Quest for the QCD phase diagram in extreme environments

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How "extreme" typically?

High Temperature

up to $T \sim \Lambda_{\mathrm{QCD}} \sim 200\mathrm{MeV}$

Relativistic Heavy-Ion Collision

High Baryon Density

up to $\rho_{\mathrm{B}} \sim (\Lambda_{\mathrm{QCD}})^3 \sim 1\mathrm{fm}^{-3}$

Relativistic Heavy-Ion Collision, Neutron Star

Strong Magnetic Field

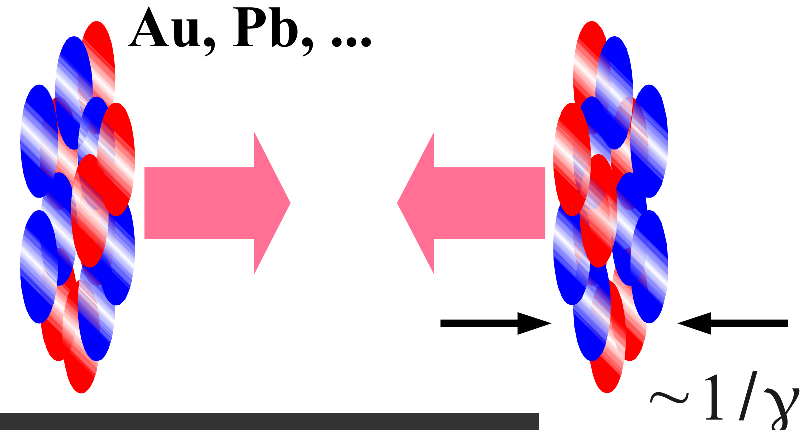
up to $eB \sim (\Lambda_{\mathrm{QCD}})^2 \sim 10^{18}$ gauss

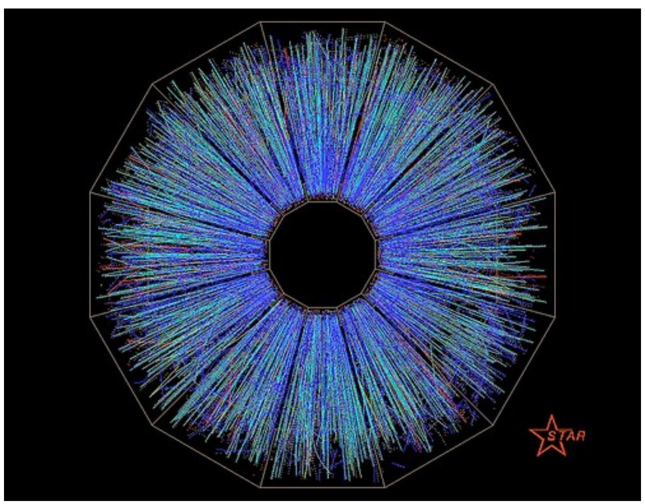
Relativistic Heavy-Ion Collision, Neutron Star

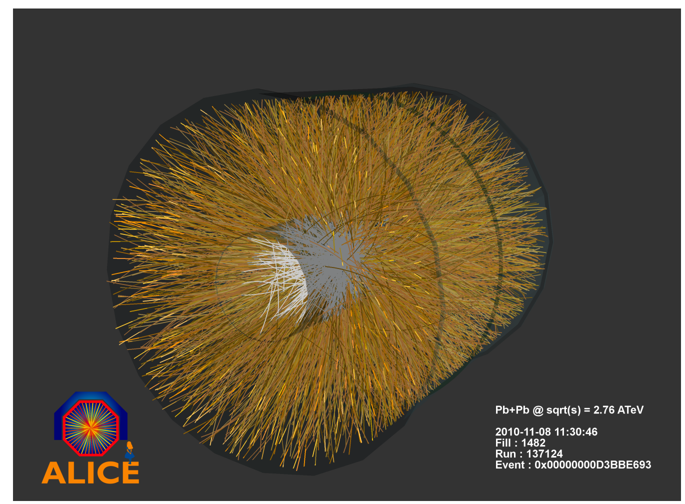
Relativistic Heavy-Ion Collision

LHC: $\sqrt{s_{NN}} = 2.7\,\mathrm{TeV} \;\; \rightarrow \gamma \sim 1400$

RHIC: $\sqrt{s_{NN}} = 200\,\mathrm{GeV} \rightarrow \gamma \sim 100$



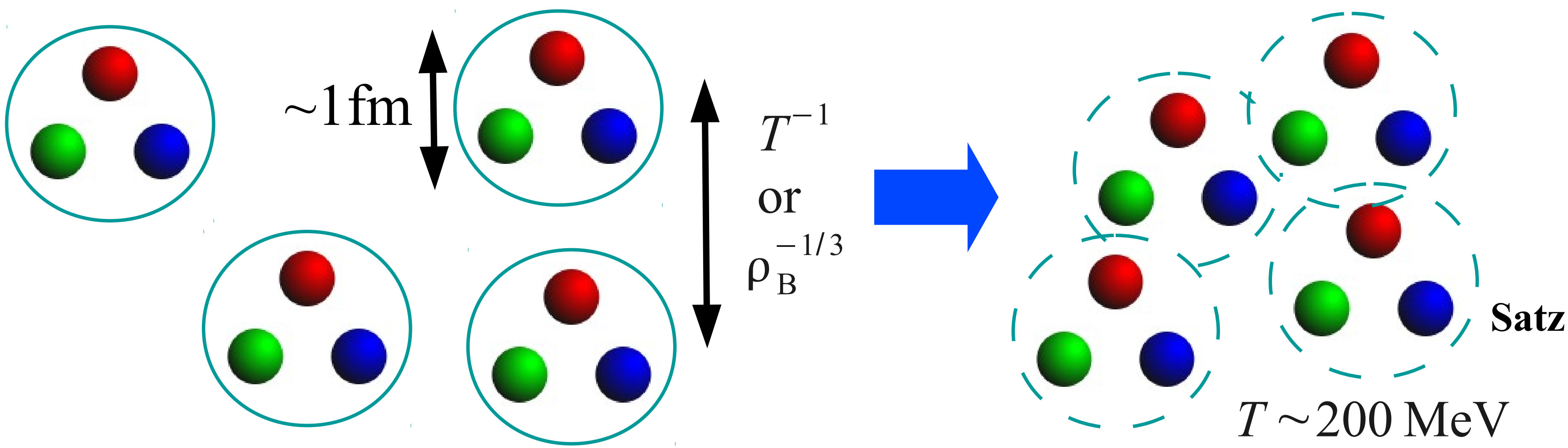




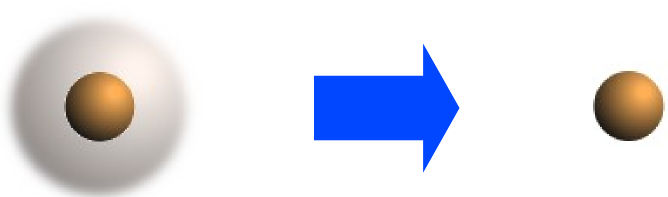
Thermalization achieved (elliptic flow by a hydro-model)
Initial temperature ~ 400MeV (distribution of direct photon)

Two Major Phase Transitions in QCD

Quark Deconfinement Transition (Polyakov Loop)



Chiral Phase Transition (Chiral Condensate)



"Bare" Quarks
$m_q \sim$ 3-6 MeV

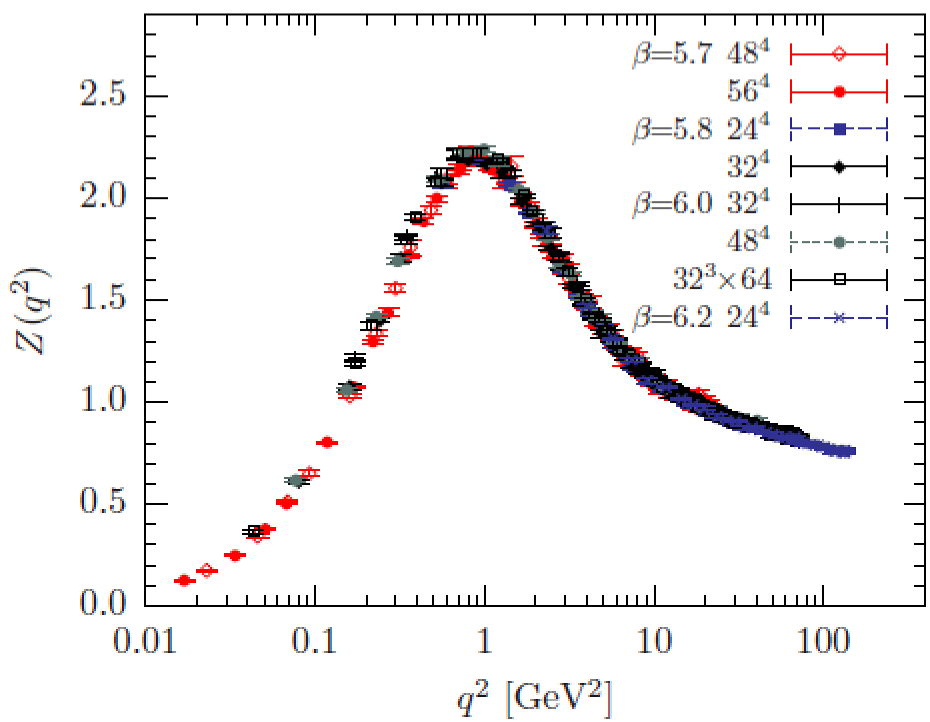
"Constituent" Quarks
$M_q \sim 350$ MeV

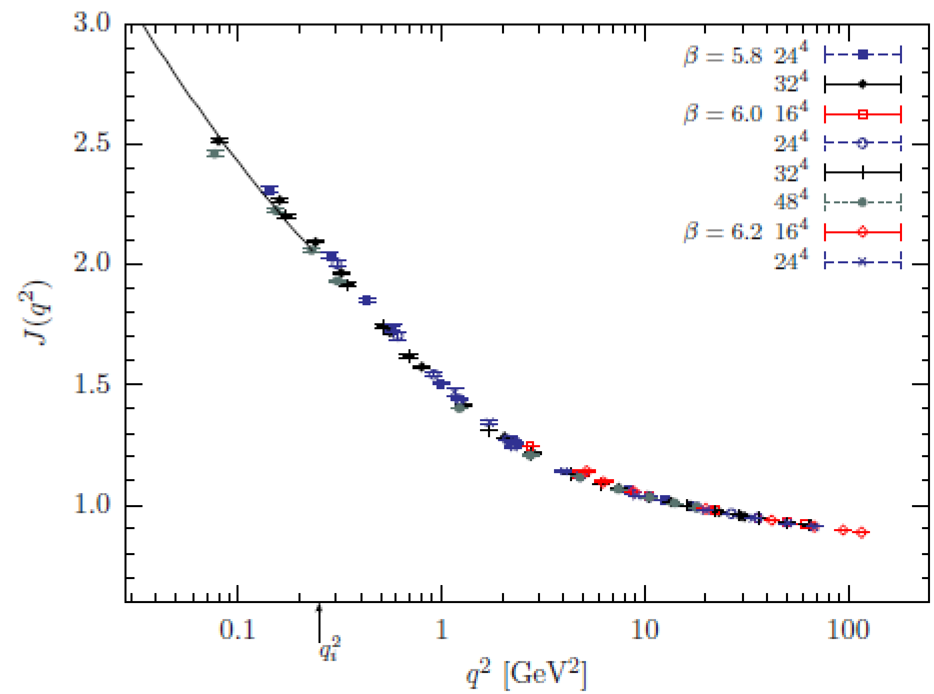
Nambu-Jona-Lasinio

How to understand deconfinement?

Confinement understood from the non-perturbative propagators of gluons and ghosts in the Landau gauge

Ilgenfrits-Muller-Preussker-Sternbeck-Schiller-Bogolubsky (2007)





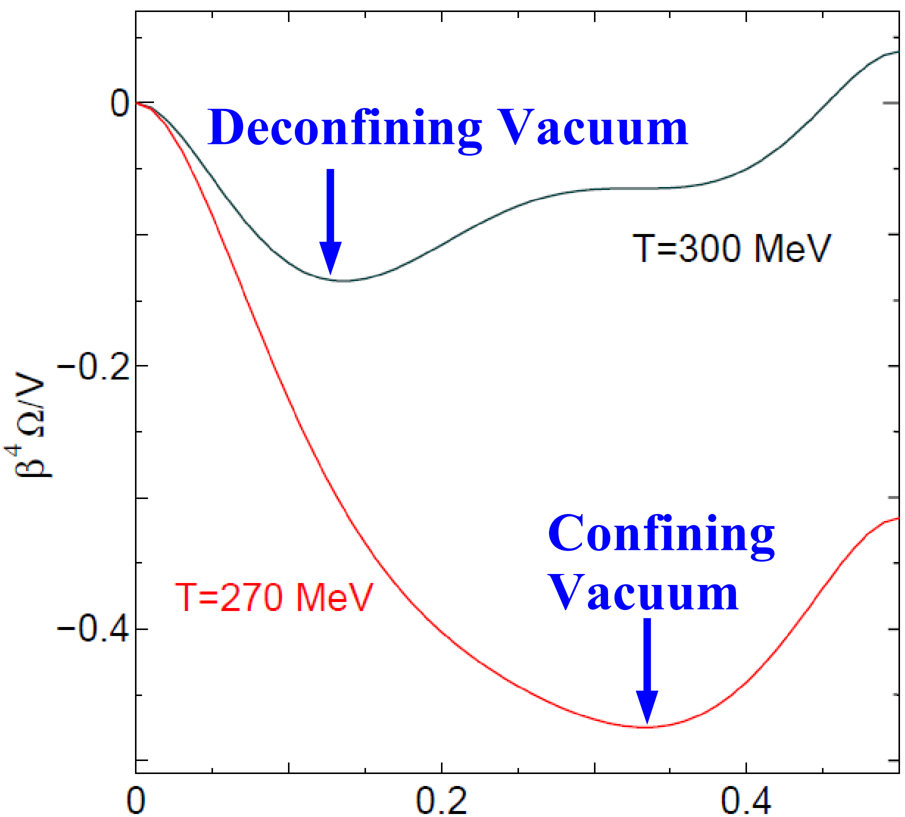
Gluons IR suppressed

Ghosts IR enhanced (confinement)

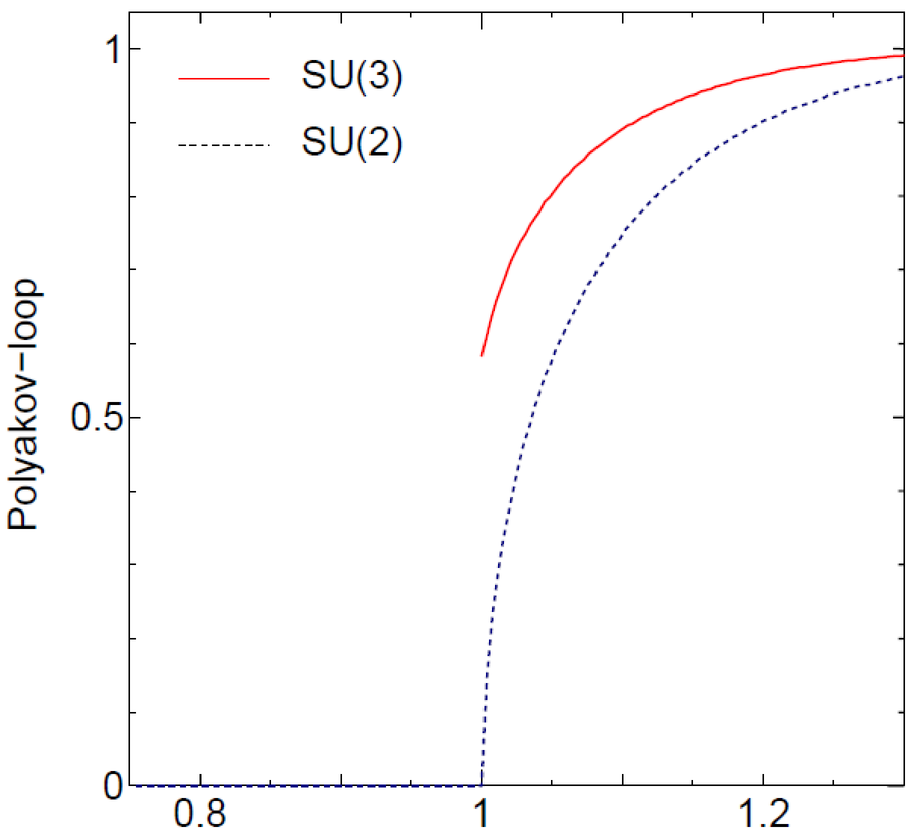
Gribov's scenario

Deconfinement from Propagators

$$\ln Z = -\frac{1}{2}\,\mathrm{tr}\,\ln D_A^{-1}(A_4) + \mathrm{tr}\,\ln D_C^{-1}(A_4) + \cdots$$



Phase of the Polyakov Loop ~ A_4



Temperature / T_c

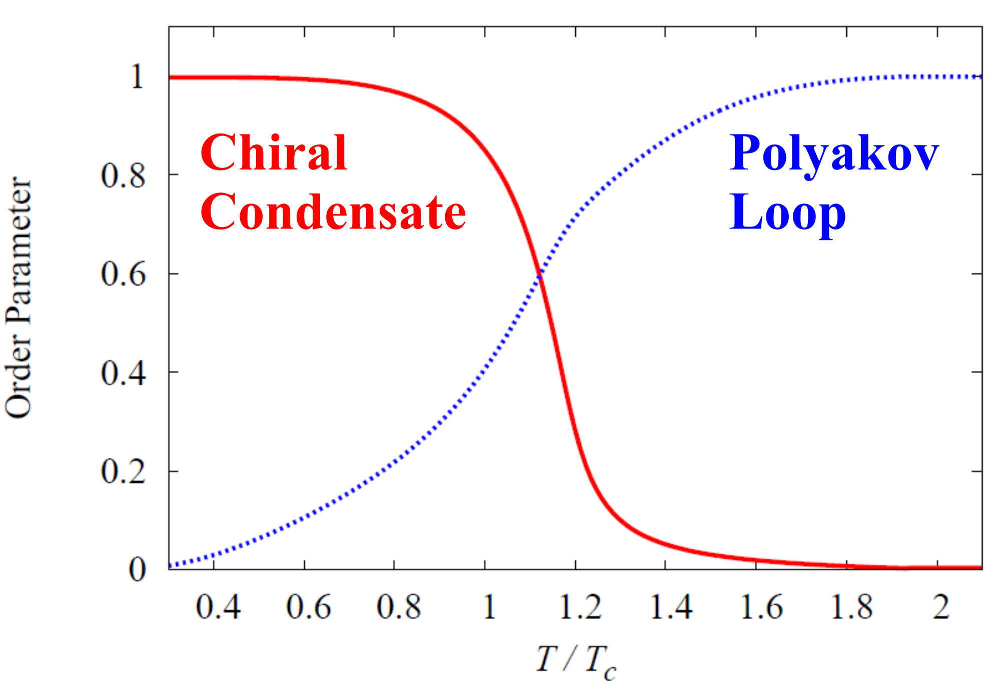
Confinement → Deconfinement

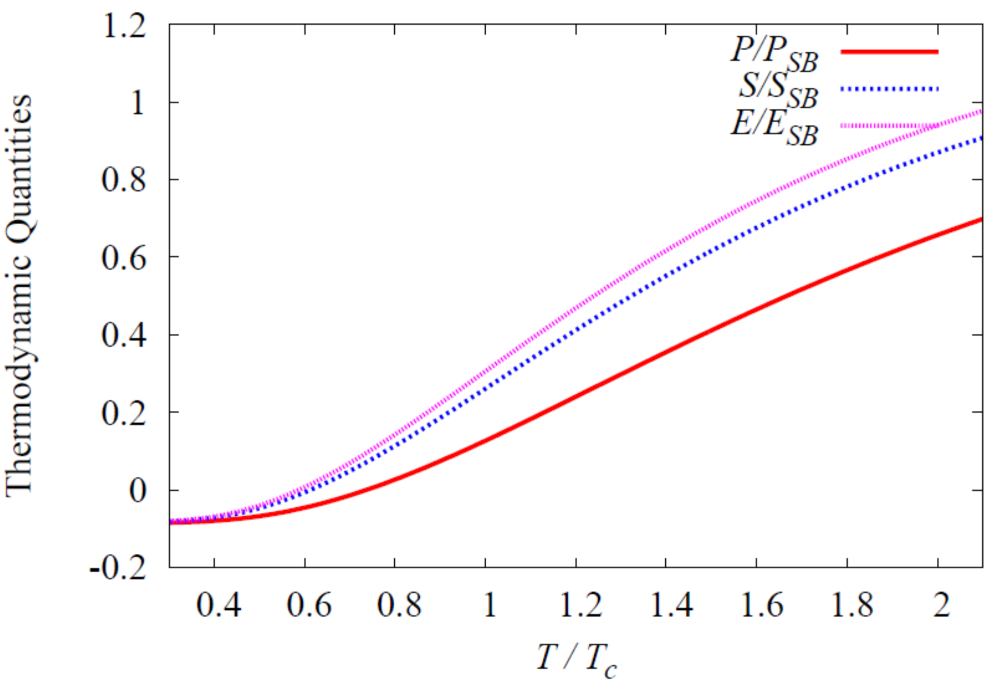
Braun-Gies-Pawlowski, KF-Kashiwa

Coupled with Dynamical Quarks

$$\ln Z_F = \mathrm{tr}\ln\left(1+e^{-(E-\mu)/T}\right) + \mathrm{tr}\ln\left(1+e^{-(E+\mu)/T}\right)$$

$$\rightarrow \mathrm{tr}\ln\left(1+L\,e^{-(E-\mu)/T}\right) + \mathrm{tr}\ln\left(1+L^\dagger e^{-(E+\mu)/T}\right)$$



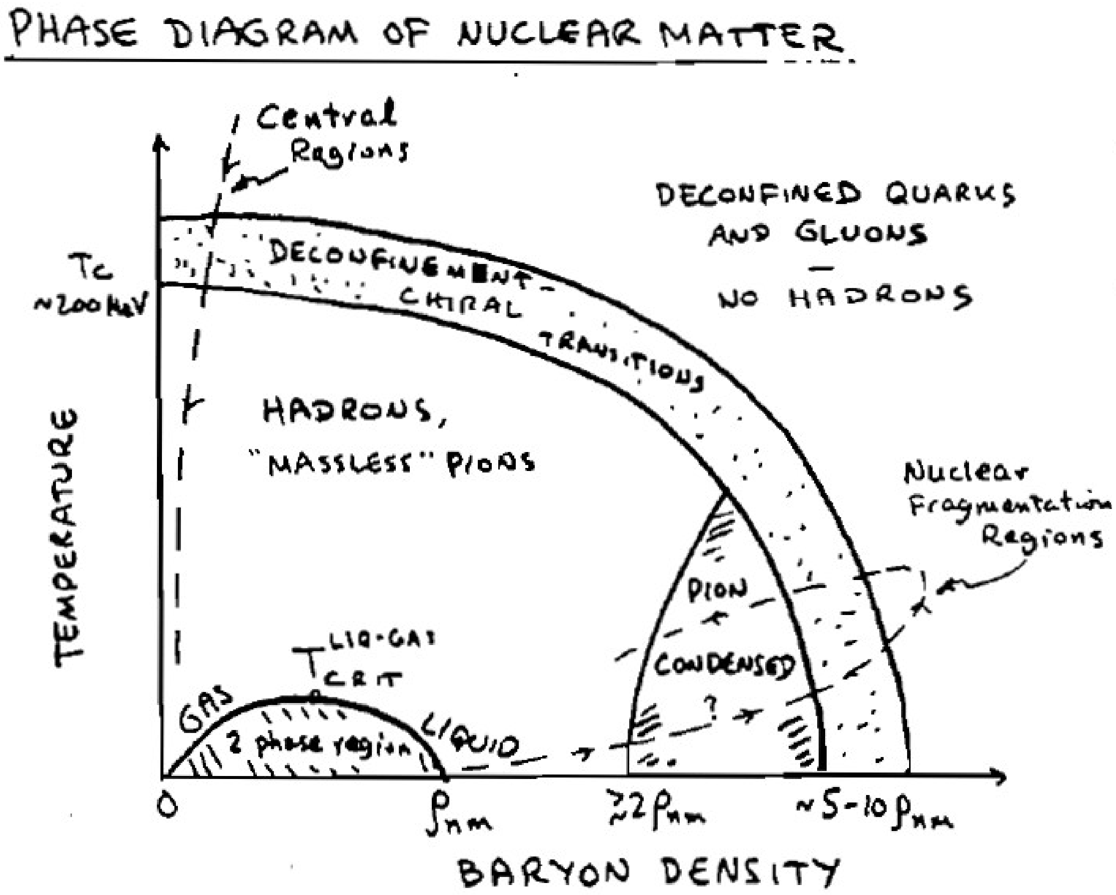


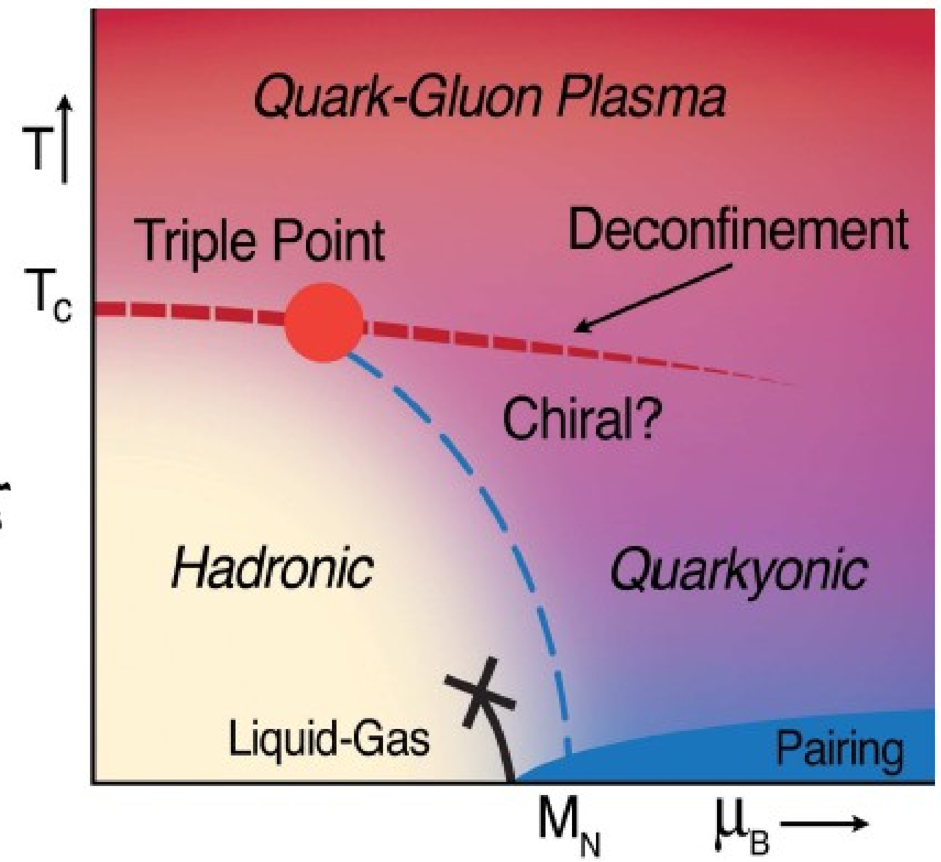
Polyakov-loop potential determined unambiguously

KF-Kashiwa (2012)

Phase Diagrams including Density

Prototype in 1983 **and** **Update in 2009**

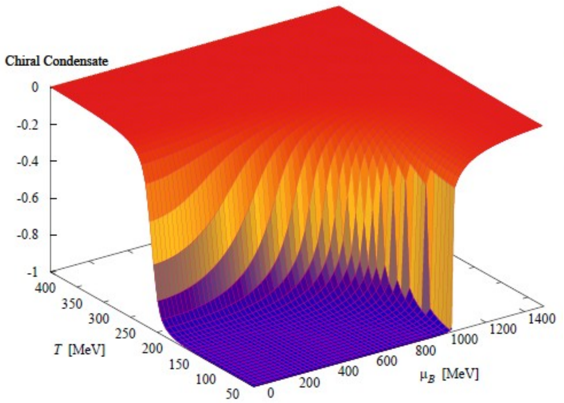




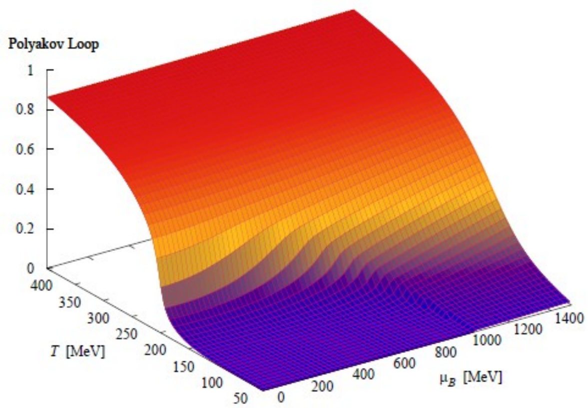
Modern View

Effective Model Results

Chiral Condensate

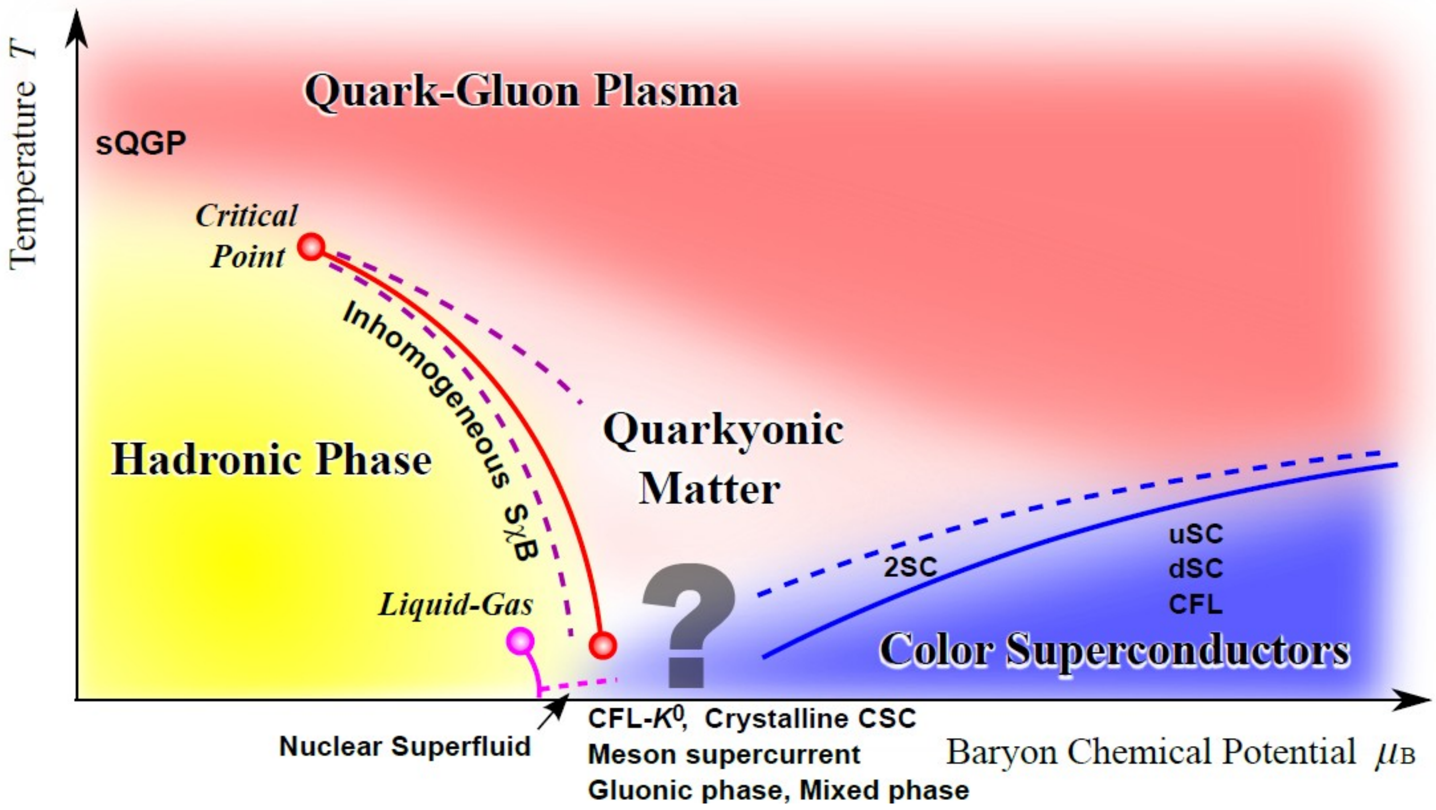


Polyakov Loop



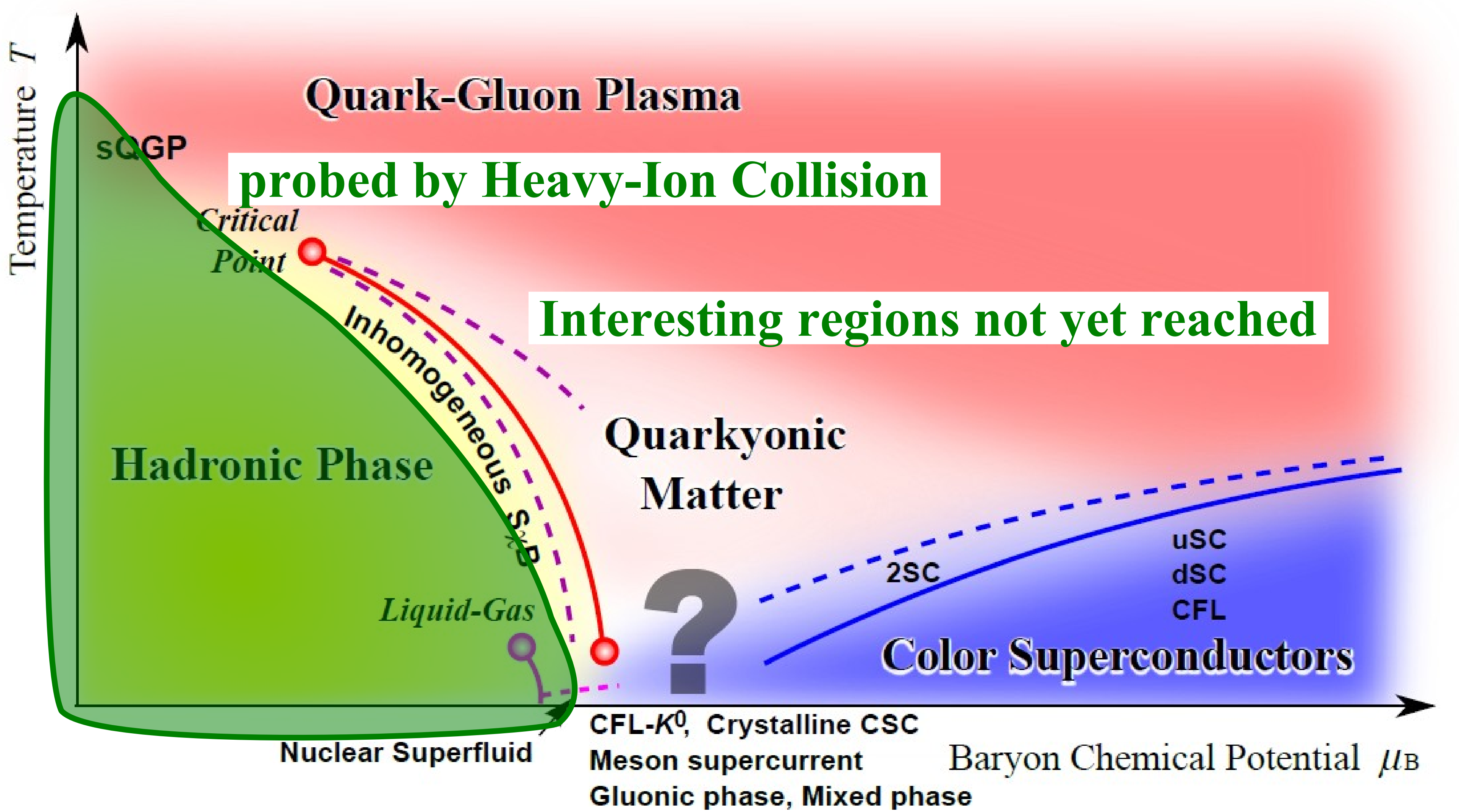
KF (2008)

Conjectured Phase Structure



KF-Hatsuda (2010)

Experimentally Confirmed

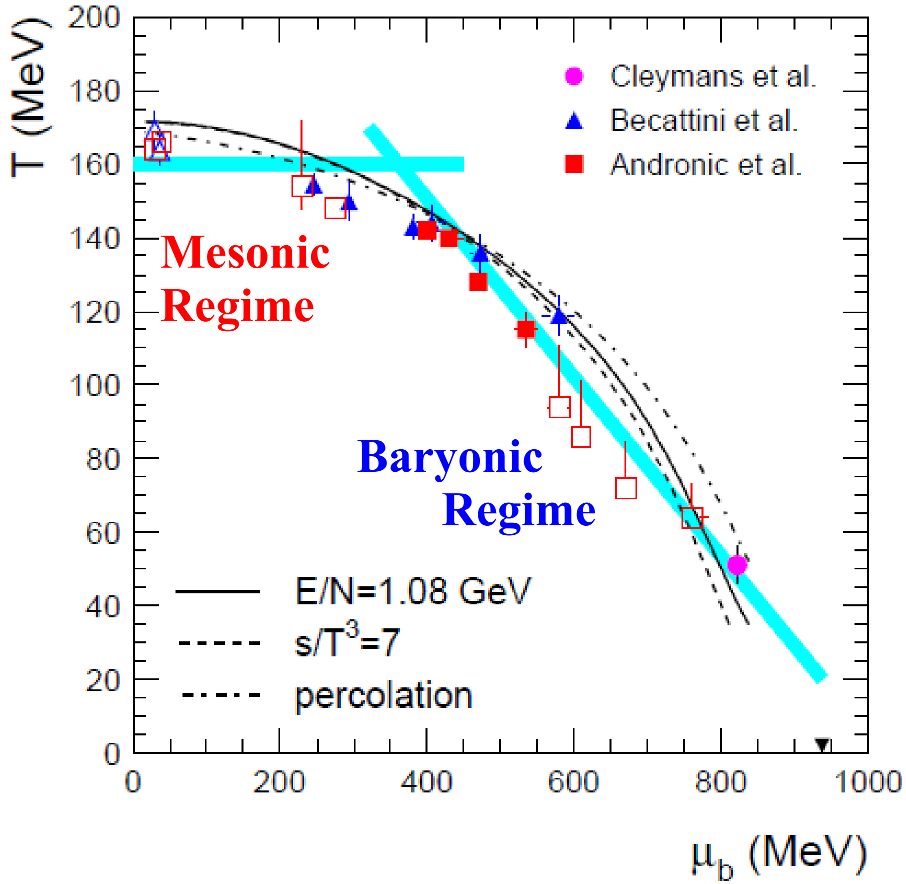


KF-Hatsuda (2010)

Experimental Data

Freeze-out points are located by the particle yields

Two regimes in **meson-dominance** and **baryon-dominance**



Mesonic Hagedorn Transition

$$Z \sim \int dm\, \rho(m)\, e^{-m/T}$$

$$\rho(m) \sim e^{m/T_H}$$

$$T_c = T_H$$

Baryonic Hagedorn Transition

$$Z \sim \int dm\, \rho_B(m)\, e^{-(m_B - \mu_B)/T}$$

$$\rho(m) \sim e^{m_B/T_B}$$

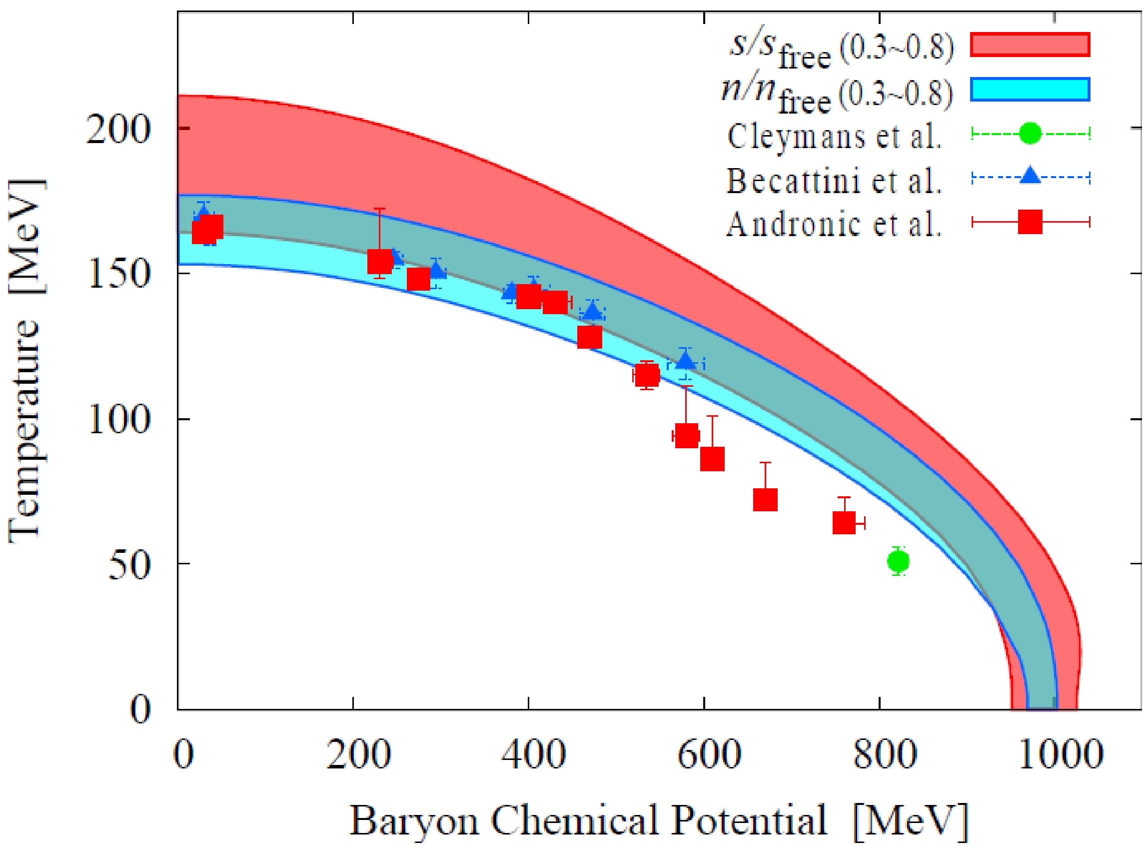
$$T_c = (1 - \mu_B/m_B)\, T_B$$

Andronic-Blaschke-Braun-Munzinger-Cleymans-KF -McLerran-Oeschler-Pisarski-Redlich-Sasaki (2010)

Thermodynamics

Statistical Model Interpretation

KF (2010)

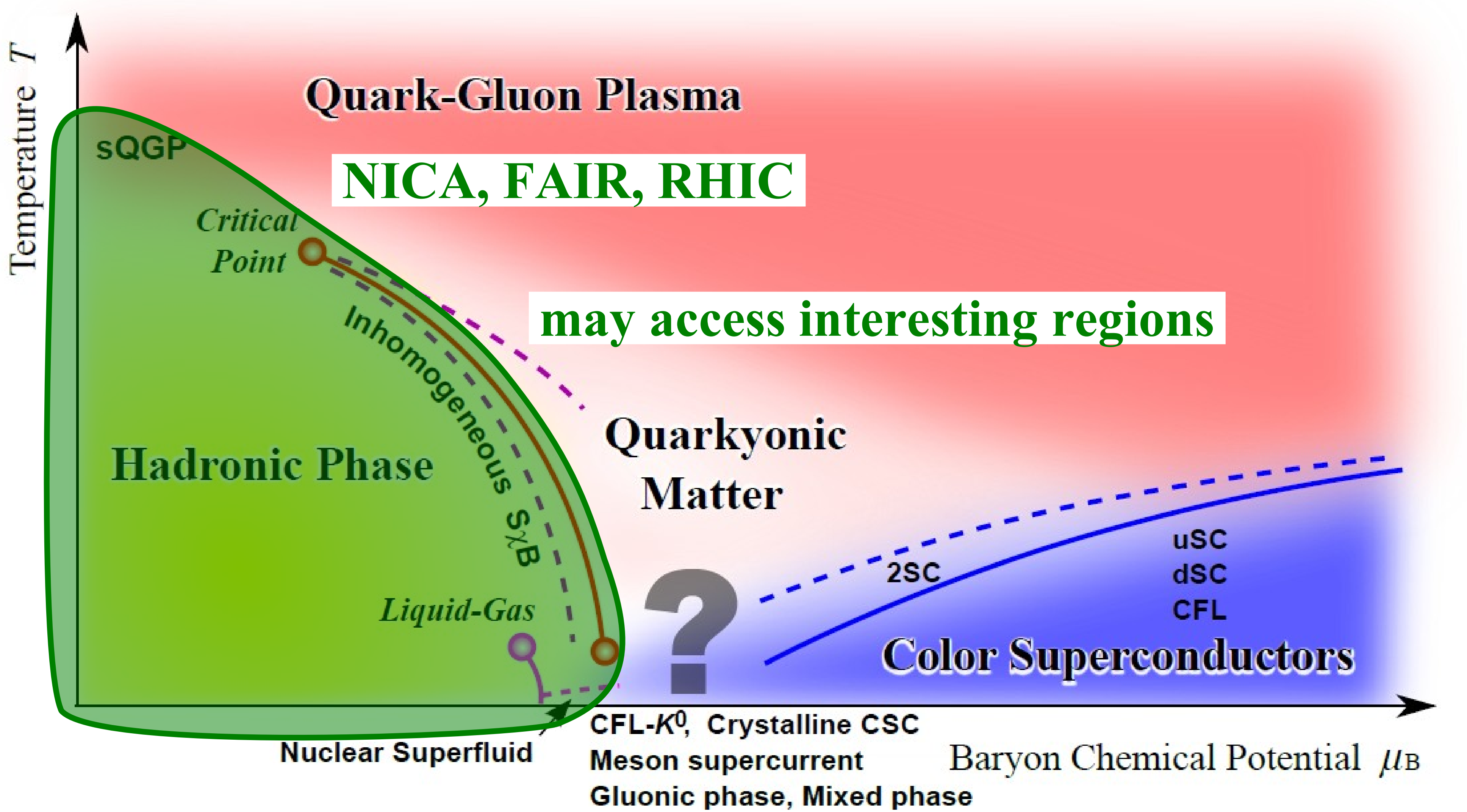


Gluon Deconfinement
~ Increasing entropy

Quark Deconfinement
~ Increasing density

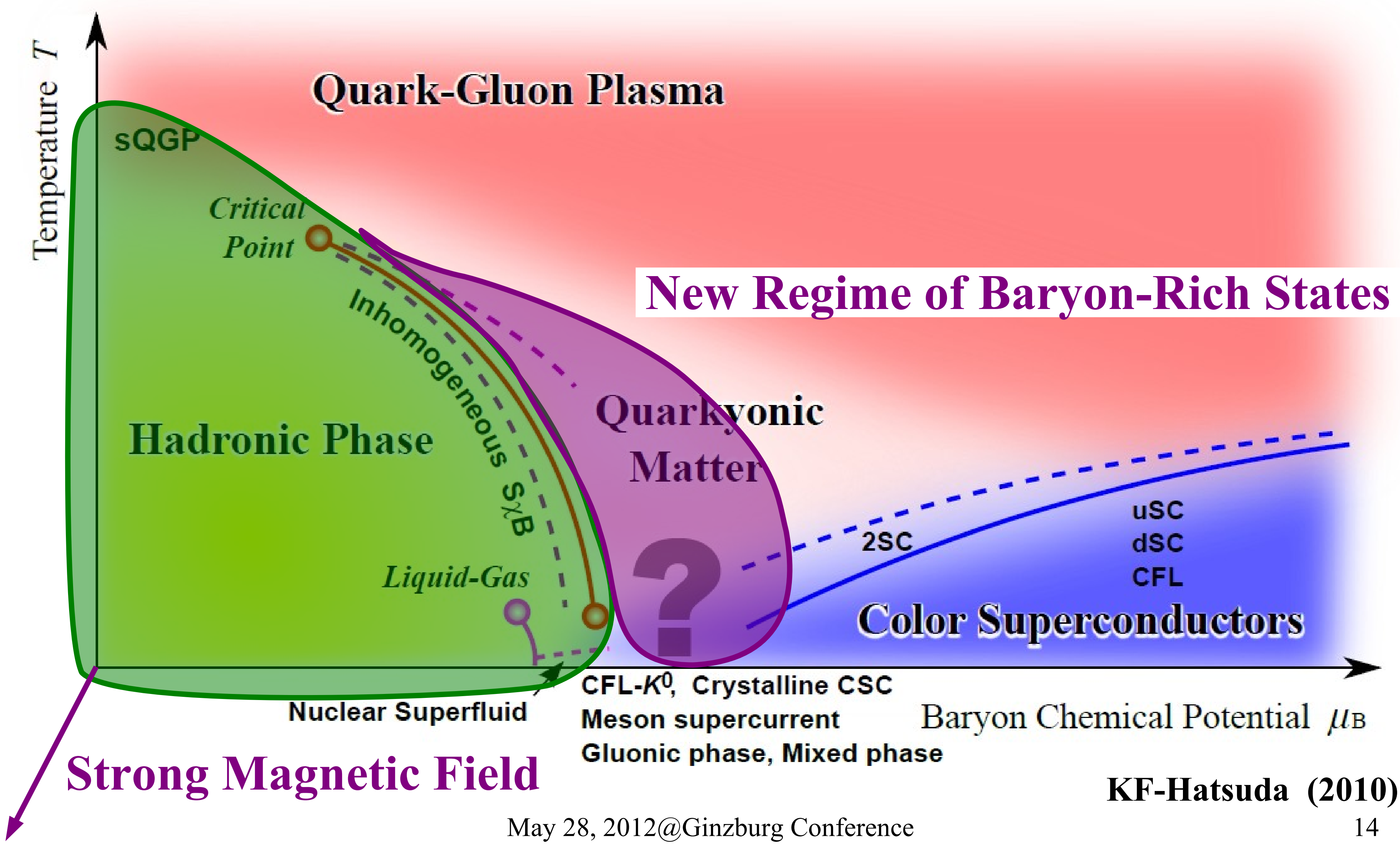
Thermodynamic quantities taken over by (quasi-)gluons and (quasi-)quarks (beyond the Hagedorn limit)

Experimentally Expected



KF-Hatsuda (2010)

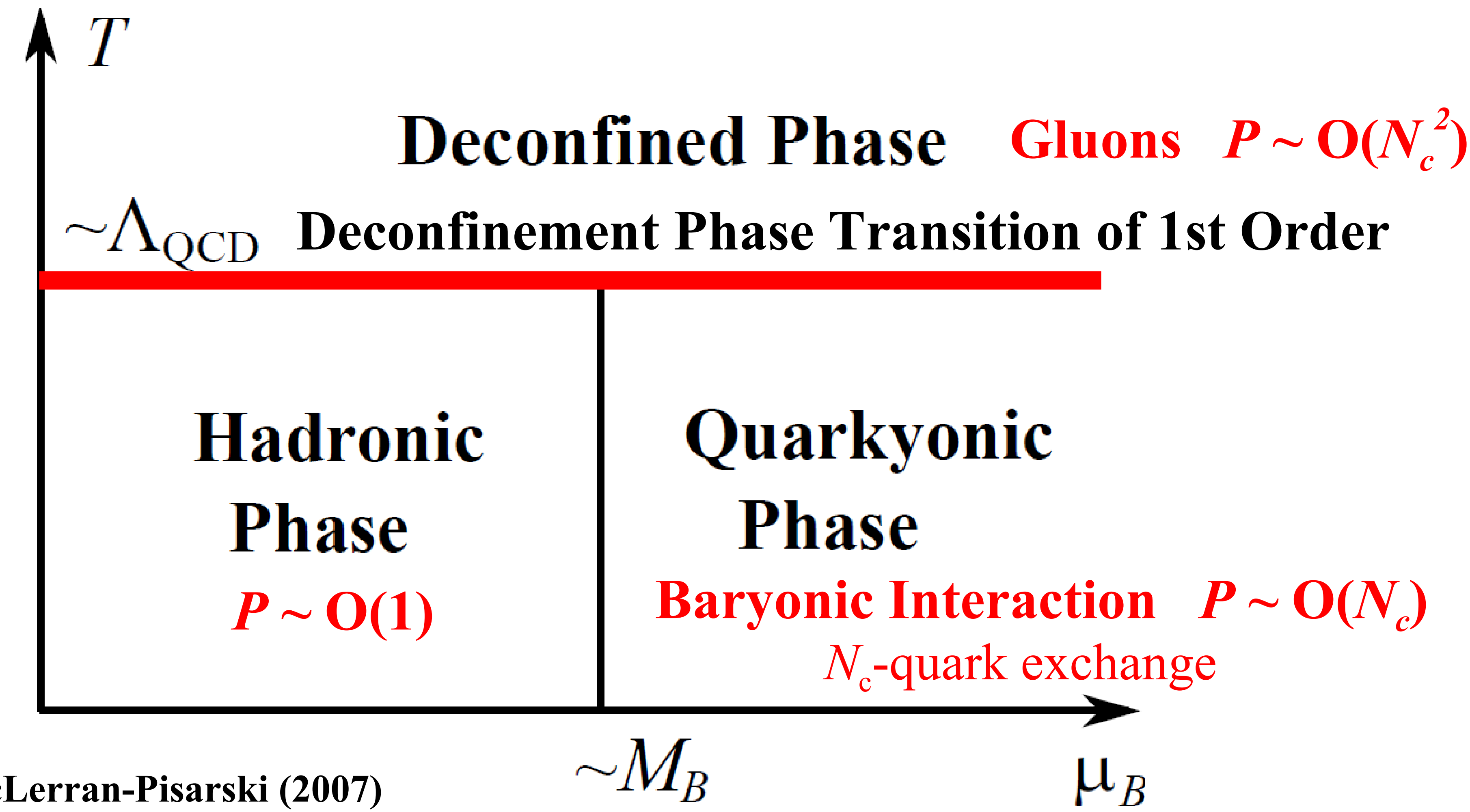
Theoretically Speculated



KF-Hatsuda (2010)

New Regime at Large μ_q and N_c

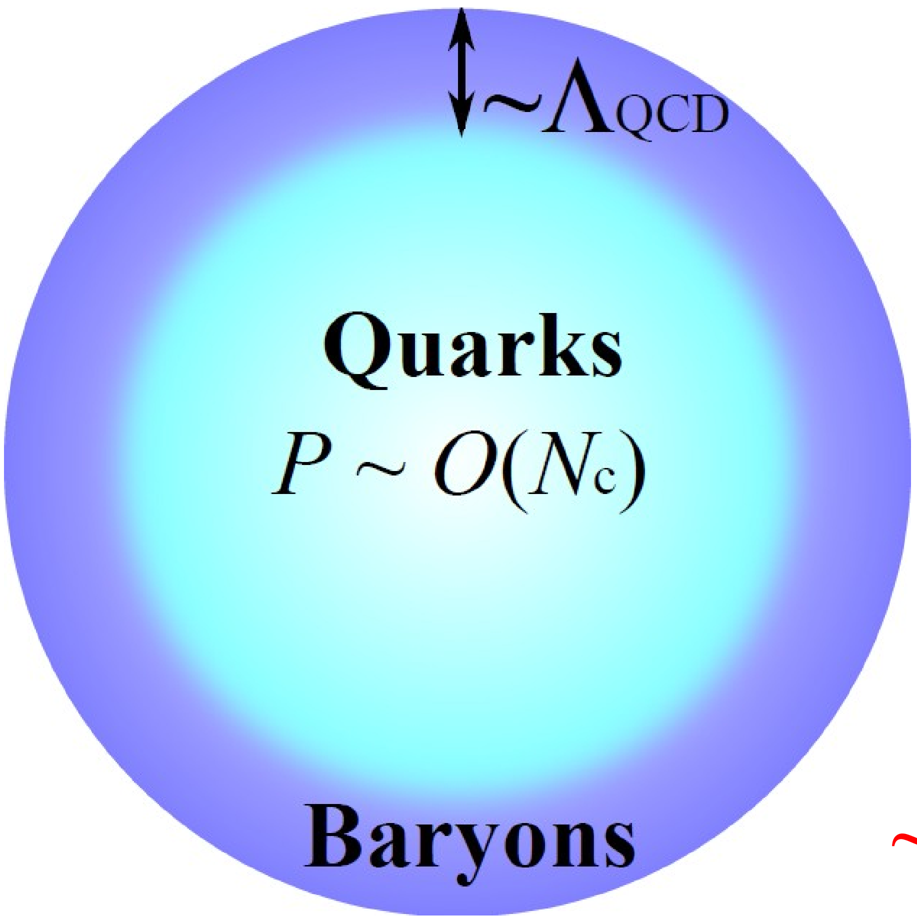
Phase Diagram of Large-N_c QCD



McLerran-Pisarski (2007)

Quarkyonic Matter

Structure of the Fermi Sphere



Ground state of large-N_c quark matter at $\mu_q >> \Lambda_{QCD}$

McLerran, Pisarski
Hidaka, Kojo

Interacting Baryon Crystal
~ Quasi-quark Gas

Quarkyonic Chiral Spiral $(\mu_q >> \Lambda_{QCD})$

Choose one direction z with $p_z \sim \mu_q$ ($p_x, p_y \sim \Lambda_{QCD}$)
(1+1)D system effectively

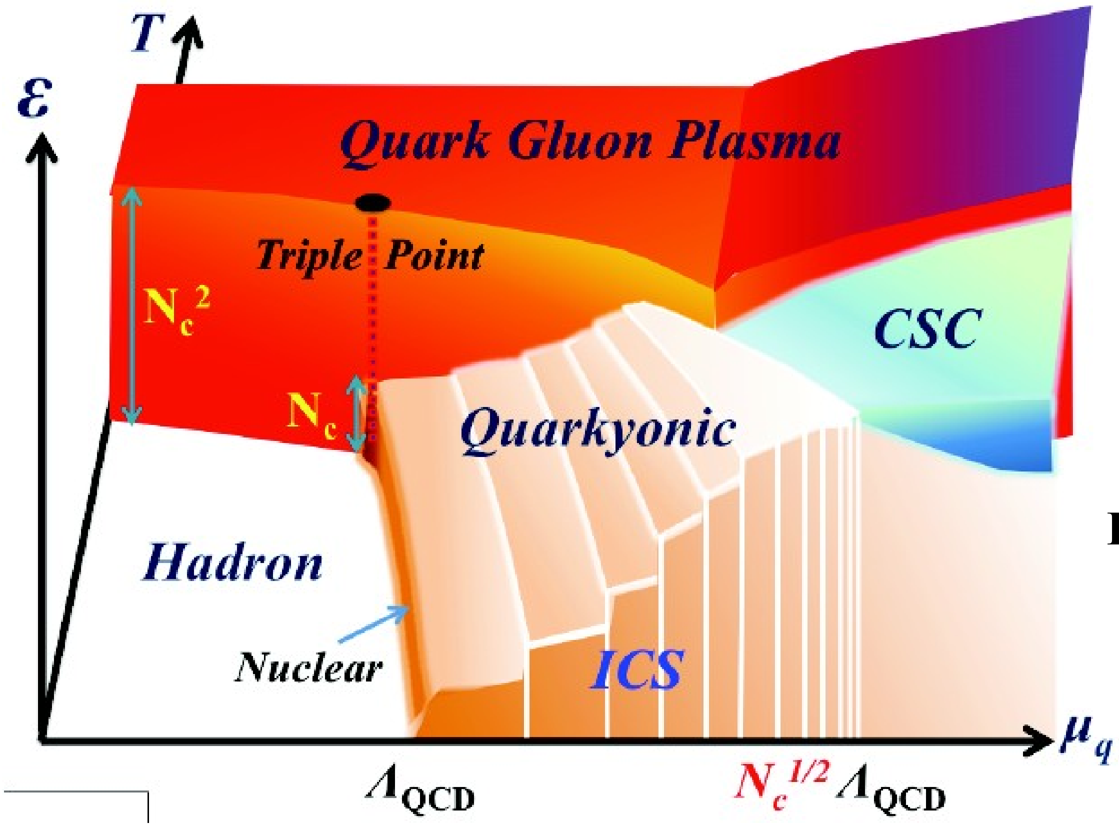
$$\bar{\psi}(i\gamma^z \partial_z + \mu\gamma^0)\psi$$
$$= \bar{\psi}'(i\gamma^z \partial_z)\psi' \qquad \psi = e^{i\gamma^0\gamma^z \mu z}\psi'$$

$\langle \bar{\psi}'\psi' \rangle =$ Homogeneous condensate at zero density

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}'\psi' \rangle \cos(2\mu z)$$
$$\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle = \langle \bar{\psi}'\psi' \rangle \sin(2\mu z)$$

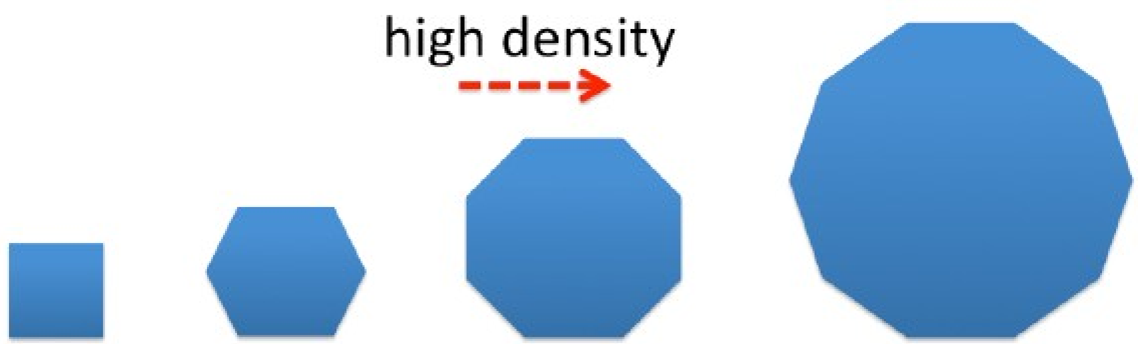
This quasi-(1+1)D system forms "one patch"

Interweaving Chiral Spirals

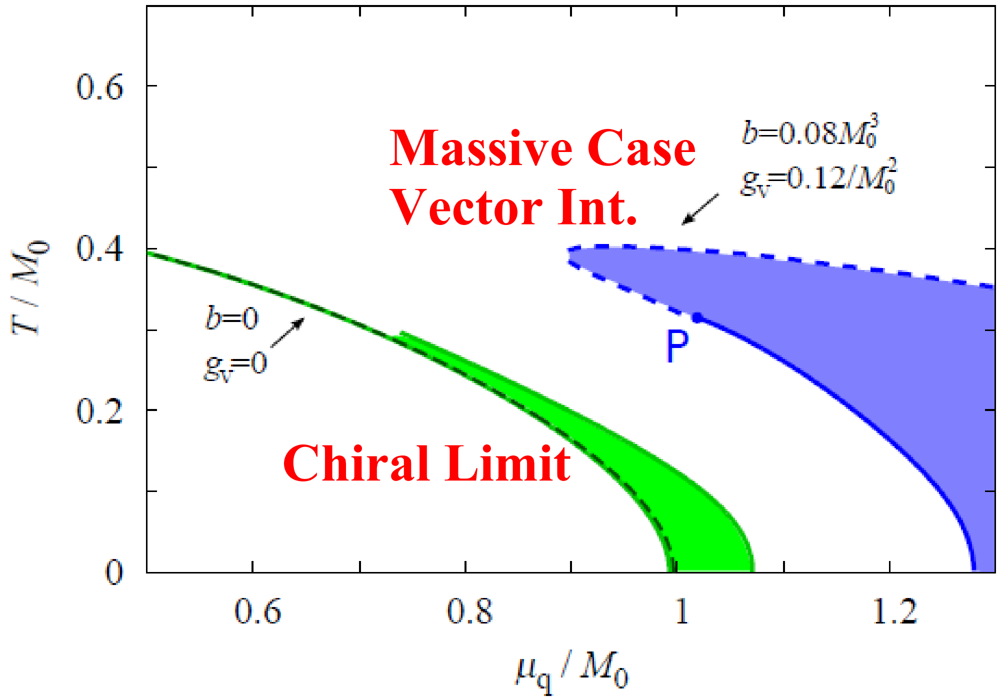


As the Fermi sphere enlarges, the patch number increases, forming a chiral quasi-crystal.

Kojo-Hidaka-KF-McLerran-Pisarski (2011)



Some Model Results



$$E_p = \sqrt{p_x^2 + p_y^2 + \left(\sqrt{p_z^2 + M^2} - q\right)^2}$$

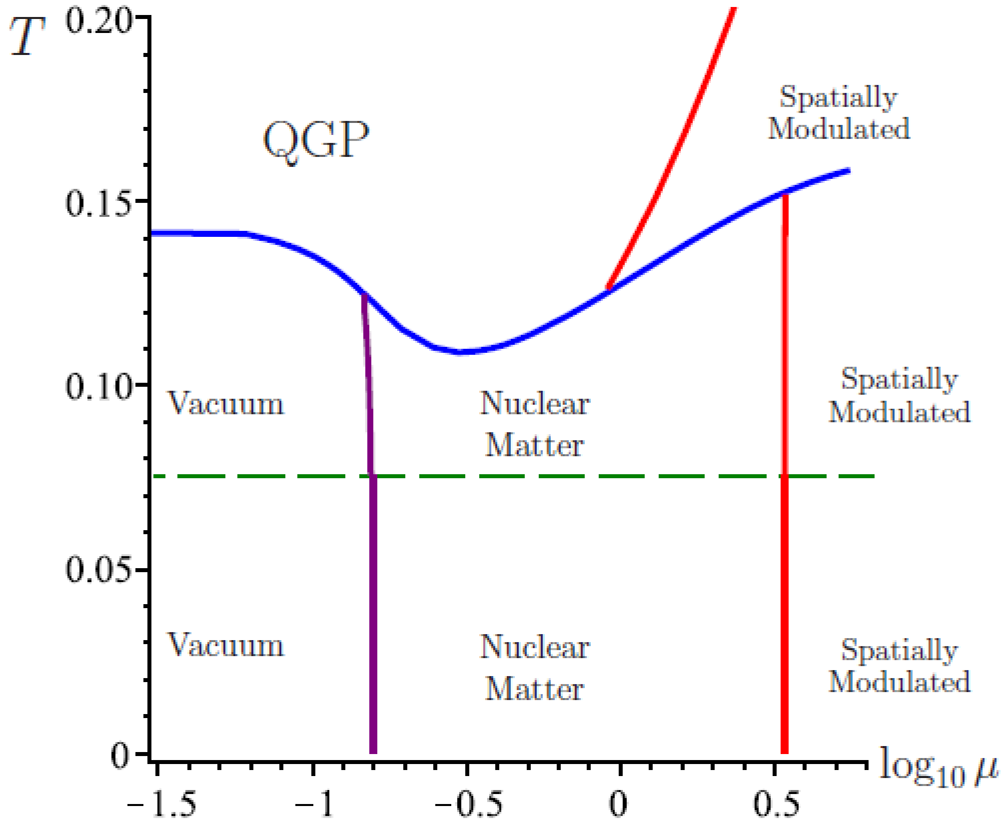
Effect of the dynamical mass M is partially canceled by q

Even when N_c and μ_q are not infinitely large, the chiral spiral is favored near the phase boundary of chiral symmetry

Nakano-Tatsumi (2003), KF (2012)

Holographic Evidence

State-of-the-art phase diagram in holographic model



Instability to inhomogeneous states is seen (in a different way from QCD...)

Nakamura-Ooguri-Park, Chuang-Dai-Kawamoto-Lin-Yeh (2010)

Density Effect ~ Magnetic Field Effect

Energy dispersion relation in *B*

$$\omega^2 = p_z^2 + \underline{2|eB|(n+1/2)} + m^2 - 2seB$$

Transverse motion = Harmonic Oscillator

Fermions (*s*=1/2) have zero mode – dominant at large *B*
Quasi-(1+1)D system is realized along the *B* direction.

Very strong *B* + Any μ_q → Chiral Spiral

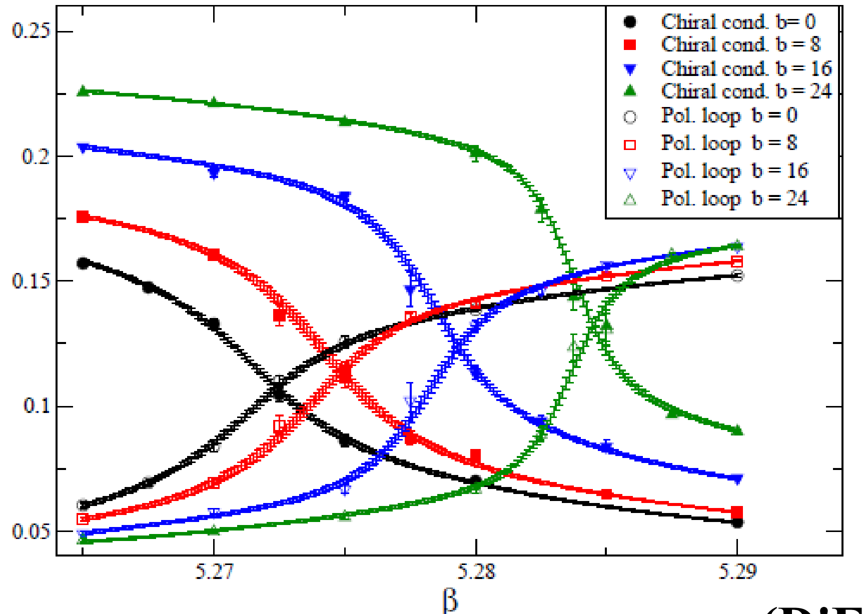
Basar-Dunne-Kharzeev

Very strong *B* + Attractive Int.
→ Cooper Instability → Magnetic Catalysis

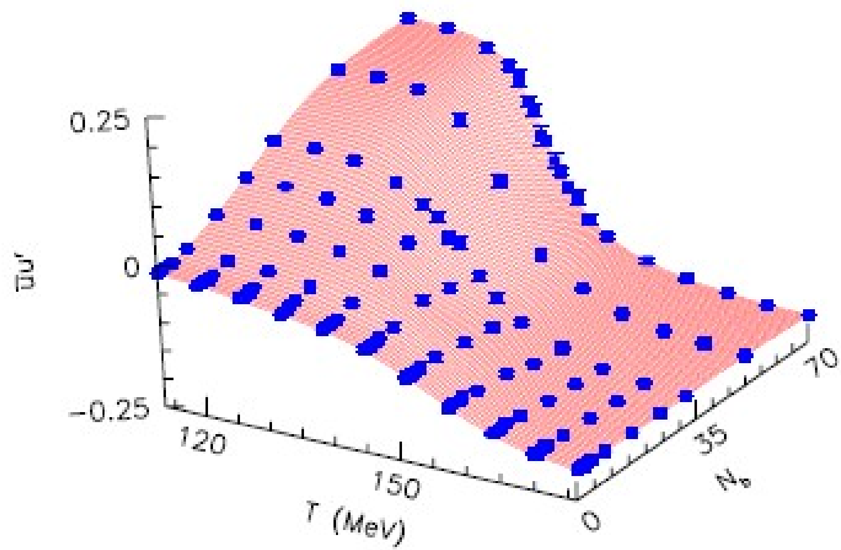
Klimenko, Gyusynin-Miransky-Shovkovy

B Effect on the Phase Diagram

QCD phase transitions affected by *B*



(D'Elia et al)



(Fodor et al)

Monte-Carlo simulation is possible (no sign problem)

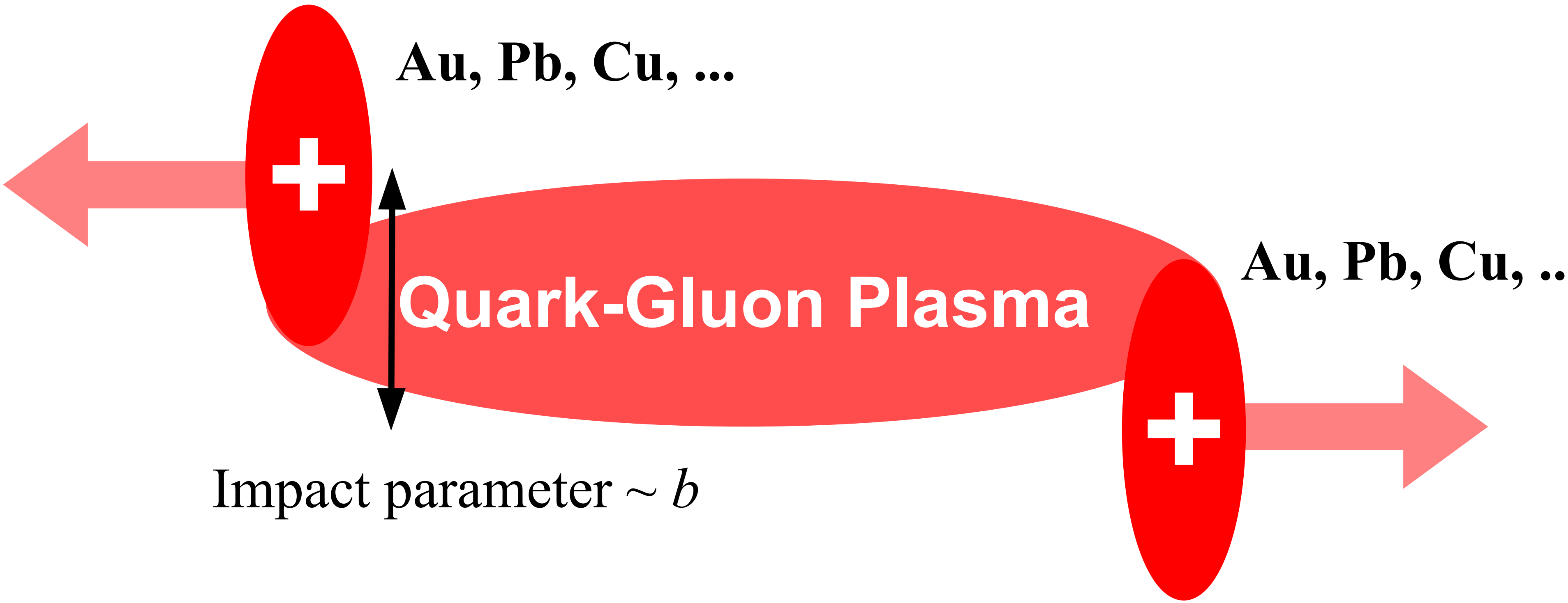
T_c increases or decreases?

Contradictory results from two groups!

Not Only Theoretical Analogue

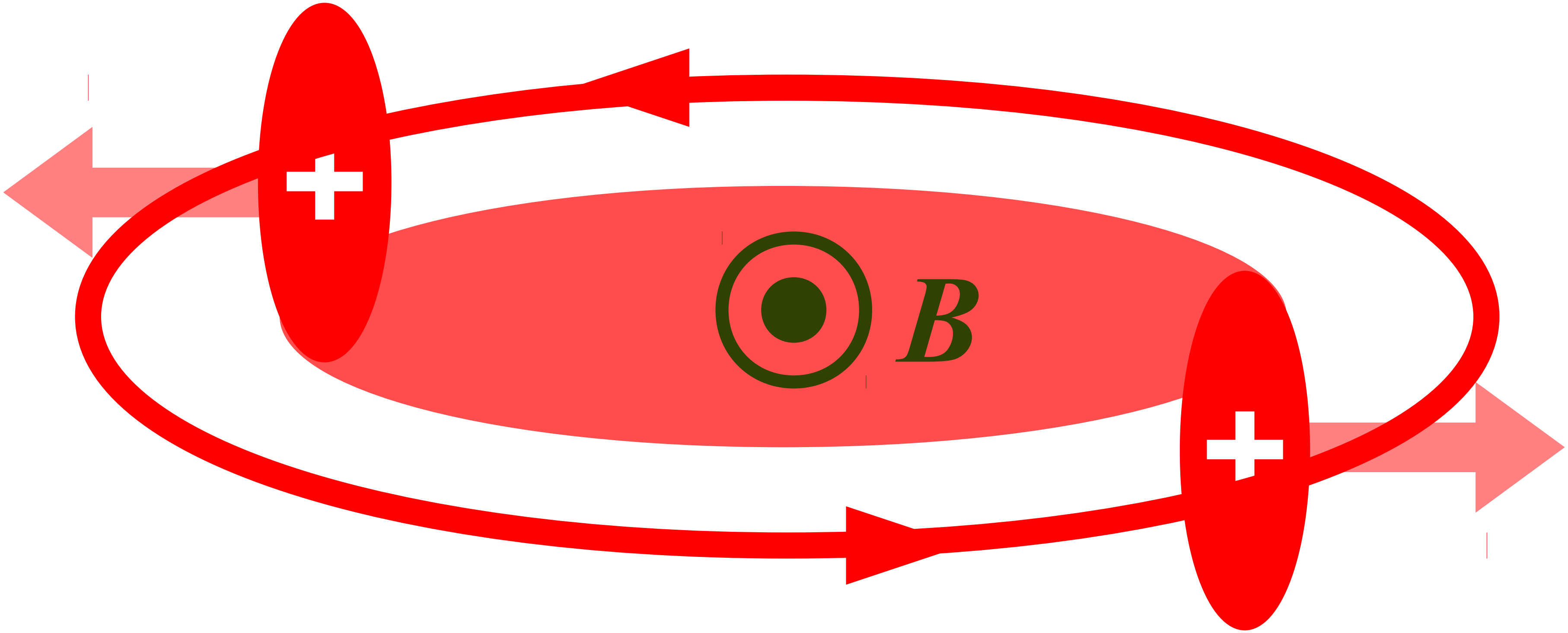
Relativistic Heavy-Ion Collision

Moving almost at the speed of light



Origin of the Magnetic Field

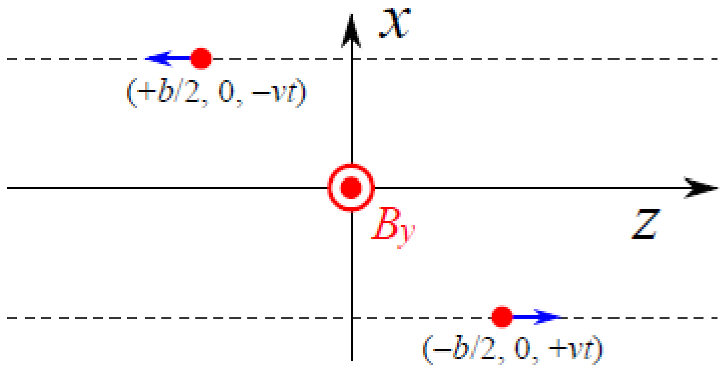
Strong B generated due to Electrodynamics

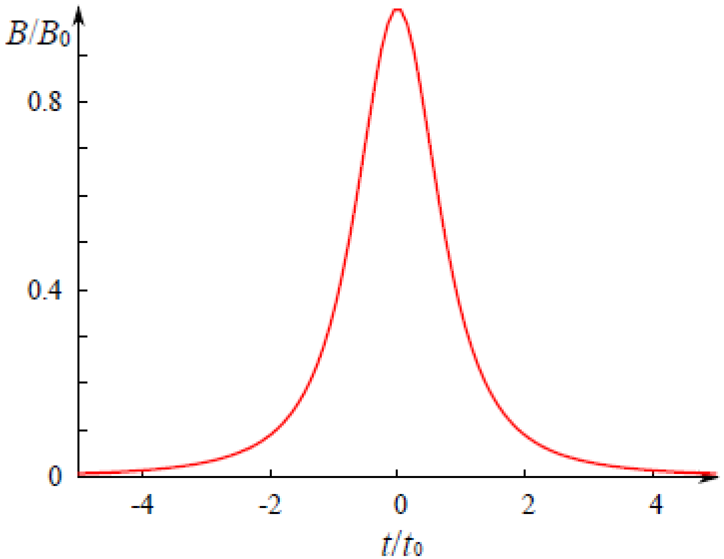


on top of the Quark-Gluon Plasma

Point-charge Approximation

Lienard-Wiechert potential





$$eB(t) = \frac{eB_0}{[1+(t/t_0)^2]^{3/2}}$$

$$eB_0 = (47.6\,\mathrm{MeV})^2 \left(\frac{1\,\mathrm{fm}}{b}\right)^2 Z \sinh(Y) \quad , \qquad t_0 = \frac{b}{2\sinh(Y)}$$

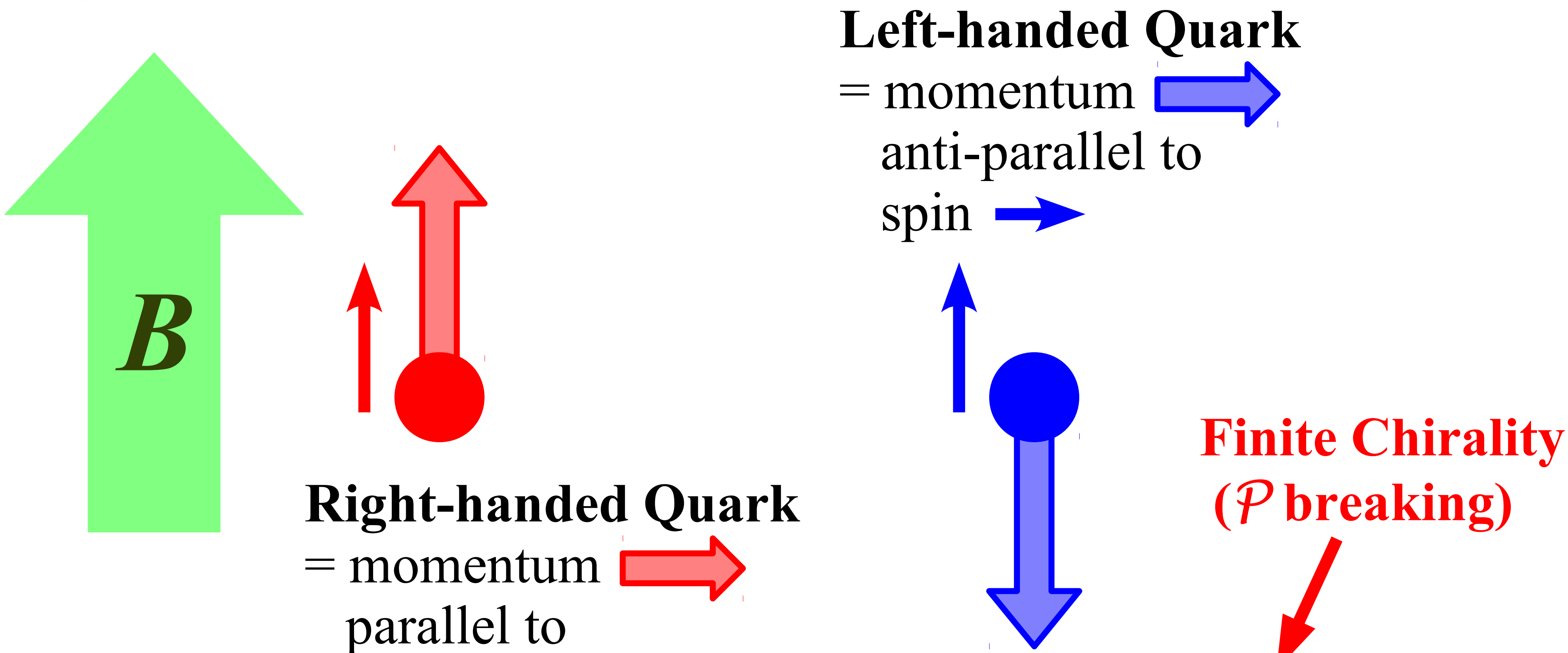
$\sim 10^{18}$ gauss

Interesting phenomena expected!

Discussed by Rafelski, Mueller, … (~1976)

Chiral Magnetic Effect

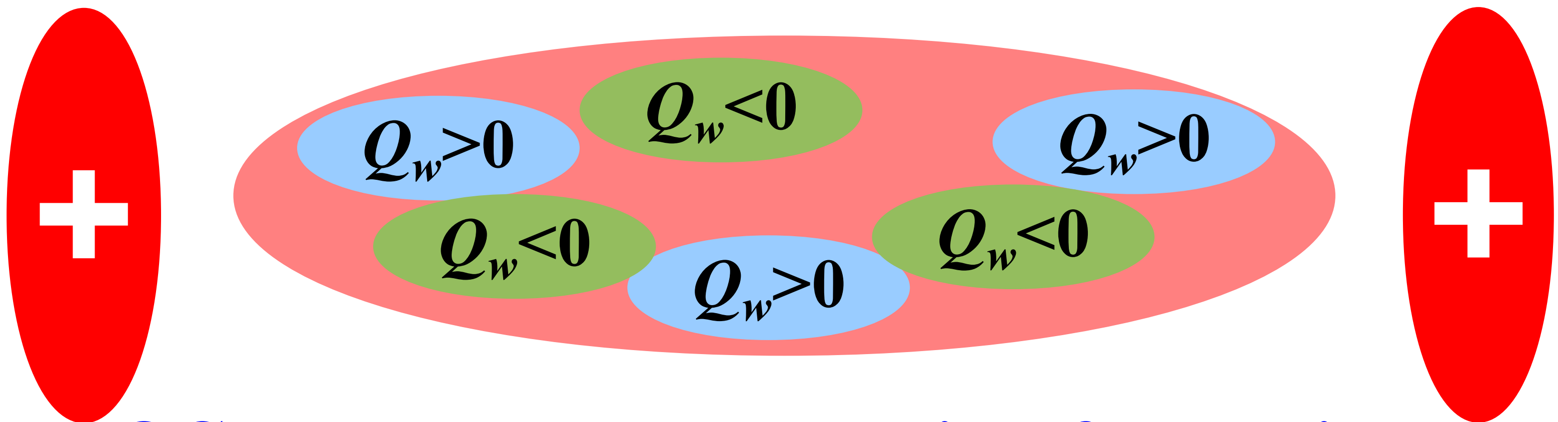
Classical Picture



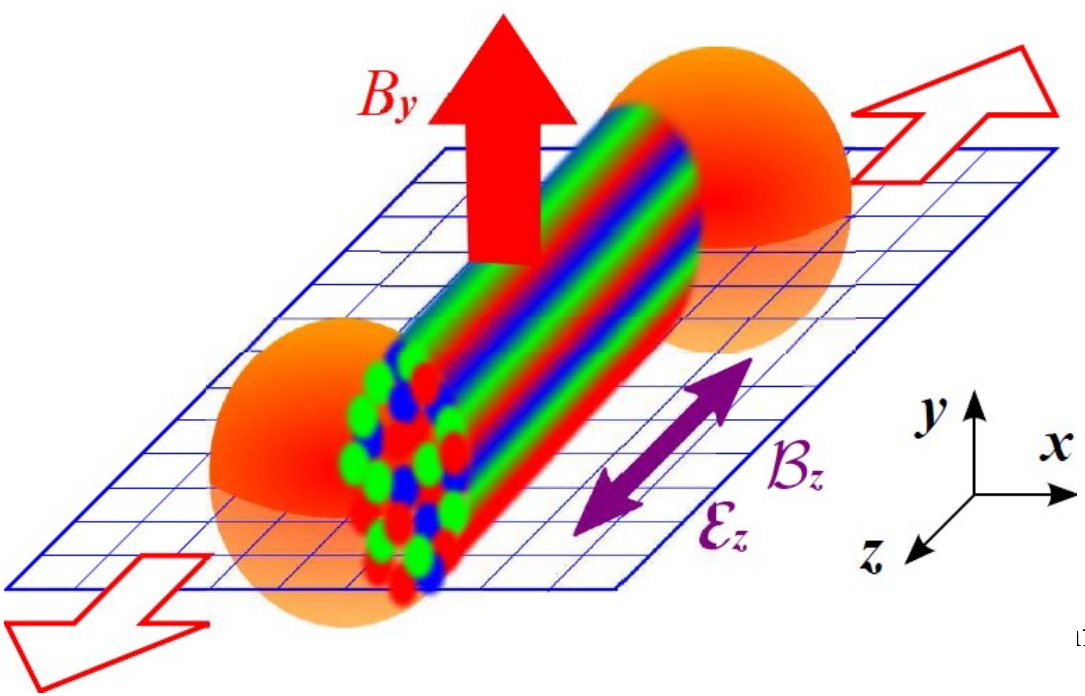
$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)

Local Parity Violation (LPV)



QCD vacuum has topological fluctuations



Initial condition of HIC (Color Glass Condensate) accommodates topological fluctuations

Current Generation through Anomaly

Chiral Magnetic Effect

$$\boldsymbol{j} = N_c \sum_{\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \boldsymbol{B}$$

Vilenkin (1980), Metlitski-Zhitnitsky, KF-Kharzeev-Warringa

Wess-Zumino-Witten Action

WZW term without *U* fields (contact term)

$$L_P = \frac{N_c}{8N_f\pi^2}\epsilon^{\mu\nu\rho\sigma}\left\{\mathrm{tr}\left[v_\mu\left(\partial_\nu v_\rho - \frac{i}{3}[v_\nu, v_\rho]\right)\right]\partial_\sigma\theta + \mathrm{tr}\left(a_\mu D_\nu a_\rho\right)\left[\frac{4}{3}\mathrm{tr}(a_\sigma) + \partial_\sigma\theta\right]\right\} - \frac{N_c}{12N_f^2\pi^2}\mathrm{tr}(a_\mu)\,\mathrm{tr}(\partial_\nu a_\rho)\partial_\sigma\theta$$

QED fields: $v_\mu = eQA_\mu = e\begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} A_\mu$

Kaiser-Leutwyler

Current Generation through WZW

$$\Gamma_p = \int \frac{N_c}{8 N_f \pi^2} \epsilon^{\mu\nu\sigma\rho} \mathrm{tr}[v_\mu \partial_\nu v_\rho] \partial_\sigma \theta + \cdots$$

$$j_z = \frac{d\,\Gamma_p}{d\,A_z} = \frac{N_c}{4 N_f \pi^2} \epsilon^{zxyt} \mathrm{tr}(Q^2) B_z \partial_t \theta$$

$$= N_c \sum_f \frac{q_f^2 B_z}{2\pi^2} \cdot \frac{\partial_t \theta}{2 N_f}$$

produces non-zero chirality

μ_5

Similar Effects

$$j_\mu \propto \epsilon_{\mu\nu\sigma\rho} \left(\partial^\nu \phi\right) F^{\sigma\rho}$$

Derivative of a pseudo-scalar quantity
η' condensate
pion condensates / profile
Strong θ angle

2nd-rank tensor
Field strength tensor
Angular momentum
Angular velocity

These effects under investigations in HIC

Summary

QCD phase diagram with chiral and deconfinement phase transitions is investigated:

- *High Temperature* – Phase transitions well understood from the zero-T properties of confinement.
- *High Baryon Density* – Inhomogeneous states favored near the phase boundary of homogeneous states.
- *Strong B Field* – Effects on the phase diagram not yet understood. Many interesting anomalous effects expected.

Experimental efforts focused on the baryon-rich matter and the visible effects of the strong *B*:

- Systematic fluctuation measurements to confirm the local parity violation / critical point / inhomogeneity