Spectral Redistribution of Gyroresonant Photons and Cyclotron Line Formation in Magnetized Atmospheres of Compact Stars

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Atmospheres of compact stars

- Radii: $R_{\rm WD} \sim 10^3$ km, $R_{\rm NS} \sim 10$ km.
- Scale heights: $H_{\rm NS} \sim 1 - 10$ cm, $H_{\rm WD} \sim 10^4 - 10^5$ cm.
- Strong magnetic fields:

$$B_{\rm NS} \sim 10^7 - 10^{14}\,\mathrm{G}, \quad B_{\rm WD} \sim 10^6 - 10^9\,\mathrm{G}.$$

- Photospheric number densities:

$$N_{\rm NS} \sim 10^{16}\,\mathrm{cm}^{-3}, \quad N_{\rm WD} \sim 10^9\,\mathrm{cm}^{-3}.$$

- Photospheric temperatures:

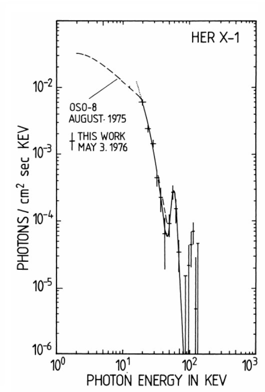
$$T_{\rm NS} \sim 50 - 1000\mathrm{eV}, \quad T_{\rm WD} \sim 1 - 5\mathrm{eV}.$$

- Strong domination of scattering over absorption in the outer layers:

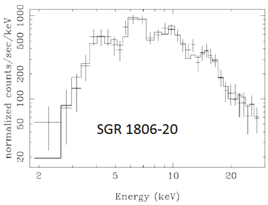
$$\frac{P_{\rm abs}}{P_{\rm sc}} = \frac{\nu_{\rm eff}}{\gamma} \sim 10^{-8} - 10^{-4}.$$

Cyclotron lines observations for compact stars

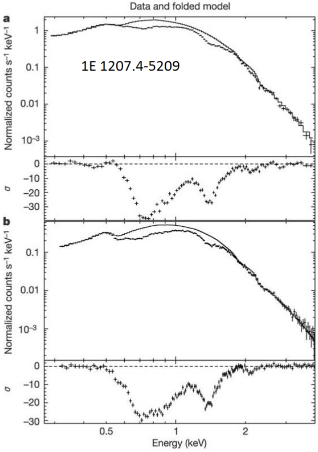
Is spectral redistribution important in 1D case?



Trumper et al. 1978



Ibrahim et al. 2002



Sanval, Pavlov, Zavlin 2002

Spectral redistribution is important for the line formation in atmospheres of main-sequence stars (3D case) [Mihalas, 1980].

Approximations

- Rarified plasma: $|n_{1,2} - 1| \ll 1$.
- All electrons on the ground Landau level.
- Isothermal atmosphere with constant temperature T, which corresponds to the Maxwellian distribution of electrons over longitudinal (with respect to the magnetic field) velocities

$$f(\beta) = \left(\frac{c^2}{2\pi m T}\right)^{1/2} \exp\left(-\frac{\beta^2}{2\beta_T^2}\right), \quad (1)$$

  where $\beta = v/c$ is the dimensionless longitudinal velocity, $\beta_T = (T/(mc^2))^{1/2}$ the thermal velocity.
- Plane-parallel atmosphere:

$$H = \frac{2kT}{m_p g} \ll R.$$

Spectral redistribution of radiation

A photon with a given frequency ω and propagation angle θ with respect to magnetic field is scattered resonantly by electrons whose longitudinal velocities are β:

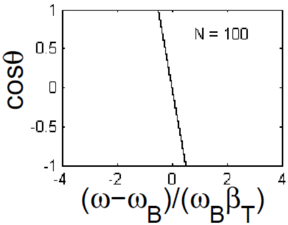
$$\omega'(1-\beta\cos\theta') = \omega(1-\beta\cos\theta).$$

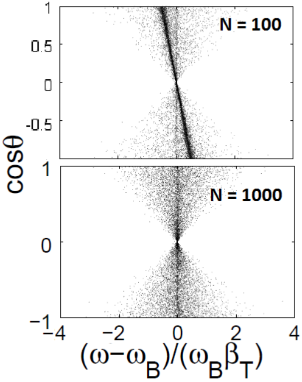
The cross section of this process is [Wang, Wasserman, Salpeter, 1980]

$$\sigma_{\rm sc} = \frac{3}{8}\frac{\gamma}{\pi\omega_B}\frac{\sigma_{\rm T}(1+\cos^2\theta)}{\left[1-\beta\cos\theta-\frac{\omega_B}{\omega}\sqrt{1-\beta^2}\right]^2+\left(\frac{\gamma}{\omega_B}\right)^2}, \qquad (2)$$

where $\omega_B = eB/(mc)$ is the cyclotron frequency, $\sigma_{\rm T}$ — Thomson scattering cross section, $\gamma = 2e^2\omega_B^2/(3mc^3)$ — radiative cyclotron line width, $\varkappa = \hbar\omega_B/(2mc^2)$ — recoil parameter (typically $\varkappa \ll \beta_T$).
The probability to scatter on the electrons with velocity in the interval (β, $\beta + \mathrm{d}\beta$) is

$$P(\beta) \sim f(\beta)\sigma_{\rm sc}\mathrm{d}\beta, \quad \Lambda \sim |\cos\theta|\exp\left(\frac{\beta_*^2}{2\beta_T^2}\right).$$





Resonance condition. Nonrelativistic approximation. Quasicoherent scattering (Zheleznykov, Litvinchuk, 1987)

$$\omega(1 - \beta \cos\theta) = \omega_B.$$

$$\beta_* = (\omega - \omega_B)/(\omega \cos\theta)$$

$$(\omega, \theta) \Longleftrightarrow (\frac{\omega - \omega_B}{\omega \cos\theta}, \theta).$$

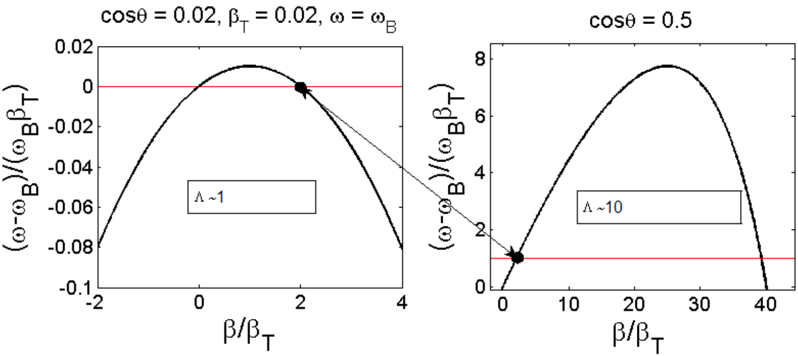
Mildly relativistic approximation

$$\omega(1 - \beta \cos\theta + \frac{\beta^2}{2}) = \omega_B.$$

Two resonance velocities:

$$\beta_{1,2} = \cos\theta \pm \sqrt{\cos^2\theta - 2\left(1 - \frac{\omega_B}{\omega}\right)}.$$

Escape due to "relativistic jumps"



The probability to increase the mean free path of a photon in q times due to relativistic jumps

$$P_{\text{jump}} \approx \frac{\beta_T}{q\sqrt{8\ln q}}.$$

Finite line width effects

Conservation laws for the scattering event

$$\omega(1 - \beta \cos\theta) = \omega'(1 - \beta \cos\theta').$$

Resonance conditions before and after the scattering event

$$\omega(1 - \beta_1 \cos\theta + \frac{\beta_1^2}{2}) = \omega_B, \tag{3}$$

$$\omega'(1 - \beta_x \cos\theta' + \frac{\beta_x^2}{2}) = \omega_B. \tag{4}$$

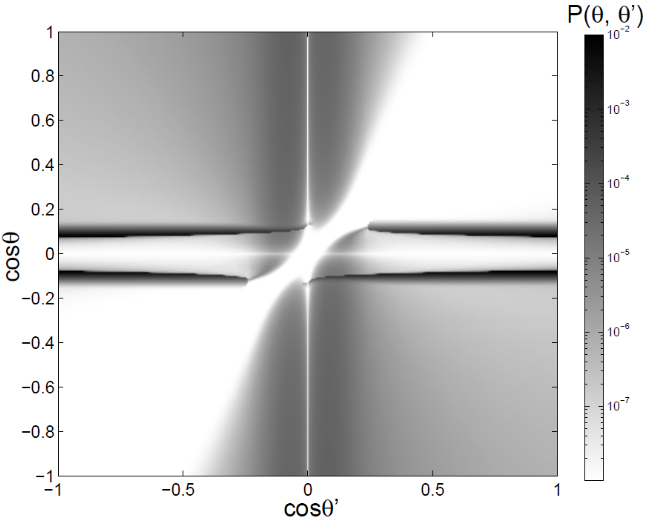
Resonance velocities β_x after the scattering

$$\frac{\beta_x^2}{2} - \beta_x \cos\theta' + \left[1 - \frac{(1 - \beta\cos\theta')}{(1 - \beta\cos\theta)}\left(1 - \beta_1 \cos\theta + \frac{\beta_1^2}{2}\right)\right] = 0. \tag{5}$$

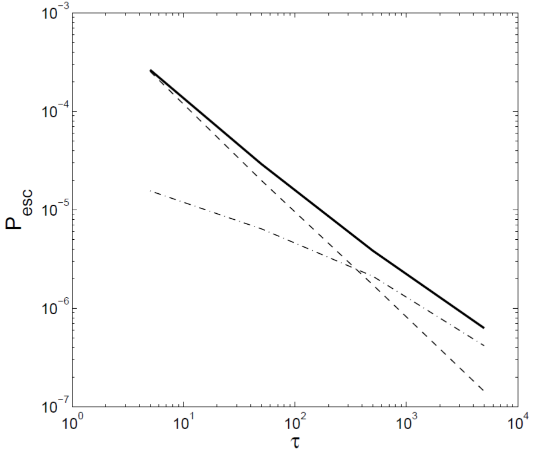
If $|\cos\theta'| \gg \beta$ then

$$\beta_x - \beta = (\beta_1 - \beta)\frac{\cos\theta}{\cos\theta'} \lesssim \gamma \ln(\beta_T/\gamma)\frac{\cos\theta}{\beta_T}. \tag{6}$$

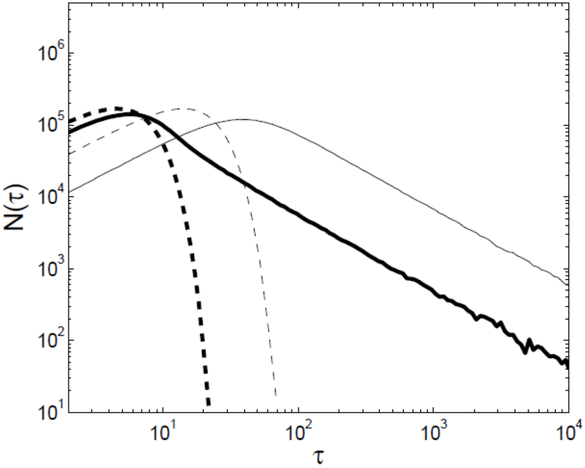
Escape from the cyclotron line core



The probability of escape from the plasma slab with optical depth τ=200 in one scattering, T = 1keV, $\gamma/\omega_B = 10^{-6}$.

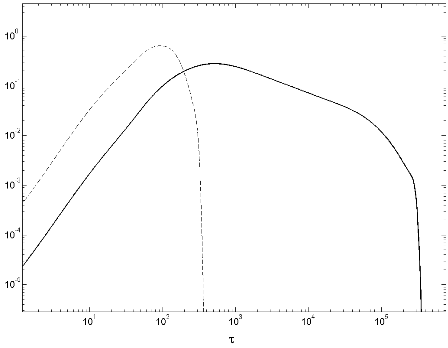


Photon escape probabilities in a single scattering versus optical depth: the solid, dashed, and dash-dotted lines indicate the total escape probability P_Σ, the escape probability due to relativistic jumps $P_{\rm jump}$, and the escape probability related to the finite natural line width P_γ, respectively. The parameters $\beta_T = 0.02$ and $\gamma/\omega_B = 10^{-6}$.



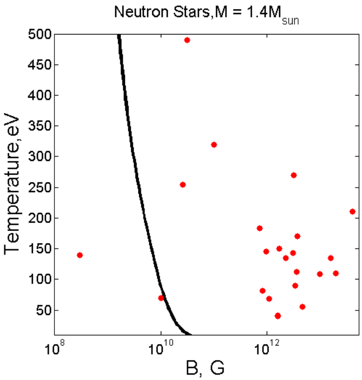
Dynamics of the smearing of the photon delta distribution $N(0) = N_0\delta(\tau)$ over the optical depths as a function of the number of scatterings. The solid thick and thin lines correspond to the distributions after 100 and 1000 scatterings for the case where the effects leading to the escape of photons from the resonance line were taken into account. The dashed lines correspond to quasi-coherent scattering. The parameters $\beta_T = 0.05$ and $\gamma/\omega_B = 10^{-6}$.

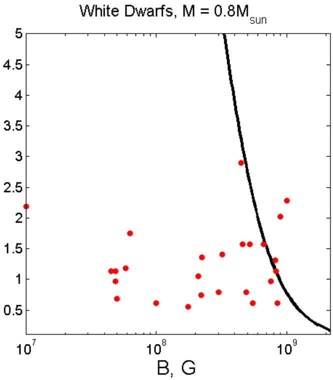
Semiinfinite atmosphere with absorption



Relative fraction of photons emitted at optical depth τ in the emergent spectra. Solid line — with redistirbution effects; dashed –– without (quasicoherent scattering). Atmospheric parameters: $T = 50\,\mathrm{eV}$, $\gamma/\omega_{\boldsymbol{B}} = 10^{-\mathbf{6}}$, $P_{\mathrm{abs}}/P_{\mathrm{sc}}(\tau = 1) = 10^{-\mathbf{6}}$.

Semiinfinite atmosphere with absorption





Quasicoherent approximation works well only to the left from the solid line. Redistribution effects become important in the right zone. Dots represent some known white dwarfs and neutron stars.

Conclusions related to the spectral redistribution

Statistically, the redistribution of photons out of the cyclotron line results in a boosted probability of their escape from a large optical depth. As our simulations show, the emerging radiation is gathered over a large interval of optical depths, spanning one or two orders of magnitude. Potentially, this causes all sorts of inhomogeneities to show up in the resulting spectrum in a more pronounced way, and the radiation transfer equation in these situations should be solved over a range of optical depths sufficiently large to capture the origin of the major part of outgoing photons.

Cyclotron wind in the atmospheres of compact stars

Radiation transfer in the atmospheres of compact stars

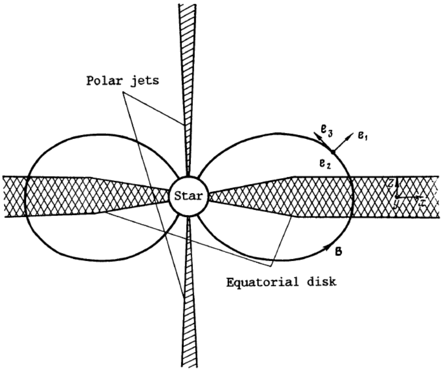


FIG. 1. Model of radiative discon.

The model of radiation diskon [Zheleznyakov, Bespalov 1990]

- Hot magnetic white dwarf or neutron star
- Cyclotron wind from the photosphere due to cyclotron radiation pressure
- Extended plasma envelope
- Polar jets along the magnetic axis

Observational appearence:

- Wide and deep depression band in spectra
- Bipolar plasma outfow
- Quasiperiodic oscillations of radiation flux

Vacuum polarisation

Dielectric and magnetic permittivities

$$\epsilon_{ik}^{(vac)} = \delta_{ik}(1-2a) + 7a\frac{B_i B_k}{B^2},$$
$$\mu_{ik}^{-1(vac)} = \delta_{ik}(1-2a) - 4a\frac{B_i B_k}{B^2},$$

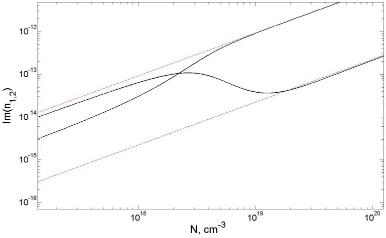
also it is assumed that $a \ll 1$, where

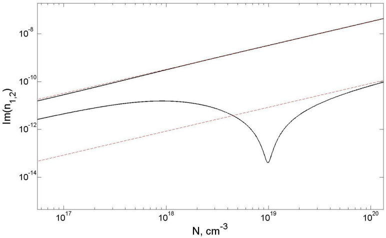
$$a = \frac{1}{45\pi}\frac{e^2}{\hbar c}\left(\frac{B}{B_{cr}}\right)^2, \qquad B_{cr} = m^2 c^3/\hbar e \simeq 4.4 \cdot 10^{13}\text{G}.$$

Dielectric permittivity of mildly relativistic plasma

$$\begin{aligned}
\epsilon_{xx} &= \epsilon_{yy} = 1 + i(b_- + b_+), \quad \epsilon_{xy} = -\epsilon_{yx} = b_- - b_+, \\
\epsilon_{xz} &= \epsilon_{zx} = -i\epsilon_{yz} = i\epsilon_{zy} = c_-, \\
\epsilon_{zz} &= 1 - \frac{\omega_{\mathrm{L}}^2}{\omega^2(1 + i\gamma/\omega)} + d_-, \\
b_+ &= \frac{i\omega_{\mathrm{L}}^2}{2\omega^2(1 + (\omega_B + i\gamma)/\omega)}, \quad b_- = \frac{\sqrt{\pi}\omega_{\mathrm{L}}^2}{2\omega^2\beta_T^2} \times \frac{\varpi(\xi_1) - \varpi(\xi_2)}{\xi_2 - \xi_1}, \\
c_- &= i\sqrt{\frac{\pi}{2}}\frac{\omega_{\mathrm{L}}^2 \sin\theta}{\omega\omega_B\beta_T} \times \frac{\xi_1\varpi(\xi_1) - \xi_2\varpi(\xi_2)}{\xi_2 - \xi_1}, \\
d_- &= \frac{\omega_{\mathrm{L}}^2 \sin^2\theta}{\omega_B^2} \times \frac{\xi_1(1 + i\sqrt{\pi}\xi_1\varpi(\xi_1)) - \xi_2(1 + i\sqrt{\pi}\xi_2\varpi(\xi_2))}{\xi_2 - \xi_1}, \\
\xi_{1,2} &= \frac{1}{\sqrt{2}\beta_T}\left(\cos\theta \pm \sqrt{\cos^2\theta - 2\left(1 - \frac{\omega_B(1 - \varkappa\sin^2\theta)}{\omega + i\gamma}\right)}\right), \\
\varpi(Z) &= \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{e}^{-u^2}\,\mathrm{d}u}{Z - u}.
\end{aligned}$$

Normal waves. Influence of vacuum polarization





Dependence of opacity coefficients $\mathrm{Im}\, n_{1,2}$ on electron number density. Left figure is for $\omega = 0.96\omega_B$, $\cos\theta = 0.5$, $B = 2.56 \times 10^{11}$G, $T = 1$keV, right one is for $\omega = 1.04\omega_B, \cos\theta = 0.5, B = 2.56 \cdot 10^{11}$G, $T = 1$ keV. Solid lines — solution with vacuum polarization, dashed — pure plasma without vacuum polarization. Top curves reffers for the extraordinary waves, bottom for the ordinary ones.

Opacity coefficients

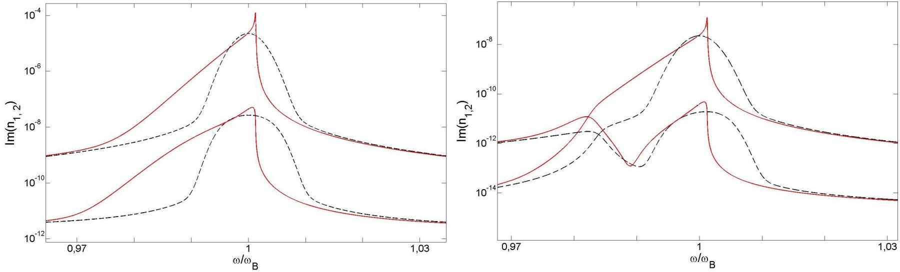


Рис.: Spectral dependence of opacity coefficients $\mathrm{Im}\, n_{1,2}$: (a) – extraordinary wave, (b) – ordinary wave. Right figure corresponds to $N_e = 10^{18}\mathrm{cm}^{-3}, \cos\theta = 0.05, B = 2.56 \cdot 10^{11}\mathrm{G},\ \ T = 1\,\mathrm{keV}$, left figure corresponds to $N_e = 10^{21}\mathrm{cm}^{-3}, \cos\theta = 0.05, B = 2.56 \cdot 10^{11}\mathrm{G},\ \ T = 1\,\mathrm{keV}$. Solid lines show calculations with relativistic effects included, dashed without (quasicoherent approximation).

Transfer equations. General view

The intensity vector $\mathbf{J}$:

$$\mathbf{J} = \frac{1}{2}\begin{pmatrix} I+Q \\ I-Q \\ 2U \\ 2V \end{pmatrix}. \tag{7}$$

The evolution of intensity vector is described by transfer equations:

$$\frac{\mathrm{d}\mathbf{J}}{\mathrm{d}s} = -\mathbf{M}\cdot\mathbf{J} + \mathbf{S}_{\mathrm{em}} + \mathbf{S}_{\mathrm{sc}}, \tag{8}$$

where s is the coordinate along the ray. Source functions $\mathbf{S}_{\mathrm{em}}$ and $\mathbf{S}_{\mathrm{sc}}$ describe emission of plasma and rescattering respectively. $\mathbf{M}$ is the transfer matrix, which describes absorption, scattering and evolution of polarization.

The transfer matrix for mildly relativistic plasma

$$\mathbf{M} = \frac{1+\varepsilon}{2}\left[\begin{array}{cccc} 2\mathrm{Im}\,\varsigma_{11} & 0 & \mathrm{Im}\,\varsigma_{12} & -\mathrm{Re}\,\varsigma_{12} \\ 0 & 2\mathrm{Im}\,\varsigma_{22} & -\mathrm{Im}\,\varsigma_{12} & -\mathrm{Re}\,\varsigma_{12} \\ -2\mathrm{Im}\,\varsigma_{12} & 2\mathrm{Im}\,\varsigma_{12} & \mathrm{Im}\,(\varsigma_{11}+\varsigma_{22}) & \mathrm{Re}\,(\varsigma_{11}-\varsigma_{22}) \\ -2\mathrm{Re}\,\varsigma_{12} & -2\mathrm{Re}\,\varsigma_{12} & \mathrm{Re}\,(\varsigma_{22}-\varsigma_{11}) & \mathrm{Im}\,(\varsigma_{11}+\varsigma_{22}) \end{array}\right]. \quad (9)$$

$$\begin{aligned} \varsigma_{11} &= E_{xx} + 2a, \\ \varsigma_{12} &= -\varsigma_{21} = E_{xy}, \\ \varsigma_{22} &= E_{yy} + 2a + 4a\sin^2\theta, \end{aligned}$$

Rescattering source function

$$\mathbf{S}_{\rm sc} = N_e \left(\frac{e^2}{mc^2}\right)^2 \int\int \mathrm{d}\,\Omega' \mathrm{d}\omega' \int \mathrm{d}\,\beta f(\beta) \mathbf{R}(\mathbf{k}' \to \mathbf{k}) \cdot \mathbf{J}(\mathbf{k}') \\ \cdot \delta(\omega'(1 - \beta\cos\theta') - \omega(1 - \beta\cos\theta)), \quad (10)$$

where ω is the frequency of a photon, $\mathrm{d}\Omega$ the element of solid angle ($\mathrm{d}\Omega = 2\pi \mathrm{d}\phi \mathrm{d}\cos\theta$), θ the angle with respect to magnetic field. Apostrophed quantities correspond to the values before the scattering. Scattering matrix $\mathbf{R}$ is

$$\mathbf{R}(\mathbf{k}' \to \mathbf{k}) = \begin{bmatrix} |a|^2 & |b|^2 & \mathrm{Re}\,(ab) & \mathrm{Im}\,(ab) \\ |c|^2 & |d|^2 & \mathrm{Re}\,(cd) & \mathrm{Im}\,(cd) \\ 2\mathrm{Re}\,(ac) & 2\mathrm{Re}\,(bd) & \mathrm{Re}\,(ad+bc) & \mathrm{Im}\,(ad-bc) \\ -2\mathrm{Im}\,(ac) & -2\mathrm{Im}\,(bd) & -\mathrm{Im}\,(ad+bc) & -\mathrm{Re}\,(ad-bc) \end{bmatrix} \quad (11)$$

Here

$$
\begin{aligned}
a &= \cos\theta' \cos\theta\, C^{(e)}(\Delta\phi) + \sin\theta' \sin\theta, \\
b &= \cos\theta\, S^{(e)}(\Delta\phi), \\
c &= -\cos\theta\, S^{(e)}(\Delta\phi), \\
d &= C^{(e)}(\delta\phi),
\end{aligned}
$$

where $\Delta\phi = \phi - \phi'$, ϕ is the asimuthal angle.

$$
\begin{aligned}
C^{(e)}(\Delta\phi) &= \frac{1}{2}\left(\frac{\omega' e^{i\Delta\phi}}{\omega'(1-\beta\cos\theta'_B + \beta^2/2) - \omega_B(1-\varkappa\sin^2\theta'_B) + i\gamma} + \frac{\omega' e^{-i\Delta\phi}}{\omega' + \omega_B + i\gamma}\right), \\
S^{(e)}(\Delta\phi) &= \frac{1}{2i}\left(\frac{\omega' e^{i\Delta\phi}}{\omega'(1-\beta\cos\theta'_B + \beta^2/2) - \omega_B(1-\varkappa\sin^2\theta'_B) + i\gamma} - \frac{\omega' e^{-i\delta\phi}}{\omega' + \omega_B + i\gamma}\right).
\end{aligned}
$$

Emission

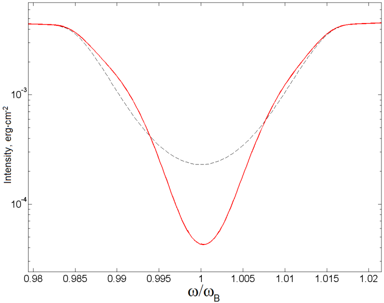
Source function due to emission is

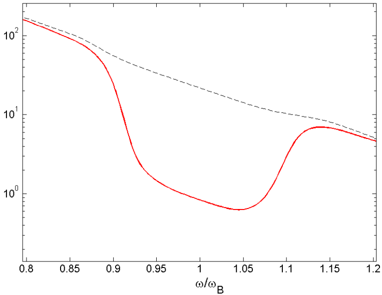
$$\mathbf{S}_{\rm em} = \frac{\varepsilon k_0 B_\omega}{2} \begin{pmatrix} \mathrm{Im}\,\varsigma_{11} \\ \mathrm{Im}\,\varsigma_{22} \\ 0 \\ -2\mathrm{Re}\,\varsigma_{12} \end{pmatrix},$$

where $B_\omega = \hbar\omega^3/(4\pi^3 c^2)(\exp(\hbar\omega/T) - 1)^{-1}$ is the Planck function. Parameter ε is the probability of absorption in the scattering event. According to Pavlov, Panov, 1976; Nagel, Ventura, 1982; Potekhin, 2008

$$\varepsilon = \frac{\nu^{\rm ff}}{\gamma}, \quad \nu^{\rm ff} = \frac{4}{3}\sqrt{\frac{2\pi}{mT}}\frac{N_e e^4}{\hbar\omega}(1 - \mathrm{e}^{-\hbar\omega/T})\Lambda^{\rm ff}. \qquad (12)$$

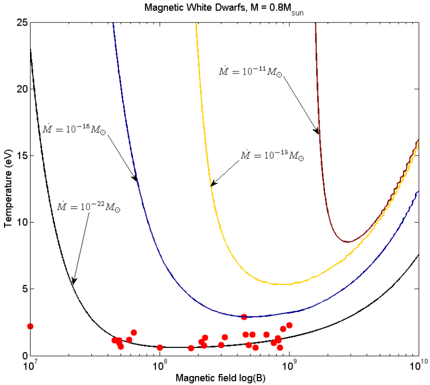
Examples of calculated emergent spectra





Left figure — typical spectrum of the emergent radiation in extraordinary wave for a hydrogen white dwarf atmosphere. The dashed and solid lines represent solutions obtained without and with frequency redistribution due to relativistic jumps, respectively. Parameters are $T = 5$eV, $B = 10^9$G, $M = 0.8M_\odot$. Right figure — spectrum of the emergent radiation for a hydrogen neutron star atmosphere. The dashed and solid lines represent solutions obtained without and with vacuum polarization, respectively. Parameters are $T = 500$ eV, $B = 1 \cdot 10^{11}$ G, $M = 1.4M_\odot$.

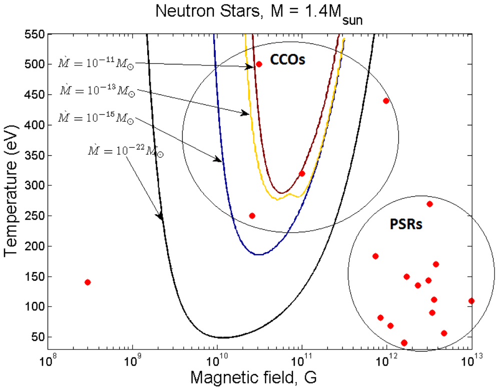
Cyclotron wind in the atmospheres of white dwarfs



- Pure hydrogen atmosphere.
- $M = 0.8 M_\odot$, $R \approx 10^9$ cm.
- Vacuum polarization and redistiribution of radiation
- $\dot{M} = 4\pi R^2 N_{\mathrm{S}} c_{\mathrm{S}}$

Points represent parameters of known white dwarfs (Kulebi et al., 2009; Kawka et al., 2004). Candidates: EUVE J0317-855, SDSS J100356.32+053825.6, HE 1043-0502, SDSS J234605.44+385337.7, GD 229

Cyclotron winds in the atmospheres of neutron stars



- Pure hydrogen atmosphere.
- $M = 1.4 M_\odot$,
  $R = 1.2 \cdot 10^6$см.
- Vacuum polarization and redistiribution of radiation
- $\dot M = 4\pi R^2 N_s c_s$

Candidates: RX J0821-43, 1E 1207.4-5209, CXOU J185238.6+004020 and other CCOs

Conclusions related to the cyclotron wind

Under LTE assumption, the cyclotron wind forms in atmospheres of magnetic white dwarfs for $T \sim 2 - 10\,\mathrm{eV}$ and $B \sim 10^8 - 10^9$ G and in atmospheres of neutron stars for $T \sim 200 - 500$ eV and $B \sim 10^{10} - 10^{11}$ G. The value of a mass loss rate is up to $10^{-11} M_{\odot}/yr$. The outflowing plasma can freely move along the magnetic field lines under the influence of radiation driven force. The motion across the field is strongly limited. Some part of the ejected plasma forms two polar jets along the open field lines. The rest of the wind is accumulated in the closed domain of the magnetosphere and may appear as a dense plasma disk near the magnetic equator.

Publications

1. M.A. Garasyov, E.V. Derishev, Vl.V. Kocharovsky. Influence of relativistic effects and vacuum polarization on the transfer of gyroresonance radiation and the stability of the atmospheres of compact stars // Astronomy Letters, 2008, V. 34, P. 305.
2. M.A. Garasyov, E.V. Derishev, Vl.V. Kocharovsky. The influence of frequency redistribution on the transfer of gyroresonant photons in the atmospheres of compact stars: Monte-Carlo analysis // Radiophysics and Quantum Electronics, 2011, V. 53, P. 679.
3. M. Garasyov, E. Derishev, V. Kocharovsky and Vl. Kocharovsky. Spectral redistribution of gyroresonant photons in magnetized atmospheres of isolated compact stars // Astronomy & Astrophysics, 2011, V. 531, P. L14.
4. M.A. Garasyov, E.V. Derishev, Vl.V. Kocharovsky. Statistics of the frequency redistribution for gyroresonance radiation in the atmospheres of compact stars // Astronomy Letters, 2011, V. 37, P. 699.