}_1 d\tilde{q}_2...d\tilde{q}_M\times$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_2;\bar{q}_1,\bar{q}_2...\bar{q}_M;\tilde{q}_1,\tilde{q}_2...\tilde{q}_M;i\beta h\},$$

$$\Psi\{p_1,q_1,Q_1;p_2,q_2,Q_2;\bar{q}_1,\bar{q}_2...\bar{q}_M;\tilde{q}_1,\tilde{q}_2...\tilde{q}_M;i\beta h\}=$$

$$Z^{-1}< q_1|\exp(-\varepsilon K)|\bar{q}_1>\exp(-\varepsilon V(\bar{q}_1,Q_1))<\bar{q}_1|\exp(-\varepsilon K)|\bar{q}_2>$$

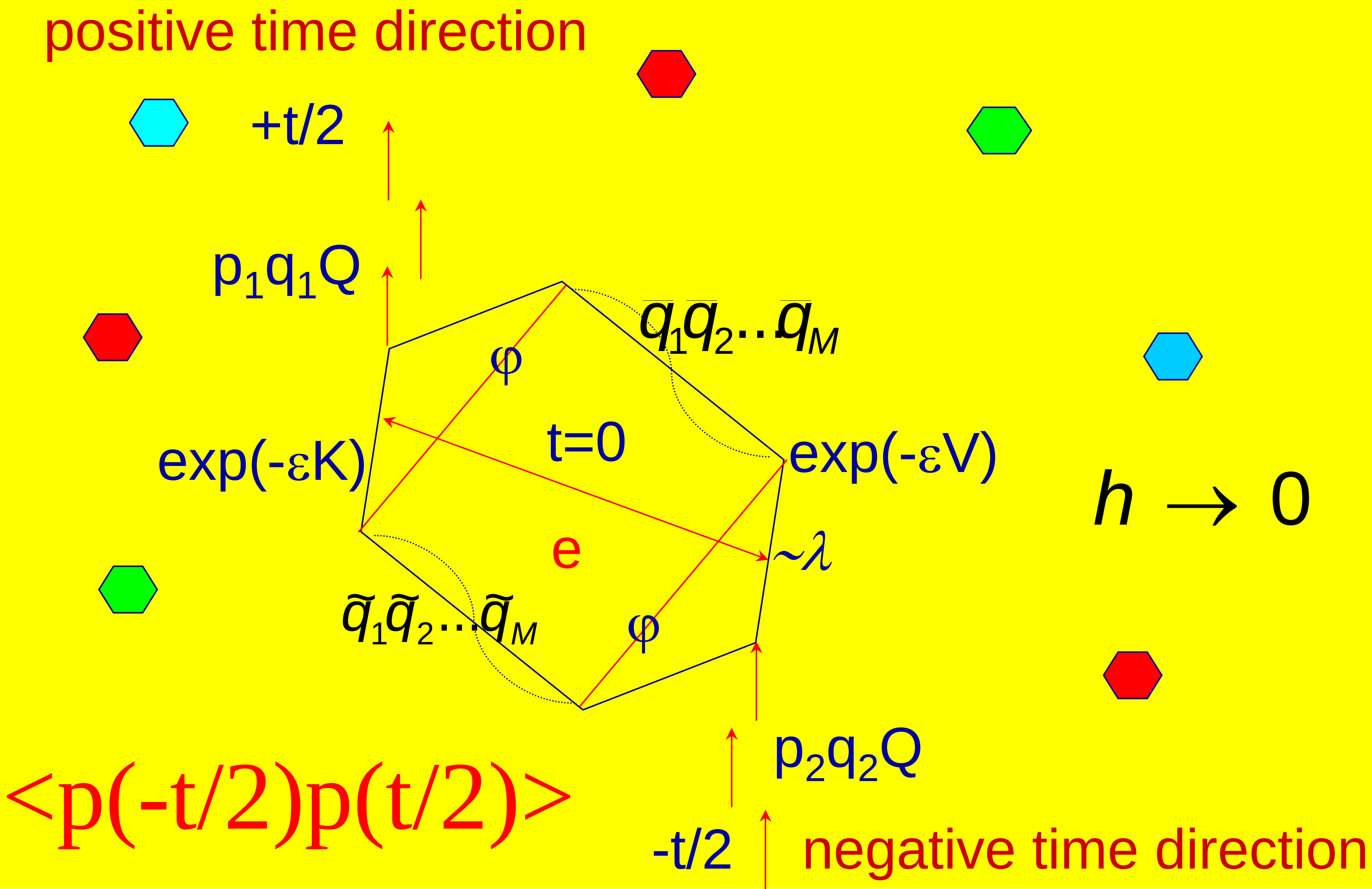
$$\exp(-\varepsilon V(\bar{q}_2,Q_1))...\exp(-\varepsilon V(\bar{q}_M,Q_1))<\bar{q}_M|\exp(-\varepsilon K)|q_2>\phi(p_2,\bar{q}_M,\tilde{q}_1)\times$$

$$< q_2|\exp(-\varepsilon K)|\tilde{q}_1>\exp(-\varepsilon V(\tilde{q}_1,Q_2))<\tilde{q}_1|\exp(-\varepsilon K)|\tilde{q}_2>$$

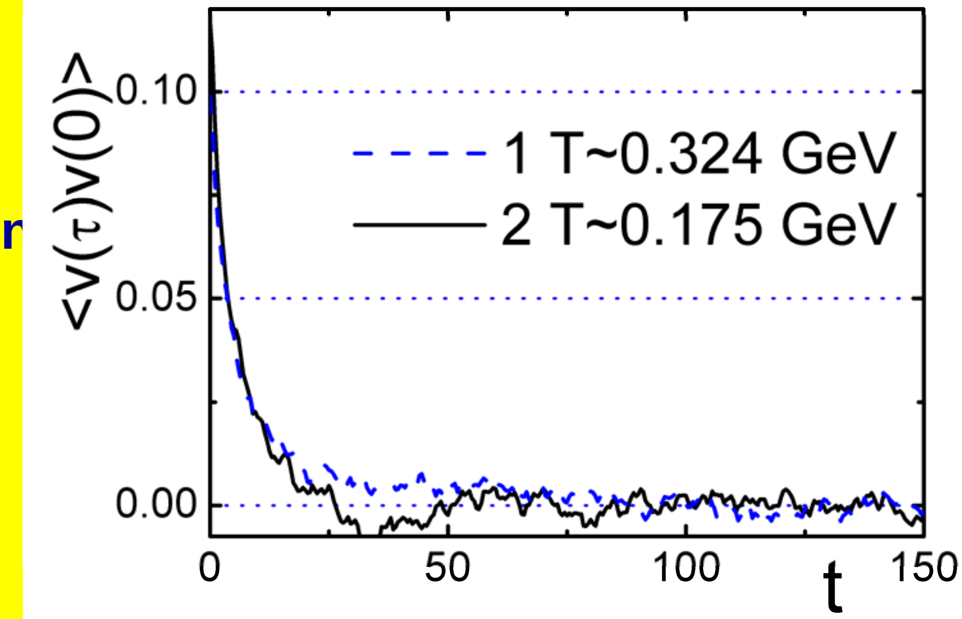
$$\exp(-\varepsilon V(\tilde{q}_2,Q_2))...\exp(-\varepsilon V(\tilde{q}_M,Q_2))<\tilde{q}_M|\exp(-\varepsilon K)|q_1>\phi(p_1,\tilde{q}_M,\bar{q}_1)$$

$$\phi(p,\bar{q},\tilde{q})\sim\lambda^{\nu}\exp(\frac{<\frac{p\lambda}{h}+i\pi\frac{\bar{q}-\tilde{q}}{\lambda}|\frac{p\lambda}{h}+i\pi\frac{\bar{q}-\tilde{q}}{\lambda}>}{2\pi}),\lambda^2=\frac{2\pi h^2\beta}{2Mm},$$

Schematic snapshot for color phase space dynamics



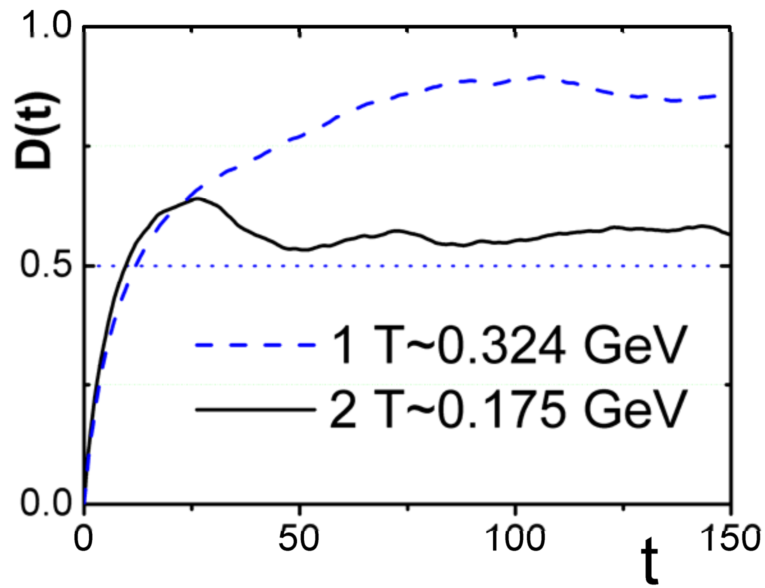
Velocity autocorrelation function and diffusion constant QGP



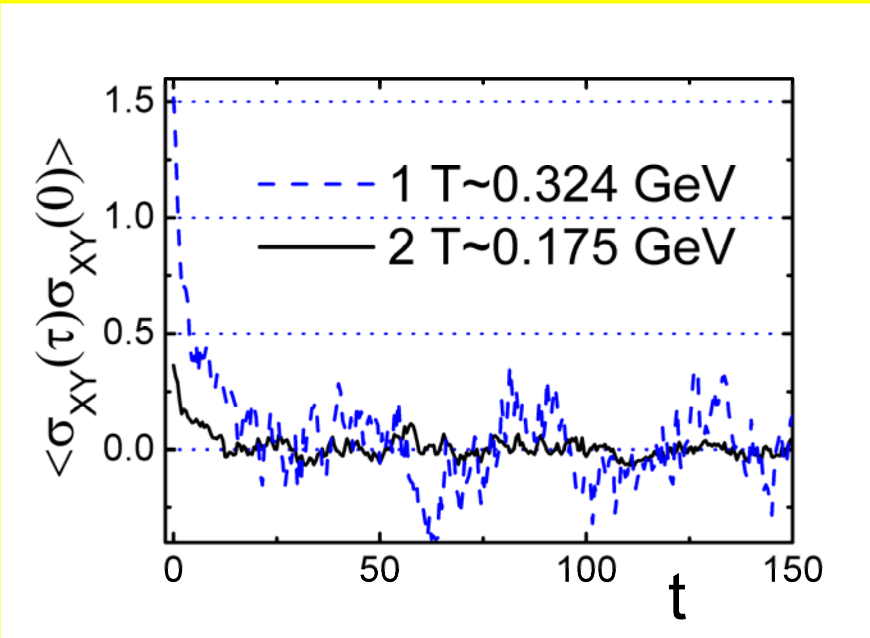
$$D = \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \int_0^t d\tau D(\tau)$$

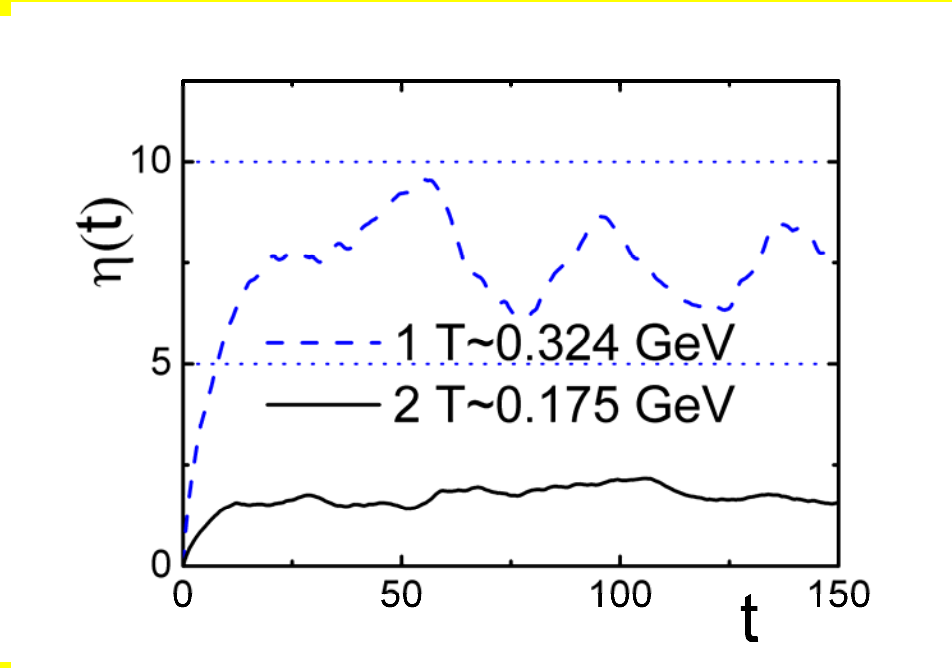
$$D(\tau) =< v(\tau/2)v(-\tau/2) >=$$

$$= \frac{1}{3N} < \sum_{i=1}^{N} \vec{v}_i(\tau/2) \bullet \vec{v}_i(-\tau/2) >$$



Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

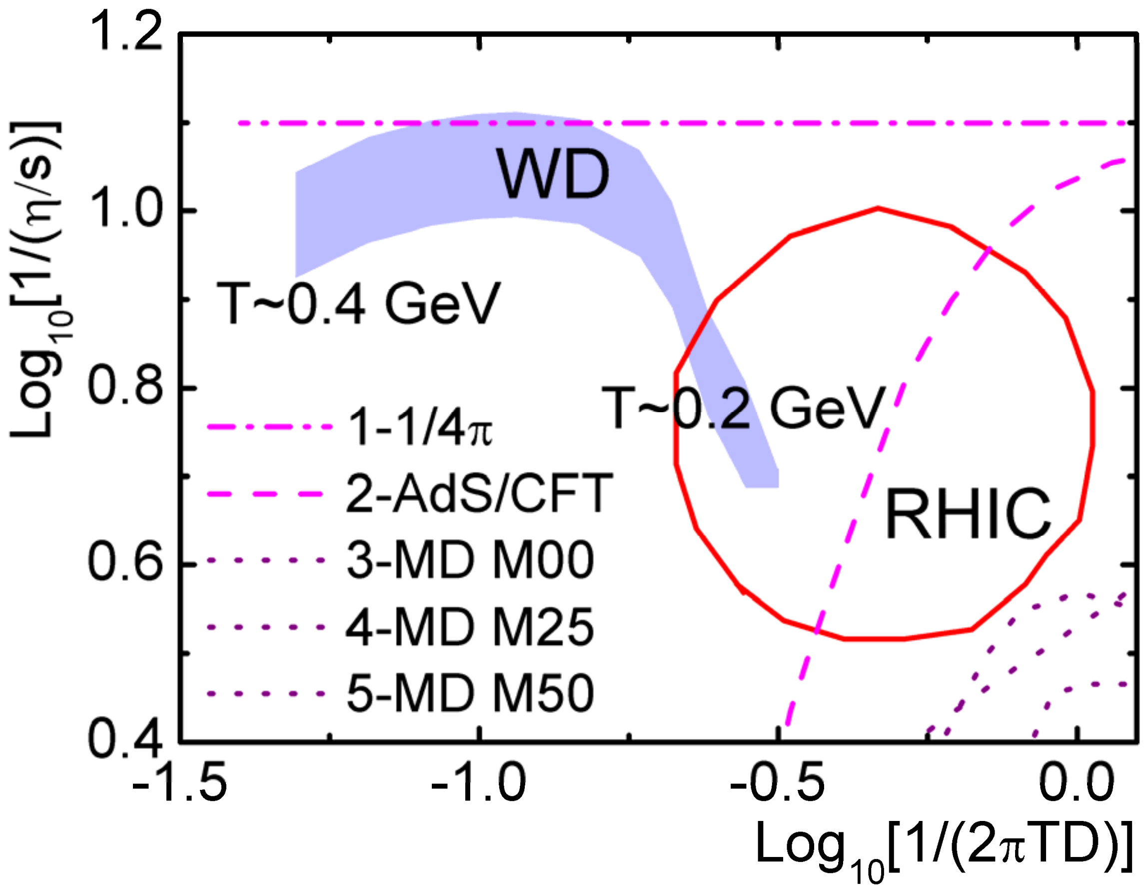




$$\eta = \lim_{t\to\infty} \int_0^t \eta(\tau)d\tau, \eta(\tau) = \frac{n}{3k_BT}\left\langle \sum_{X<Y} \sigma_{XY}(\tau/2)\sigma_{XY}(-\tau/2) \right\rangle$$

$$\sigma_{XY}(\tau) = \frac{1}{N}\left(\sum_{i=1}^{N} p_{ix}p_{iy} / \sqrt{p_i^2 + m_i^2} + \frac{1}{2}\sum_{i\neq j} r_{ij,x}F_{ij,y} \right)$$

Diffusion coefficient and shear viscosity



CONCLUSIONS

• Path integral Monte Carlo is a reliable and very fast method of simulation thermodynamic properties in a wide range of plasma parameters

•Results of simulations agree with available theoretical and experimental data.

•Combination of path integral MC with Wigner and Wong dynamics can be applied to treatment transport properties of QGP.

Thank you for attention.

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