Chern-Simons vector models and higher spins

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Outline

- The Klebanov-Polyakov-Sezgin-Sundell conjectures:
  - HS gravity in AdS_4 $\leftrightarrow$ 3d vector models
- Vasiliev's higher spin gauge theory in 4d
  - the "Type A" and "Type B" models
  - Parity violating models
- Chern-Simons theory with vector fermion matter
  - Exact planar thermal free energy on R^2
  - Higher spin symmetry at large N and conjectural AdS dual
- Summary and conclusions

Based on work with S. Minwalla, S. Prakash, S. Trivedi, S. Wadia, X. Yin

Klebanov-Polyakov-Sezgin-Sundell ('02) conjecture:

Vasiliev's minimal bosonic HS gravity in AdS_4 is dual to free or critical *3d* $O(N)$ *vector model*, in the $O(N)$ singlet sector.

$$S = \frac{1}{2}\int d^3x \partial_\mu \phi^i \partial_\mu \phi^i \qquad \leftrightarrow \qquad \text{"type A" HS gravity}$$

$$(\Delta, S) = (1,0)^+ + \textstyle\sum_{s \text{ even}} (s+1, s)$$

$$S = \int d^3x \psi^i \gamma^\mu \partial_\mu \psi^i \qquad \leftrightarrow \qquad \text{"type B" HS gravity}$$

$$(\Delta, S) = (2,0)^- + \textstyle\sum_{s \text{ even}} (s+1, s)$$

- Critical theories: interacting fixed points reached after perturbing these free theories by quartic interaction. Correspond to change of boundary condition on the bulk scalar field in the HS gravity side.
- Non-minimal versions (all integer spins): vector models with complex fields in $U(N)$ singlet sector.

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- Why vector models? A free gauge theory of SYM type also has HS cunserved currents $J_s \sim \mathrm{Tr}\Phi\partial^s\Phi$. But in addition there are many more single trace operators $\mathrm{Tr}\,\Phi\partial^{k_1}\Phi\partial^{k_2}\Phi\cdots\partial^{k_n}\Phi$, which should be dual to massive fields in the bulk.
- In a vector theory, operators of the form $(\phi^i\partial\cdots\partial\phi^i)(\phi^j\partial\cdots\partial\phi^j)$ are analogous to multi-trace operators and should be thought as multi-particle states from bulk point of view.
- A vector model has precisely the right spectrum to be dual to a *pure* HS gauge theory!

Klebanov-Polyakov-Sezgin-Sundell ('02) conjecture:

Vasiliev's minimal bosonic HS gravity in AdS_4 is dual to free/critical *3d* $O(N)$ *vector model*, in the $O(N)$ singlet sector.

- The restriction to singlet sector is important to match boundary and bulk spectrum. It may be implemented by gauging the $O(N)$ symmetry and taking a limit of zero gauge coupling. In practice, we may couple the vector field to a Chern-Simons gauge field at level k, and take the limit $k \to \infty$.
- This suggests it may be interesting to study more generally vector models coupled to Chern-Simons at finite coupling (i.e. finite $\lambda = N/k$ in the large N limit).

The Vasiliev's equations

- Master fields:
  1. $W(x|y,\bar{y},z,\bar{z}) = W_\mu dx^\mu$ — 1-form in space-time
  2. $S(x|y,\bar{y},z,\bar{z}) = S_\alpha dz^\alpha + S_{\dot\alpha} d\bar{z}^{\dot\alpha}$ — 1-form in $(z,\bar{z})$-space
  3. $B(x|y,\bar{y},z,\bar{z})$ — scalar

  x^μ: spacetime, $y_\alpha, \bar{y}_{\dot\alpha}, z_\alpha, \bar{z}_{\dot\alpha}$: twistor variables.

- Collecting W and S into the 1-form $\mathcal{A} = W_\mu dx^\mu + S_\alpha dz^\alpha + S_{\dot\alpha} d\bar{z}^{\dot\alpha}$, Vasiliev's equation can be written as

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = \mathcal{V}(B * \kappa) dz^2 + \bar{\mathcal{V}}(B * \bar{\kappa}) d\bar{z}^2$$

$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$

The Vasiliev's equations

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = \mathcal{V}(B * \kappa) dz^2 + \bar{\mathcal{V}}(B * \bar{\kappa}) d\bar{z}^2$$
$$dB + \mathcal{A} * B - B * \pi(\mathcal{A}) = 0$$

- Up to field redefinitions, $\mathcal{V}(X)$ can be put in the form

$$\mathcal{V}(X) = X \, exp_*(i\Theta(X)) \,,$$
$$\Theta(X) = \theta_0 + \theta_2 X * X + \theta_4 X * X * X * X + \ldots$$

An infinite family of HS gravity theories in 4d. Same spectrum, but a choice of $\Theta(X)$ characterizes the *interactions* in the theory.
(e.g. θ_0 affects 3-point interactions. $\theta_2, \theta_4, \ldots$ enter in higher-point functions)

Parity

- If we impose that the theory has a parity symmetry only two inequivalent choices are left

  - $\Theta(X) = 0, \quad \text{i.e.} \quad \mathcal{V}(X) = X \qquad \text{if} \quad B \quad \text{is parity even}$
  - $\Theta(X) = \frac{\pi}{2}, \quad \text{i.e.} \quad \mathcal{V}(X) = iX \qquad \text{if} \quad B \quad \text{is parity odd}$

  which correspond respectively to the "type A" and "type B" models, conjecturally dual to scalar/fermion vector models (free or critical).
- If we do not require parity symmetry, we have a large class of possible parity breaking HS gravity theories parameterized by a choice of the function $\Theta(X)$, or parameters $\theta_0, \theta_2, \ldots$.
- At least classically, these are all consistent HS theories in AdS_4. One may ask what are the dual CFTs.

Chern-Simons vector model

SG, S. Minwalla, S. Prakash, S. Trivedi, S. Wadia, X. Yin 2011

- Consider the 3d theory of a fundamental massless fermion coupled to a $U(N)$ Chern-Simons gauge field at level k

$$S = \frac{k}{4\pi} S_{CS}(A) + \int d^3x \, \bar{\psi}_i \gamma^\mu D_\mu \psi^i \qquad i = 1, \ldots, N$$

- In 3d, ψ has dimension 1, and the only marginal coupling is the Chern-Simons coupling k. This cannot run because it is quantized to be integer.
- Fine-tuning the mass of the fermion to zero, we obtain a family of interacting CFT's labelled by two integers k, N.
- Taking $k \to \infty$, this reduces to the singlet sector of the free fermionic vector model dual to Vasiliev's type B theory.

Chern-Simons vector model

$$S = \frac{k}{4\pi} S_{CS}(A) + \int d^3x\, \bar{\psi}_i \gamma^\mu D_\mu \psi^i \qquad i = 1, \ldots, N$$

- We will be interested in the large N 't Hooft limit

$$N \to \infty, k \to \infty \qquad \text{with} \quad \lambda = \frac{N}{k} \quad \text{fixed}$$

- In this limit, we effectively have a *continuous line* of non-susy CFT's parameterized by λ. At $\lambda = 0$ we reduce to the free fermionic vector model.
- All I said so far applies for fermion being in any representation, e.g. the adjoint. However, working with a vector fermion entails several simplifications so that exact results become possible.
- The analogous Chern-Simons bosonic vector model has been studied in parallel to our work in *Aharony et. al., 2011*. Also, interesting work in progress on susy extensions of these Chern-Simons vector models (see X. Yin talk).

Chern-Simons vector model

- I will discuss in particular two interesting results about the large N limit of this Chern-Simons vector model

1. The *exact* free energy of the theory on R^2 at finite temperature

$$F = -T \log Z_{R^2 \times S^1_\beta} = - h(\lambda) \, N V_2 T^3$$

$h(\lambda)$ is a non-trivial function which we can compute *exactly* in λ.

2. At $N \to \infty$, for all λ, the theory admits an ∞-dimensional *higher spin symmetry*, i.e. there is an infinite tower of HS currents $J_s, s = 1, 2, 3, \ldots$ which are conserved at large N, so that

$$\Delta(J_s) = s + 1 + \mathcal{O}(\frac{1}{N}) \qquad \forall \lambda$$

Exact thermal free energy

- The Chern-Simons gauge field does not carry propagating degrees of freedom, so the theory is still essentially a vector model, and we expect it to be simpler than a typical large N gauge theory.
- However, the cubic self-interaction of the CS gauge field still makes perturbation theory complicated in general.
- Drastic simplifications can be achieved in a convenient gauge. We employ the "*light-cone gauge*"

$$A_- = 0 \qquad\qquad x^\pm = x^1 \pm i x^2$$

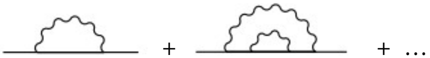
  Here x^1, x^2 are the Euclidean coordinates on R^2. The Euclidean time direction is x^3, which will be compactified on a circle of radius $\beta = 1/T$.
- In this gauge, the cubic self-interaction vanishes, and the large N free energy can be solved exactly.

Exact fermion propagator

- The basic ingredient we need to get the free energy is the exact fermion propagator

$$\langle \psi(p)^i \bar{\psi}(-p)_j \rangle = \delta_i^j \frac{1}{ip_\mu \gamma_\mu + \Sigma(p)}$$

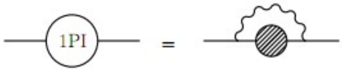
- $\Sigma(p)$ is the exact fermion self-energy. In the light-cone gauge and in the planar limit, it receives contributions only from 1PI rainbow diagrams



- Note that diagrams with matter loops do not contribute at leading order at large N, because the fermion is in the fundamental.

Exact fermion propagator

- It is not difficult to see that the sum of rainbow diagrams contributing to $\Sigma(p)$ satisfies the Schwinger-Dyson equation



$$\Sigma(p) = \frac{N}{2}\int \frac{d^3q}{(2\pi)^3}\left(\gamma^\mu \frac{1}{i\gamma^\alpha q_\alpha + \Sigma(q)}\gamma^\nu\right) G_{\mu\nu}(p-q)$$

- Here $G_{\mu\nu}(p)$ is the light-cone A_μ propagator: $G_{+3} = -G_{3+} = \frac{4\pi i}{kp^+}$.
- At finite temperature, we impose antiperiodic b.c. on the fermion, so

$$q^3 = \frac{2\pi}{\beta}(n+1/2), \qquad \int d^3q \to \int d^2q \sum_{\mathbb{Z}+1/2}$$

Exact fermion propagator

- Employing the "dimensional reduction" scheme to regulate the loop integrals (shown to be consistent in CS-matter theories by *Chen, Semenoff, Wu '92* up to 2-loops), we solved the Schwinger-Dyson equation explicitly.
- The solution takes the form

$$\Sigma(p) = f(\beta p_s)p_s + i\, g(\beta p_s)p^- \gamma^+ \qquad p_s^2 \equiv p_1^2 + p_2^2$$

with

$$f(y) = \frac{2\lambda}{y} \log\left(2\cosh\left[\frac{1}{2}\sqrt{c^2+y^2}\right]\right), \qquad g(y) = \frac{c^2}{y^2} - f(y)^2$$

$$c = 2\lambda \log\left(2\cosh\frac{c}{2}\right)$$

- The equation determining $c = c(\lambda)$ has no solutions for $|\lambda| \geq 1$. We conclude that the CFT is defined only for $0 \leq |\lambda| < 1$.

Exact thermal free energy

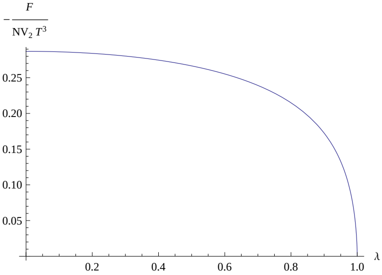
- Once we have the exact fermion self-energy Σ, one may show by path integral or diagrammatically that the free energy is given in terms of Σ by

$$F = NV_2 T \sum_n \int \frac{d^2 q}{(2\pi)^2} \mathrm{Tr} \left[\log \left[i\gamma^\mu q_\mu + \Sigma(q) \right] - \frac{1}{2} \Sigma(q) \left(\frac{1}{i\gamma^\mu q_\mu + \Sigma(q)} \right) \right]$$

- Performing the integral and sum, the final result is

$$F = -\frac{NV_2 T^3}{6\pi} \left[c^3 \frac{1-\lambda}{\lambda} + 6 \int_c^\infty dy \; y \log\left(1 + e^{-y}\right) \right] \equiv -NV_2 T^3 h(\lambda)$$

where $c = c(\lambda)$ is the constant introduced earlier.



$$h(\lambda) = \frac{3\zeta(3)}{4\pi} - \frac{2(\log 2)^3}{3\pi}\lambda^2 - \frac{(\log 2)^4}{2\pi}\lambda^4 + \ldots \qquad \lambda \ll 1$$

$$h(\lambda) \sim \frac{(1-\lambda)}{6\pi}\log^3(1-\lambda) + \ldots \qquad \lambda \to 1$$

- The function $h(\lambda)$ decreases monotonically from the free field value to zero at $\lambda = 1$. Extreme thinning of d.o.f. at "strong coupling". For comparison, in ABJM model we have $h(\lambda) \sim 1/\sqrt{\lambda}$ at $\lambda \to \infty$.

Higher spin symmetry at large N

- Recall that in the free theory ($\lambda = 0$), the spectrum of $U(N)$ invariant single trace primaries is

$$J_0 = \bar{\psi}_i \psi^i \,, \qquad J_s \sim \bar{\psi}_i \gamma_{(\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_s)} \psi^i + \ldots$$

- In the interacting theory, these can be made gauge invariant by $\partial_\mu \to D_\mu$. The CS sector does not add any further single-trace primaries, because $(F_{\mu\nu})^i_j \sim \frac{1}{k} \bar{\psi}_j \gamma^\rho \psi^i \epsilon_{\mu\nu\rho}$ by e.o.m.
- In the free theory $\partial \cdot J_s = 0$, i.e. J_s are in short representations of the conformal algebra with $(\Delta, S) = (s+1, s)$.
- Turning on the interaction, we expect the currents not to be conserved any more and to acquire anomalous dimension $\Delta_s = s + 1 + \epsilon_s(\lambda, N)$.

Higher spin symmetry at large N

- But for the currents to become non-conserved at $\lambda \neq 0$, we must have

$$\partial \cdot J_s \sim \lambda \mathcal{O}^{(s+2,s-1)}$$

In other words, there must be an operator in the $(s+2, s-1)$ representation with which J_s can combine to form a long representation.

- At $N = \infty$, single trace operators can only combine with other single trace operators. But there are no single-trace primaries in the spectrum with quantum numbers $(s+2, s-1)$!
- Therefore we conclude that at $N = \infty$, for all λ, the currents are still conserved, which implies

$$\Delta(J_s) = s + 1 + \mathcal{O}(\frac{1}{N}) \qquad \forall\ \lambda$$

- The vector nature of ψ is essential for this to work.

Higher spin symmetry at large N

- What happens is that, at finite N, J_s can (and does) combine with "multi-trace" operators. The non-conservation equation takes the schematic form

$$\partial \cdot J_s \sim \frac{f(\lambda)}{\sqrt{N}} \sum \partial^m J_{s_1} \partial^n J_{s_2} + \frac{g(\lambda)}{N} \sum \partial^m J_{s_1} \partial^n J_{s_2} \partial^p J_{s_3}$$

- The argument above implies that the HS currents do not have anomalous dimensions in the planar limit. But one can in fact argue that the scalar J_0 has protected dimension as well

$$\Delta(J_0) = 2 + \mathcal{O}(\frac{1}{N})$$

which we have checked perturbatively to two-loop order.

Comments on the holographic dual

- At $\lambda = 0$, we know that the theory should be dual to the Vasiliev's "type B" theory. So the holographic dual should be some deformation of it.
- Turning on λ, we have seen that the spectrum of "single trace" primaries is

$$(\Delta, S) = (2 + \mathcal{O}(\frac{1}{N}), 0) + \sum_{s=1}^{\infty}(s + 1 + \mathcal{O}(\frac{1}{N}), s)$$

which implies that the dual bulk spectrum should contain classically massless higher spin fields and a $m^2 = -2$ scalar.
- The HS fields (and the scalar) can acquire mass via loop-corrections, but the bulk classical equations of motion should have exact higher spin gauge symmetry (to decouple longitudinal polarizations).
- Hence, the holographic dual should still be a higher spin gauge theory (with HS symmetry broken at quantum level), and it should break parity due to the boundary Chern-Simons term.

Comments on the holographic dual

- The only parity breaking higher spin gravity theory currently known is Vasiliev's theory specified by the general "interaction phase"

$$\Theta(X) = \theta_0 + \theta_2 X * X + \dots$$

- A natural conjecture is then that our Chern-Simons vector model is dual to the parity breaking Vasiliev's theory with some specific choice

$$\theta_0(\lambda)\,, \qquad \theta_2(\lambda)\,, \qquad \dots$$

  with the condition that $\theta_0(\lambda \to 0) = \frac{\pi}{2}$, $\theta_{2,4,\dots}(\lambda \to 0) = 0$.
- We do not know a priori how to determine the phase as a function of λ. But we can in principle compute perturbatively correlators on both sides and compare.

Comments on the holographic dual

- From considerations based on the softly broken HS symmetry purely on CFT side, Maldacena-Zhiboedov showed that 3pt functions should be a sum of free boson, free fermion and a parity odd tensor structure

  $$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \cos^2\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_B + \sin^2\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_F + \sin\theta_0 \cos\theta_0 \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{odd}$$

  Confirmed by a direct 2-loop calculation in the CS-fermion theory, which gives $\theta_0(\lambda) = \frac{\pi}{2}(1-\lambda) + \mathcal{O}(\lambda^3)$.

- From the bulk calculation in Vasiliev's theory with general phase θ_0, we get such a decomposition, with $\langle JJJ \rangle_B$ and $\langle JJJ \rangle_F$ correctly coming out. However currently the coefficient of $\sin\theta_0 \cos\theta_0$ appears to vanish...
- The appearance of the odd structure should just follow from symmetries as shown by MZ, strongly suggesting that we are missing something in the bulk calculation.

Summary and conclusion

- Chern-Simons vector models define lines of interacting CFT's with lagrangian description. They have approximate higher spin symmetry at large N.
- We proposed a generalization of the KPSS conjecture which involves a parity breaking version of Vasiliev's HS gravity. Partial evidence, still work in progress.
- Some future directions
  - Higher-point functions from bulk and CFT
  - Study of exact solutions and their CFT interpretation
  - Free energy from the bulk HS theory? (Bulk action?)
  - Susy extensions and relation to string theory
  - Extensions to higher dimensions
  - ...