Constructing AdS/DS HS Gravity From Vector Model QFT

GINZBURG CONFERENCE,
May 2012, MOSCOW

Antal Jevicki (Brown University)

With: S.Das,R.d.Mello Koch,K.Jin,J.Rodrigues,Qibin Ye

TALK

- Direct Construction of Higher Spin Gravity(Vasiliev) from QFT in terms of Bi-Local Collective Fields:

- Applied to Higher Spin AdS Case:
  S. Das + A.J. 03; R. d M Koch, A.J., K. Jin, J. Rodrigues 10,11
  Bi-local Map to HS AdS
- Recent studies: Coleman-Mandula Theorem:S=1
  and Implications

- Recent extension to de Sitter HS Theory
S.Das,D.Das,A.J and Q.Ye

Unified Description AdS/dS :Similarities and Differences

CFT d=3

- **O(N): Boson**
- $\phi_i = \phi_i(t, \vec{x}) = \phi_i(x^+, x^-, x)$, with $i = 1, 2, 3, ..., N$

  $\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi\cdot\phi)$ → **Null Plane**
- **Fixed Points : g=0 UV CFT Klebanov Polyakov 02**

  g≠0 IR CFT AdS4

- **Sp(2N): Fermion**

$$I_{Sp(N)} = \frac{1}{8\pi}\int d^3x\,\left(\Omega_{ab}\delta^{ij}\partial_i\chi^a\partial_j\chi^b + m^2\chi\cdot\chi + \lambda(\chi\cdot\chi)^2\right)$$

Strominger, Hartman, Anninos 11

dS4

- **Infinite Sequence of HS Currents**

$$J_{\mu_1\mu_2\cdots\mu_s} = \sum(-)^k(\ )(\ )\partial_{\mu_1\cdots\mu_k}\phi(x)\partial_{\mu_{k+1}\cdots\mu_s}\phi(x)$$

- **Generating Function**

$$O(x^\mu, \varepsilon^\mu) = \phi_i(x,\varepsilon)\sum\frac{1}{(2n)!}(\ )\phi_i(x,\varepsilon)$$

3d 2d

$$\boxed{\varepsilon^2 = 0}$$ **Tracelessness**

- **Currents and Boundary Duals to AdS HS Fields of Vasiliev: Coupling to HS: Bekaert**

$$\boxed{J_{\mu_1\mu_2\cdots\mu_s}(x)} \Longleftrightarrow \boxed{H_{\hat\mu_1\hat\mu_2\cdots\hat\mu_s}(x^\mu, z\to 0)}$$

$$ds^2 = \frac{dx^2+dz^2}{z^2}$$

Bilocal Field Theory

- Exact construction

- Change from field $\vec{\phi}(x) = (\phi_1, \phi_2, ...\phi_N)$ to the bilocal field:

$$\boxed{\Phi(x,y) = \phi(x) \cdot \phi(y) = \sum_{a=1}^{N} \phi^a(x)\phi^a(y)}$$

O(N) invariant

3d + 3d

- Represents a more general set than the conformal fields: $\mathcal{O}(x, \epsilon)$ $\epsilon^2 = 0$

3d + 2d

The collective (effective) action:

- **Partition function:**

$$Z = \int [d\phi^i(x)] e^{-S[\phi]} = \int \prod_{x,y} d\Phi(x,y) \mu(\Phi) e^{-S_c[\Phi]}$$

- **The collective(effective) action:**

$$S_{eff} = \mathrm{Tr}[-(\partial_x^2 + \partial_y^2)\Phi(x,y) + m^2\Phi(x,y) + V] + \boxed{\frac{N}{2}\mathrm{Tr}\ln\Phi}$$

- **Origin of the** $\ln\Phi$ **interaction: Jacobian**

$$\int d\vec{\phi} e^{-S} \to \int d\Phi \det\left|\frac{\partial\phi^a(x)}{\partial\Phi(x_1, x_2)}\right| e^{-S}$$

- **The measure:**

$$\mu(\Phi) = (\det\Phi)^{V_x V_p}$$

$V_x = L^3$ space

$V_p = \Lambda^3$ momentum cutoff

Topology:

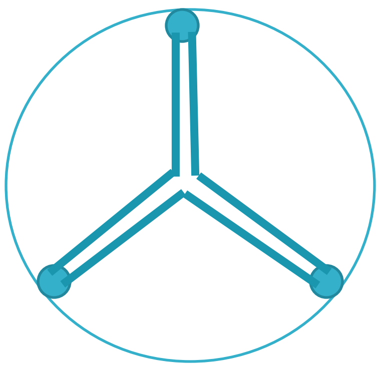
Witten diagram! (bulk AdS)

1

$$\langle \Phi(x_1, y_1)\Phi(x_2, y_2)\rangle$$

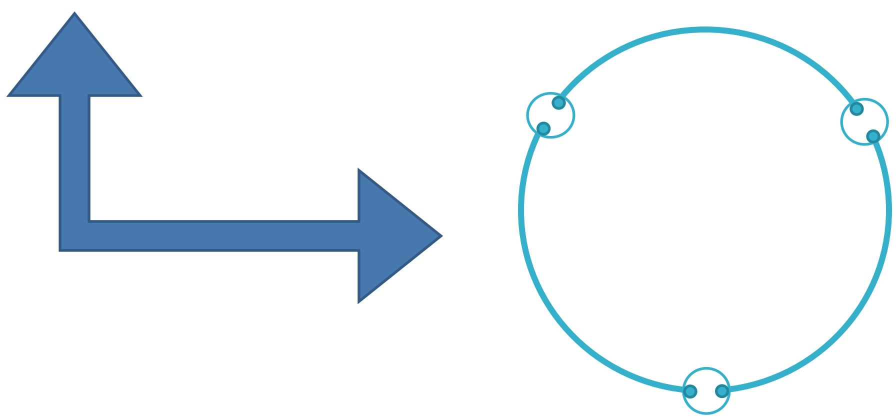
Propagator

$\frac{1}{\sqrt{N}}$



$$\langle \Phi(1)\Phi(2)\Phi(3)\rangle$$

Collective Field Theory



CFT Correlators: GIOMBI-YIN

Light-Cone Map

- CFT_3: collective bi-local fields — AdS_4: higher spin fields

$$\boxed{\Psi(x^+;(x_1^-,x_1),(x_2^-,x_2))} \Longleftrightarrow \boxed{\Phi(x^+;x^-,x,z;\theta)}$$

- **Same number of dimensions**

$$1+2+2 \qquad = \qquad 1+3+1$$

- **Representation of the conformal group SO(2,3)**
- **one does not have a coordinate transformation**
- **It is a canonical transformation**

Summary :

- **Represents a nonlinear theory with G=1/N Vertices**
- **At the linearized level agrees with Metsaev's light cone HS theory:**

$$\boxed{S_2 = \int dxdz \mathcal{H}(-2\partial_+\partial_- + \partial_z^2 + \partial_i^2 + \frac{m^2}{z^2})\mathcal{H}} \qquad m^2 = 0$$

By construction (of $H_3, \quad H_4 \quad \cdots$ **) , it reproduces the boundary z=0 correlators of the O(N) model:**

$$\langle O(x_1\varepsilon_1)\cdots O(x_n\varepsilon_n)\rangle$$

Coleman-Mandula Theorem in CFT3/AdS4

- **The simplest case of the correspondence involves CFT | $_{UV}$ with g=0, i.e. A free QFT and HS AdS4**
- **In CFT we have an infinite number of conserved charges/ currents:**

$$Q^s = \int d\vec{x} J_{0\mu_1\mu_2\cdots\mu_s}$$

or in the light-cone:

$$Q^s = \int dx^- dx J_{--\ldots-}$$

- **In such a theory, the C-M theorem implies that the S-Matrix is 1 S=1**

The Relevance (implication) of the C-M theorem in CFT3/AdS4 :

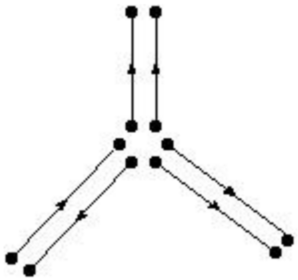
Maldacena + Zhiboedov arxiv 1112.1016

- Not having an S-Matrix (in AdS or CFT) they studied the implication on the correlation functions: Showed that the theory is (trivial) simple i.e. correlators are given by free fields
- Still: $C_n = \langle O_1 O_2 \dots O_n \rangle \neq 0$ arbitrary n-point correlators are non-zero
- 'Boundary S-Matrix', Mack, Penedones
- Non-zero S

S-Matrix

In Bi-Local Representation :One can define an S-Matrix : Scattering of DiPoles

- **Can be evaluated through bi-local Feynman rules**



- **Result: $S_3=0$, $S_4=0$**
- **And a proof to all orders.**

R.d.M.Koch, A. J, K. Jin, J. Rodrigues, Q.Ye 12

LSZ Reduction Formula

- **define an S-Matrix for scattering of "Collective DiPoles"(these would be mesons in 't Hoofts large N**
- **In a time-like gauge (single time), one has the on-shell relation**

$$\boxed{E^2 - (\sqrt{\vec{k}_1^2} + \sqrt{\vec{k}_2^2})^2 = 0}$$

Single time **Bi-Local**

$$S = \lim \prod_i (E_i^2 - (\omega_1 + \omega_2)^2) \underline{\langle \tilde{O}(t_1, x_1, x_1') \tilde{O}(t_2, x_2, x_2') \cdots \rangle}$$

In light-cone gauge, it would correspond to:

$$\lim(P^- - \frac{p_1^2}{2p_1^+} - \frac{p_2^2}{2p_2^+})\langle\tilde{O}(x^+;(x^-,x)_1,(x^-,x)_2)\tilde{O}(x'^+,(x^-,x)_{1'},(x^-,x)_{2'})\cdots\rangle$$

- **Note: Maldacena+Zhiboedov in their work reconstructed the correlatios of**

$$O(x^+;(x_1^-,x_2^-);x_1=x_2)$$

- **So here one is not in a position to consider the above defined S-Matrix**
- **Present work complements the work of MZ/extra observables:implications on the structure of the theory**

- **<u>General Theorem</u>: For all duals coming from free large N theories with infinitely many conserved charges: S=1**

- **Nonlinearities in G: 1/N can be removed by field transformations[Should we do that?]**

- **A Change of boundary conditions will result in non-trivial S-Matrix: Relevant for Vasilievs HS/CFT**

where we switch from one fixed point to another by a change of boundary conditions

Sp(2N) Fermions :de Sitter

- **The action:** $$S = \int d^dx\, dt (\partial^\mu \eta_1^i \partial_\mu \eta_2^i)$$

- **Bi-local variables introduced based on Sp(2N):**

$$\begin{aligned}
\eta &= (\eta_1^1, \eta_2^1, \eta_1^2, \eta_2^2, \cdots, \eta_1^N, \eta_2^N) \\
a(k) &= (a_{k-}^1, a_{k+}^1, a_{k-}^2, a_{k+}^2, \cdots, a_{k-}^N, a_{k+}^N) \\
\tilde{a}(k) &= (a_{k+}^{1\dagger}, -a_{k-}^{1\dagger}, a_{k+}^{2\dagger}, -a_{k-}^{2\dagger}, \cdots, a_{k\dagger}^{N+}, -a_{k-}^{N\dagger})
\end{aligned}$$

- **We will now use a more general (pseudo-spin) framework(both commuting and non-comuting bi-local operators**

Pseudo-Spin Formalism

- **ALL Sp(2N) invariant operators:**

$$A(p_1,p_2) \;=\; \frac{-i}{2\sqrt{N}} a^T(p_1)\epsilon_N a(p_2) = \frac{i}{2\sqrt{N}} \sum_{i=1}^{N} (a^i_{p_1+} a^i_{p_2-} + a^i_{p_2+} a^i_{p_1-})$$

$$A^+(p_1,p_2) \;=\; \frac{-i}{2\sqrt{N}} \tilde{a}^T(p_1)\epsilon_N \tilde{a}(p_2) = \frac{i}{2\sqrt{N}} \sum_{i=1}^{N} (a^{i\dagger}_{p_1+} a^{i\dagger}_{p_2-} + a^{i\dagger}_{p_2+} a^{i\dagger}_{p_1-})$$

$$B(p_1,p_2) \;=\; \tilde{a}^T(p_1)\epsilon_N a(p_2) = \sum_{i=1}^{N} a^{i\dagger}_{p_1+} a^i_{p_2+} + a^{i\dagger}_{p_1-} a^i_{p_2-}$$

$$\boxed{\epsilon_N = \epsilon \otimes \mathbb{I}_N,\; \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}$$

Algebra:

Invariant operators close an algebra:

$$
\begin{aligned}
[A(\vec{p}_1,\vec{p}_2), A^\dagger(\vec{p}_3,\vec{p}_4)] &= \frac{1}{2}(\delta_{\vec{p}_2,\vec{p}_3}\delta_{\vec{p}_4,\vec{p}_1} + \delta_{\vec{p}_2,\vec{p}_4}\delta_{\vec{p}_3,\vec{p}_1}) \\
&\quad + \frac{1}{4N}[\delta_{\vec{p}_2,\vec{p}_3}B(\vec{p}_4,\vec{p}_1) + \delta_{\vec{p}_2,\vec{p}_4}B(\vec{p}_3,\vec{p}_1) \\
&\qquad + \delta_{\vec{p}_1,\vec{p}_3}B(\vec{p}_4,\vec{p}_2) + \delta_{\vec{p}_1,\vec{p}_4}B(\vec{p}_3,\vec{p}_2)] \\
[B(\vec{p}_1,\vec{p}_2), A^\dagger(\vec{p}_3,\vec{p}_4)] &= \delta_{\vec{p}_2,\vec{p}_3}A^\dagger(\vec{p}_1,\vec{p}_4) + \delta_{\vec{p}_2,\vec{p}_4}A^\dagger(\vec{p}_1,\vec{p}_3) \\
[B(\vec{p}_1,\vec{p}_2), A(\vec{p}_3,\vec{p}_4)] &= -\delta_{\vec{p}_1,\vec{p}_3}A(\vec{p}_2,\vec{p}_4) - \delta_{\vec{p}_1,\vec{p}_4}A(\vec{p}_2,\vec{p}_3)
\end{aligned}
$$

- **Casimir :**

$$
\boxed{\frac{4}{N}A^+ \star A + (1-\frac{1}{N}B)\star(1-\frac{1}{N}B) = \mathbb{I}}
$$

- **Compact (infinite dimensional) pseudospin-algebra**

- **Bosonic Case:**

$$
\begin{aligned}
[A(\vec{p}_1,\vec{p}_2), A^\dagger(\vec{p}_3,\vec{p}_4)] &= \frac{1}{2}(\delta_{\vec{p}_2,\vec{p}_3}\delta_{\vec{p}_4,\vec{p}_1} + \delta_{\vec{p}_2,\vec{p}_4}\delta_{\vec{p}_3,\vec{p}_1}) \\
&\quad + \frac{1}{4N}[\delta_{\vec{p}_2,\vec{p}_3}B(\vec{p}_4,\vec{p}_1) + \delta_{\vec{p}_2,\vec{p}_4}B(\vec{p}_3,\vec{p}_1) \\
&\qquad + \delta_{\vec{p}_1,\vec{p}_3}B(\vec{p}_4,\vec{p}_2) + \delta_{\vec{p}_1,\vec{p}_4}B(\vec{p}_3,\vec{p}_2)] \\
[B(\vec{p}_1,\vec{p}_2), A^\dagger(\vec{p}_3,\vec{p}_4)] &= \delta_{\vec{p}_2,\vec{p}_3}A^\dagger(\vec{p}_1,\vec{p}_4) + \delta_{\vec{p}_2,\vec{p}_4}A^\dagger(\vec{p}_1,\vec{p}_3) \\
[B(\vec{p}_1,\vec{p}_2), A(\vec{p}_3,\vec{p}_4)] &= -\delta_{\vec{p}_1,\vec{p}_3}A(\vec{p}_2,\vec{p}_4) - \delta_{\vec{p}_1,\vec{p}_4}A(\vec{p}_2,\vec{p}_3)
\end{aligned}
$$

- **With Casimir** $-\frac{4}{N}A^+ \star A + (1+\frac{1}{N}B)\star(1+\frac{1}{N}B) = \mathbb{I}$
- **Joint Notation:**

$$\boxed{4\gamma A^+ \star A + (1-\gamma B)\star(1-\gamma B) = \mathbb{I}}$$

$$\gamma = \begin{cases} 1/N & \textbf{Fermionic} \\ -1/N & \textbf{Bosonic} \end{cases}$$

- **Collective representation:**

$$
\begin{aligned}
A(p_1p_2) &= \frac{\sqrt{-\gamma}}{2}\int dy_1dy_2e^{-i(p_1y_2+p_2y_2)}\{-\frac{2}{\kappa_{p_1}\kappa_{p_2}}\Pi\star\Psi\star\Pi(y_1y_2)-\frac{1}{2\gamma^2\kappa_{p_1}\kappa_{p_2}}\frac{1}{\Psi}(y_1y_2)\\
&+\frac{\kappa_{p_1}\kappa_{p_2}}{2}\Psi(y_1y_2)-i\frac{\kappa_{p_1}}{\kappa_{p_2}}\Psi\star\Pi(y_1y_2)-i\frac{\kappa_{p_2}}{\kappa_{p_1}}\Pi\star\Psi(y_1y_2)\}\\
A^+(p_1p_2) &= \frac{\sqrt{-\gamma}}{2}\int dy_1dy_2e^{-i(p_1y_2+p_2y_2)}\{-\frac{2}{\kappa_{p_1}\kappa_{p_2}}\Pi\star\Psi\star\Pi(y_1y_2)-\frac{1}{2\gamma^2\kappa_{p_1}\kappa_{p_2}}\frac{1}{\Psi}(y_1y_2)\\
&+\frac{\kappa_{p_1}\kappa_{p_2}}{2}\Psi(y_1y_2)+i\frac{\kappa_{p_1}}{\kappa_{p_2}}\Psi\star\Pi(y_1y_2)+i\frac{\kappa_{p_2}}{\kappa_{p_1}}\Pi\star\Psi(y_1y_2)\}\\
B(p_1p_2) &= \frac{1}{\gamma}+\int dy_1dy_2e^{-i(p_1y_2+p_2y_2)}\{\frac{2}{\kappa_{p_1}\kappa_{p_2}}\Pi\star\Psi\star\Pi(y_1y_2)+\frac{1}{2\gamma^2\kappa_{p_1}\kappa_{p_2}}\frac{1}{\Psi}(y_1y_2)\\
&+\frac{\kappa_{p_1}\kappa_{p_2}}{2}\Psi(y_1y_2)-i\frac{\kappa_{p_1}}{\kappa_{p_2}}\Psi\star\Pi(y_1y_2)+i\frac{\kappa_{p_2}}{\kappa_{p_1}}\Pi\star\Psi(y_1y_2)\}
\end{aligned}
$$

where $\kappa_p = \sqrt{\omega_p}$.

- **This implies that the perturbative 1/N expansion is identical: with an N to –N swich.**

Hilbert Space:Difference

- **Bosons:for finite N, one has relationships between bi-local variables :K+1 > N**

  Det A(k,l)=0

- **Fermions: Finite N---cutoff in Hilbert space:**

$$
\begin{aligned}
[A^+(1,2)]^4 &= 0 \\
A^+(1,2)A^+(1,3)A^+(1,3)A^+(1,4)A^+(1,1) &= 0
\end{aligned}
$$

BiLocal Hilbert :Kahler Quantization

- **These issues are resolved:**
- **Oscillator Representation:**

$$\begin{aligned} A(p_1,p_2) &= \alpha \star (1-\frac{1}{N}\alpha^\dagger \star \alpha)^{\frac{1}{2}}(p_1,p_2) \\ A^+(p_1,p_2) &= (1-\frac{1}{N}\alpha^\dagger \star \alpha)^{\frac{1}{2}} \star \alpha^\dagger(p_1,p_2) \\ B(p_1,p_2) &= 2\,\alpha^\dagger \star \alpha(p_1,p_2) \end{aligned}$$

- **Kahler Representation**

$$\begin{aligned} \alpha &= Z(1+\frac{1}{N}\bar{Z}Z)^{-\frac{1}{2}} \\ \alpha^\dagger &= (1+\frac{1}{N}\bar{Z}Z)^{-\frac{1}{2}}\bar{Z} \end{aligned}$$

- **Pseudo-spins in the Z Representation are given by:**

$$
\begin{aligned}
A(p_1,p_2) &= Z\star(1+\frac{1}{N}\bar{Z}\star Z)^{-1}(p_1,p_2)\\
A^+(p_1,p_2) &= (1+\frac{1}{N}\bar{Z}\star Z)^{-1}\star\bar{Z}(p_1,p_2)\\
B(p_1,p_2) &= 2\,Z\star(1+\frac{1}{N}\bar{Z}\star Z)^{-1}\star\bar{Z}(p_1,p_2)
\end{aligned}
$$

- **Regularization:** **x** $\longrightarrow$ **k**

  Cutoff: **K**

Quantization on Kahler Manifold

F.A.Berezin Quantization in complex symmetric spaces 75'

Quantization of a classical mechanics with nonlinear phase space 78'

A. Volovich Discrete Space-time 01'

- **Kahler scalar product in bi-local Hilbert space:**

$$(F_1, F_2) = C(N, K) \int d\mu(\bar{Z}, Z) F_1(Z) F_2(\bar{Z}) \det[1 + \bar{Z}Z]^{-N}$$

$$d\mu = \det[1 + \bar{Z}Z]^{-2K} d\bar{Z} dZ$$

Dimension of quantized Hilbert space

- **Normalization constant**

$$\boxed{(F_1, F_1) = 1 \text{ for } F = 1}$$

$$a(N,K) = \frac{1}{C(N,K)} = \int d\mu(\bar{Z}, Z) \det[1 + \bar{Z}Z]^{-N}$$

- **Next from:Tr (1) one deduces the following formula for the dim of the Hilbert Space**

$$\boxed{\text{Dim } \mathcal{H}_B = \frac{C(N,K)}{C(0,K)} = \frac{a(0,K)}{a(N,K)}}$$

Complex Matrix Integral

- **Diaganoalize**

  Z(k,l) ⟶ **Diag [** $\omega_1, \omega_2, \omega_3, \cdots, \omega_K$ **]**

- **The Matrix integral becomes:**

$$a(N,K) = \frac{\text{Vol }\ \Omega}{K!} \int \Delta(x_1, \cdots, x_K)^2 \prod_l (1+\omega_l^2)^{-2K-N} \prod_l d\omega_l$$

- **With the Vandemonde Determinant :**

$$\Delta(x_1, \cdots, x_K) = \prod_{k<l} (x_k - x_l)$$

$$\text{with } x_i = \omega_i^2$$

Evaluation:

- **Use the Selberg integral:** 1944

$$
\begin{aligned}
I(\alpha,\beta,\gamma,n) &= \int_0^1 dx_1 \cdots \int_0^1 dx_n |\Delta(x)|^{2\gamma} \prod_{j=1}^{n} x_j^{\alpha-1}(1-x_j)^{\beta-1} \\
&= \prod_{j=0}^{n-1} \frac{\Gamma(1+\gamma+j\gamma)\Gamma(\alpha+j\gamma)\Gamma(\beta+j\gamma)}{\Gamma(1+\gamma)\Gamma(\alpha+\beta+(n+j-1)\gamma)}
\end{aligned}
$$

- **Our case:**

$$\alpha = 1, \ \ \beta = N+1, \ \ \gamma = 1, \ \ n = K$$

- **Result:**

$$\boxed{\mathrm{Dim}\ \mathcal{H}_B = \prod_{j=0}^{K-1} \frac{\Gamma(j+1)\Gamma(N+K+j+1)}{\Gamma(K+j+1)\Gamma(N+j+1)}}$$

Re: Fermionic Counting

- **With** $A^\dagger(k,l) = \sum_{i=1}^{N} a_+^{i\dagger}(k) a_-^{i\dagger}(l)$ **and** $a^{\dagger 2} = 0$
- **Explicit enumeration of Sp(2N) invariant states in the fermionic Hilbert space (for low values of N and K)**
- **Examples:**

	N=2	N=3
K=1	N+1=3	N+1=4
K=2	20	50

- Bi-Local Hilbert space gave the same numbers!

- Bi-Local QUANTIZATION based on:

1. Bosonic Bi-local fields $Z(x,y)$
2. Kahler Quantization
3. Non-linear Phase Space($1/N$)

REPRODUCES the Fermionic Counting

At Large $K >> N > 1$: DimHilbert Space $=$ $2NK\log 2$

Large N Exclusion Principle : $N=1/G$

Will not speculate on deSitter Etropy

Conclusion

- I have presented some elements of a bi-local approach to Higher Spin / CFT_3 correspondence.
- For dualities involving free QFT's: S-Matrix = 1:in agreement with the Coleman-Mandula theorem
- The formulation was extended from AdS to deSitter space-time: Perturbative level N to -N
- Hilbert Space Level: Compact vs Non-compact phase space :Huge Difference
- Geometric(Khaler) Quantization
- Implementation of (finite) N Exclusion Principle: Hilbert Space for Sp(2N)/deSitter theory
- Pseudospins->Chern –Simons formulation

Thanks!