



# Recent Progress in Understanding of Astrophysical Turbulence

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Ginzburg conference on Physics, LPI, 1.6.2012

# Unsolved problems of physics and astrophysics

*Physics–Uspekhi* **45** (2) 205–211 (2002)

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*PHYSICS OF OUR DAYS*

PACS numbers: 01.55.+b, 01.90.+g

## **On some advances in physics and astronomy over the past three years**

V L Ginzburg

DOI: 10.1070/PU2002v045n02ABEH001187

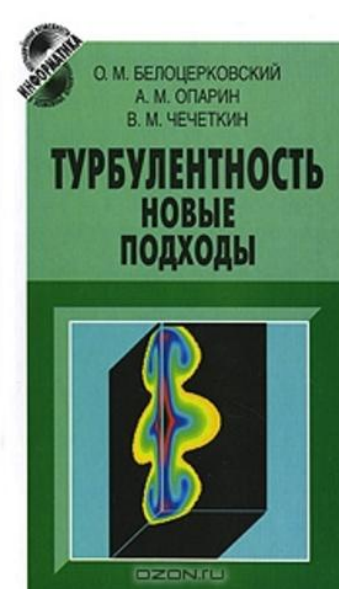
1. Controlled nuclear fusion.
2. High-temperature and room-temperature superconductivity (HTSC and RTSC).
3. Metallic hydrogen. Other exotic substances.
4. Two-dimensional electron liquid (anomalous Hall effect and some other effects).
5. Some problems of solid state physics (heterostructures in semiconductors, quantum wells and dots, metal–insulator junctions, charge and spin density waves, mesoscopics).
6. Second-order phase transitions and related transitions. Some examples of such transitions. Cooling (laser cooling, in particular) to ultralow temperatures. Bose–Einstein condensation in gases.
7. Surface physics. Clusters.
8. Liquid crystals. Ferroelectrics. Ferrotoroids.
9. Fullerenes. Nanotubes.
10. The behavior of a substance in superstrong magnetic fields.
11. Nonlinear physics. Turbulence Solitons. Chaos. Strange attractors.

# Turbulence in astrophysics

(Belotserkovsky, Oparin & Chechetkin 2002)

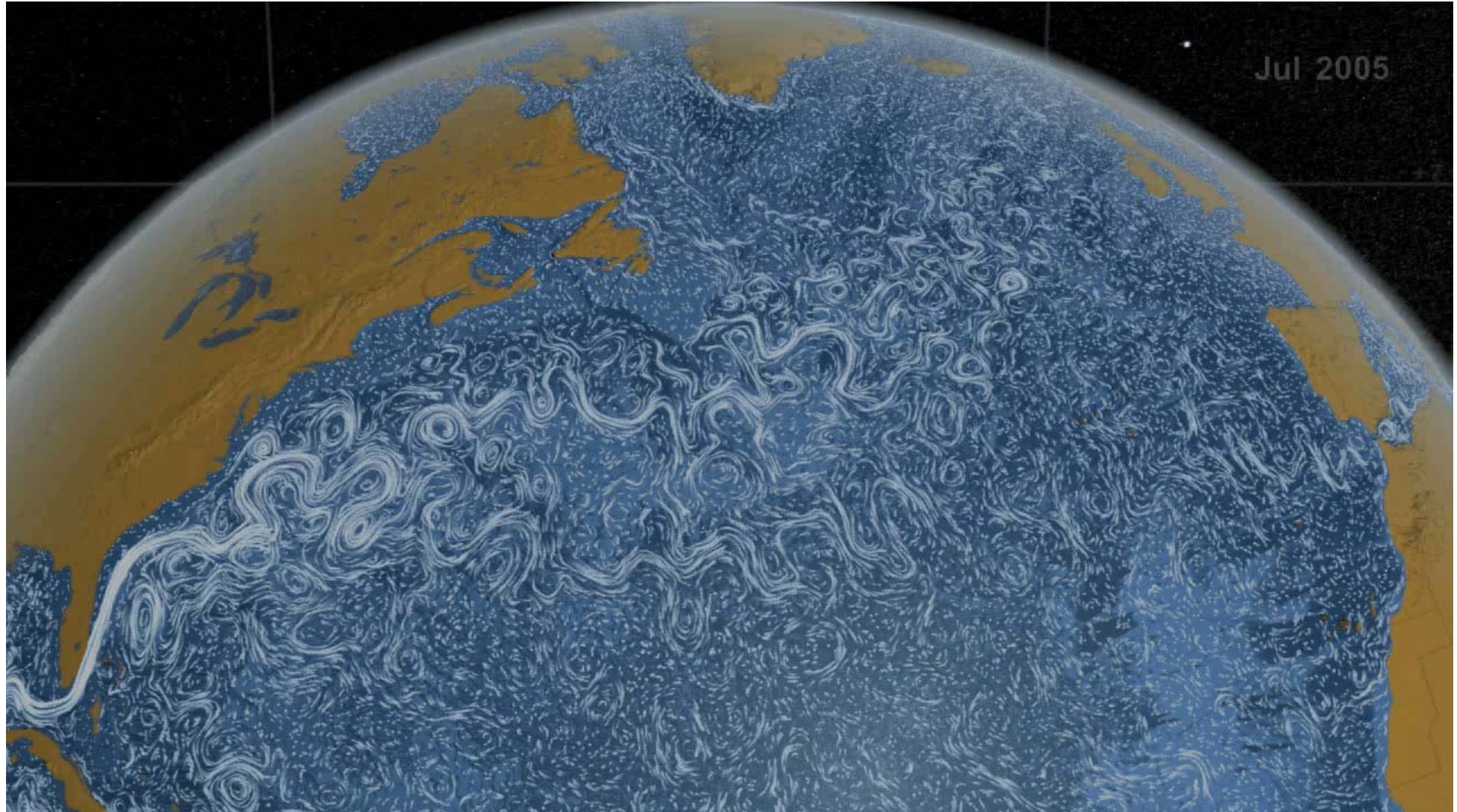
What is the primary state of aggregation of matter in the Universe?

- ✓ ~~Solid~~
- ✓ ~~Liquid~~
- ✓ ~~Gaseous~~
- ✓ ~~Plasma~~
- ✓ Turbulent





# Terrestrial Ocean (NASA 2005-2007)



# Unsolved problems of turbulence: outline

- Hydro
- MHD

# Unsolved problems of turbulence: outline

- **Microscopic aspects:**

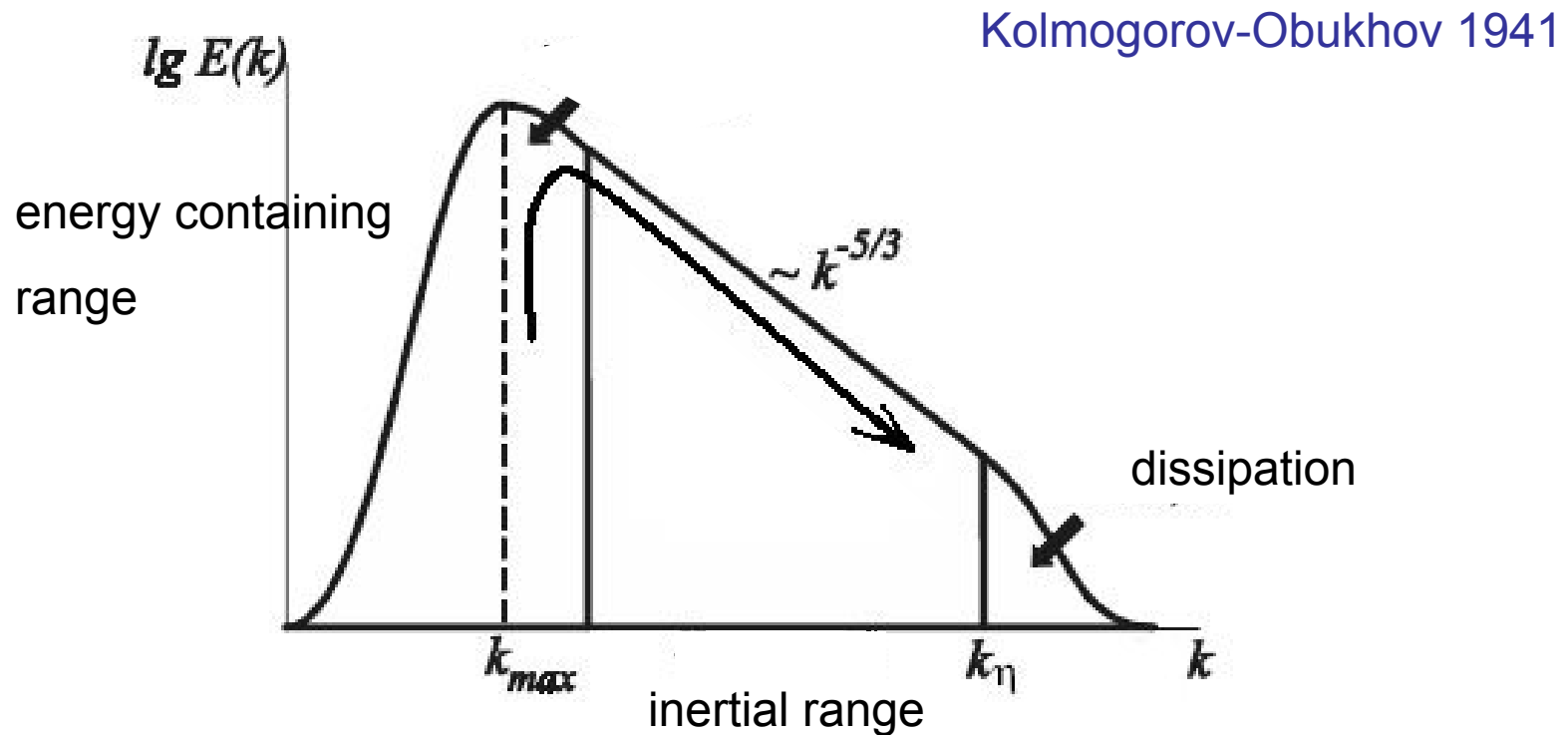
«Geometry» of turbulence => statistical properties:  
spectra, correlation, ...

Direct or inverse cascade => self-organization

- **Macroscopic aspects:**

Transport properties -> beyond turbulent viscosity

# Incompressible isotropic turbulence: direct cascade



# Turbulence in incompressible medium: monofractal

Turbulent motion = one mode = one physics

Solenoidal (incompressible fluid):

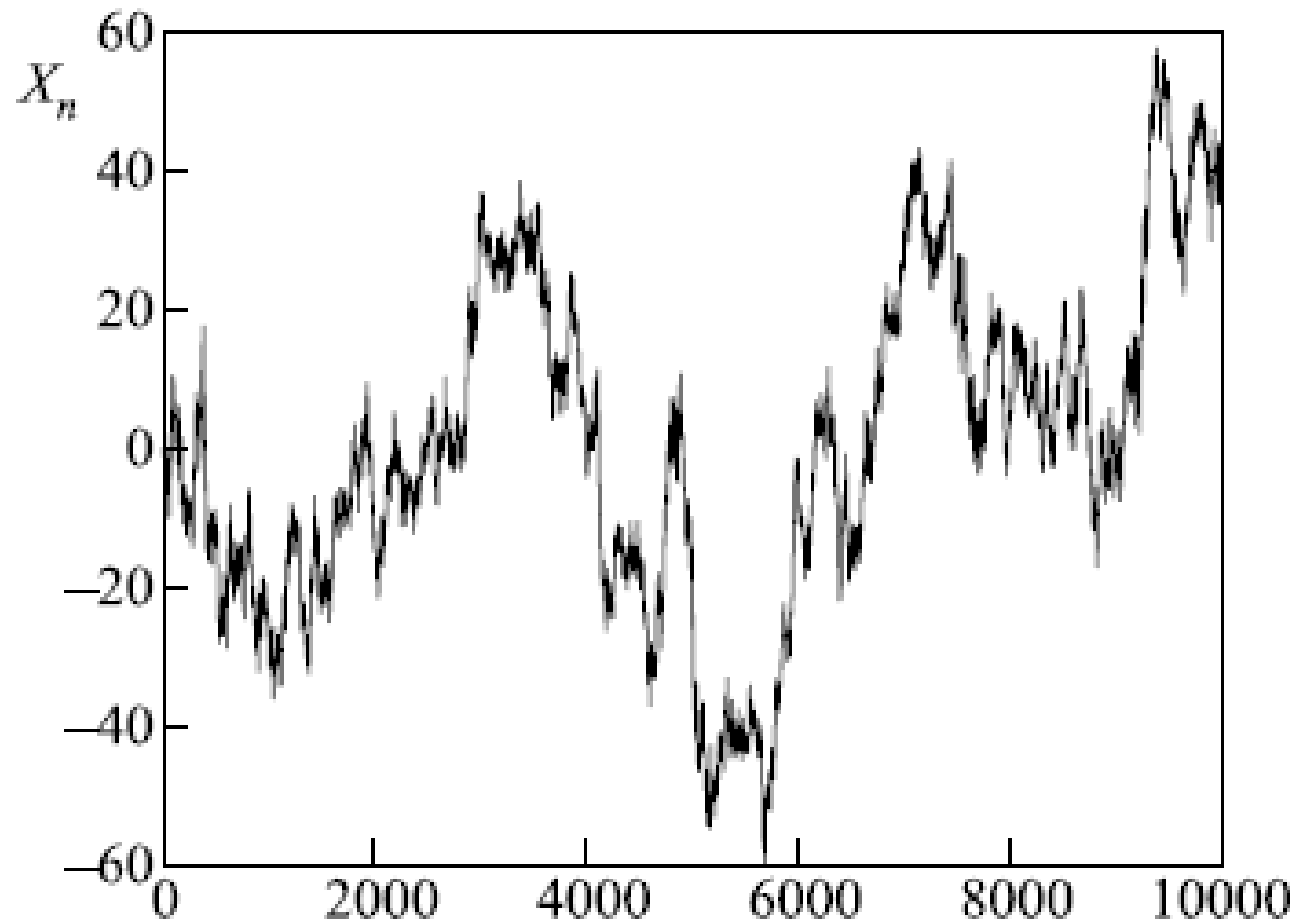
$$E(k) \sim k^{-5/3} \quad (\text{Kolmogorov, Obukhov 1941});$$

Alfvén (analogue of incompressible in MHD):

$$E(k) \sim k^{-3/2} \quad (\text{Iroshnikov, Kraichnan 1964-65}).$$



# Monofractal turbulence



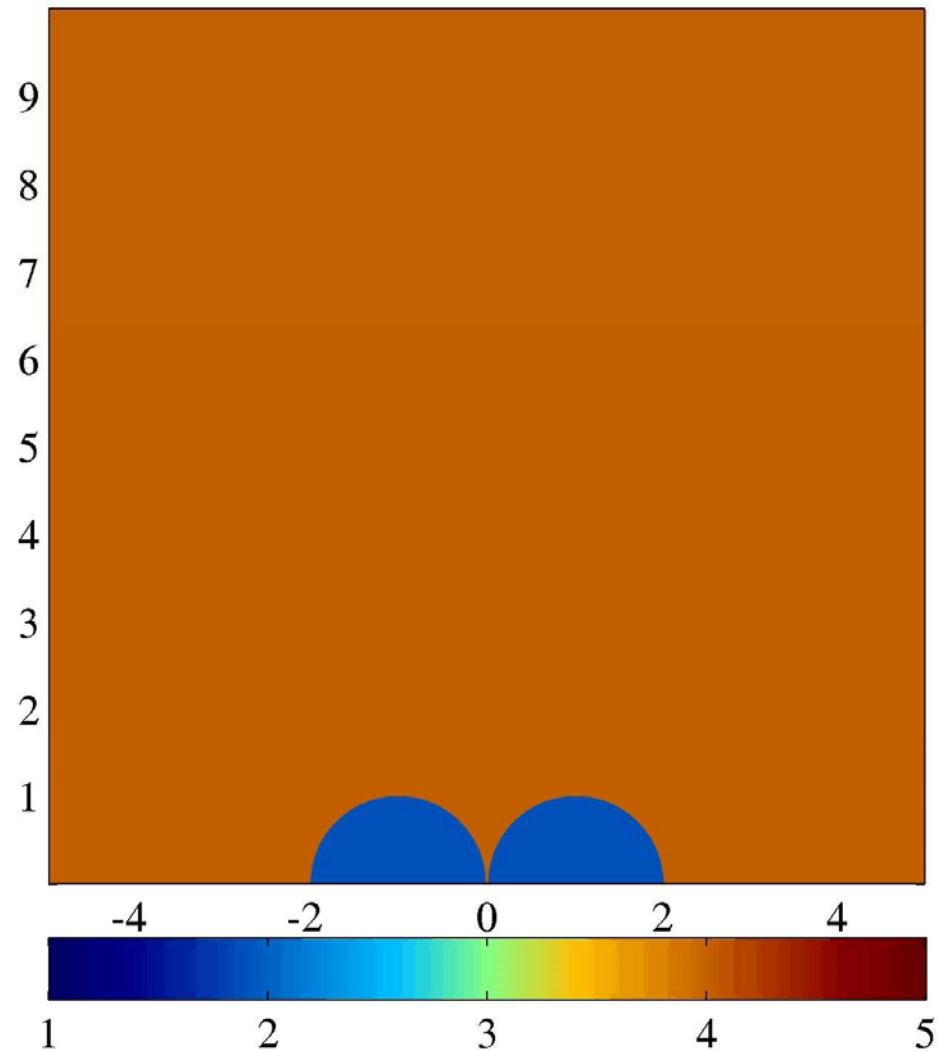
Saichev & Filimonov 08

# Compressible turbulence

- Additional dimensional parameter  $c_s \Rightarrow$  dimensional approach fails!
- For deducing of spectra (in the hope of universality) one needs other physical principles

# Compressible turbulence: intermittency

$\lg[T](x,y), t=0.000e+000$



Eremin +IK 2012

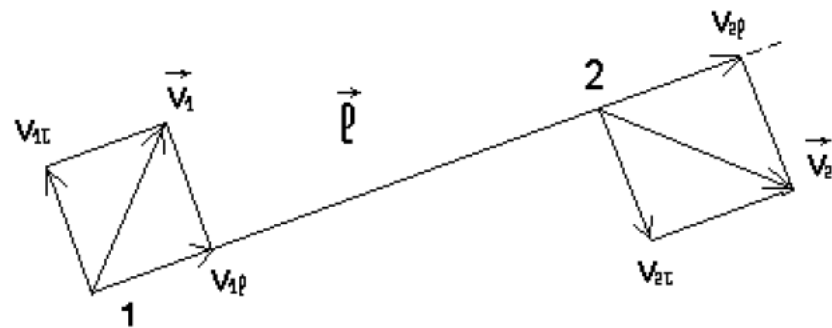
# Diagnostics of turbulence geometry: structural functions & scaling exponents

Velocity structural function

$$S_p(l) = \langle \Delta v_i^p \rangle$$

where

$$\Delta v_i = v_i(\vec{r} + \vec{l}) - v_i(\vec{r}).$$

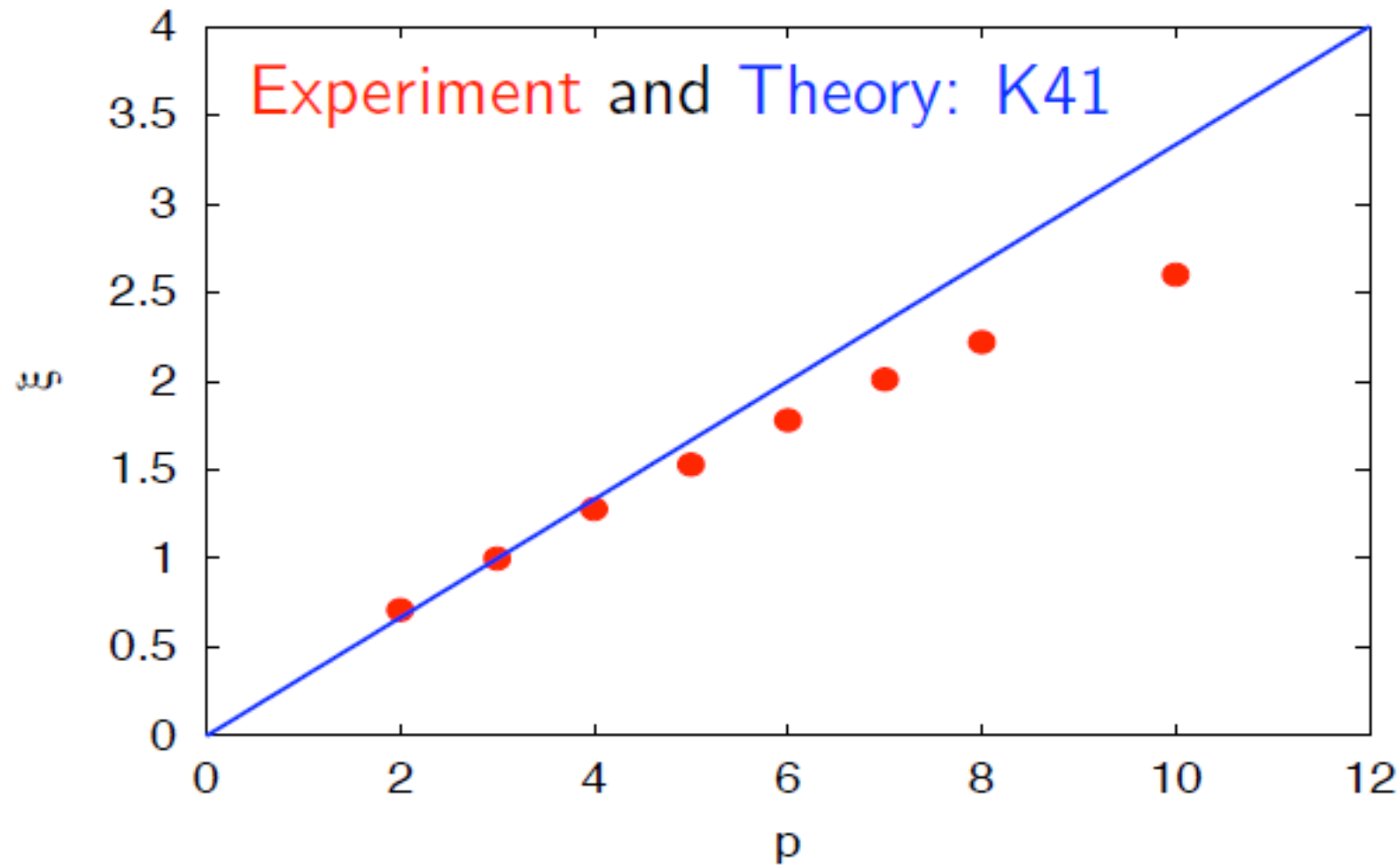


# Diagnostics of turbulence geometry: structural functions & scaling exponents

Kolmogorov theory states  $\zeta_p = p / 3$

if there is a scaling  $S_p(l) \propto l^{\zeta_p}$

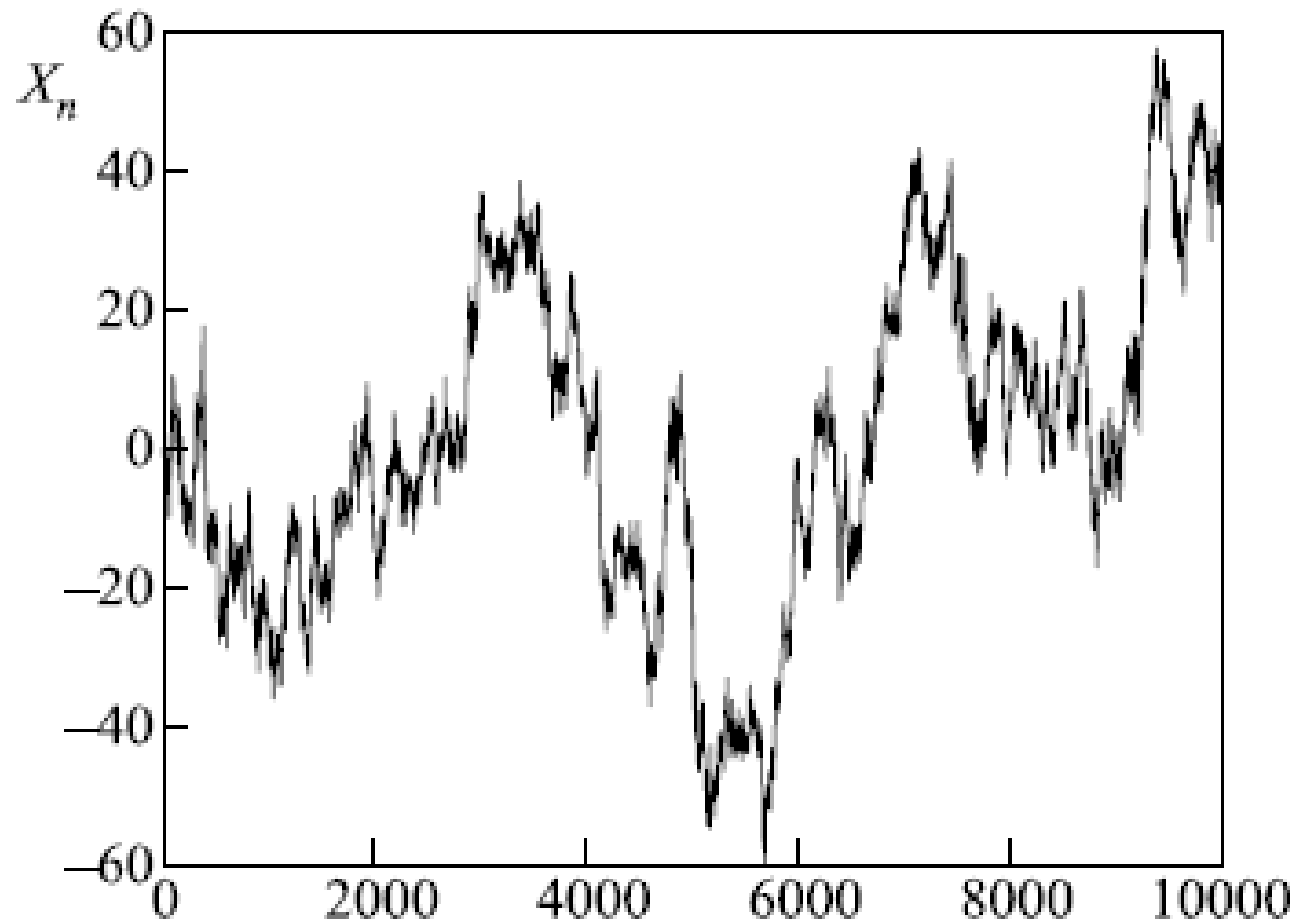
# Monofractal turbulence: Experiment confronts theory



From Grauer 09

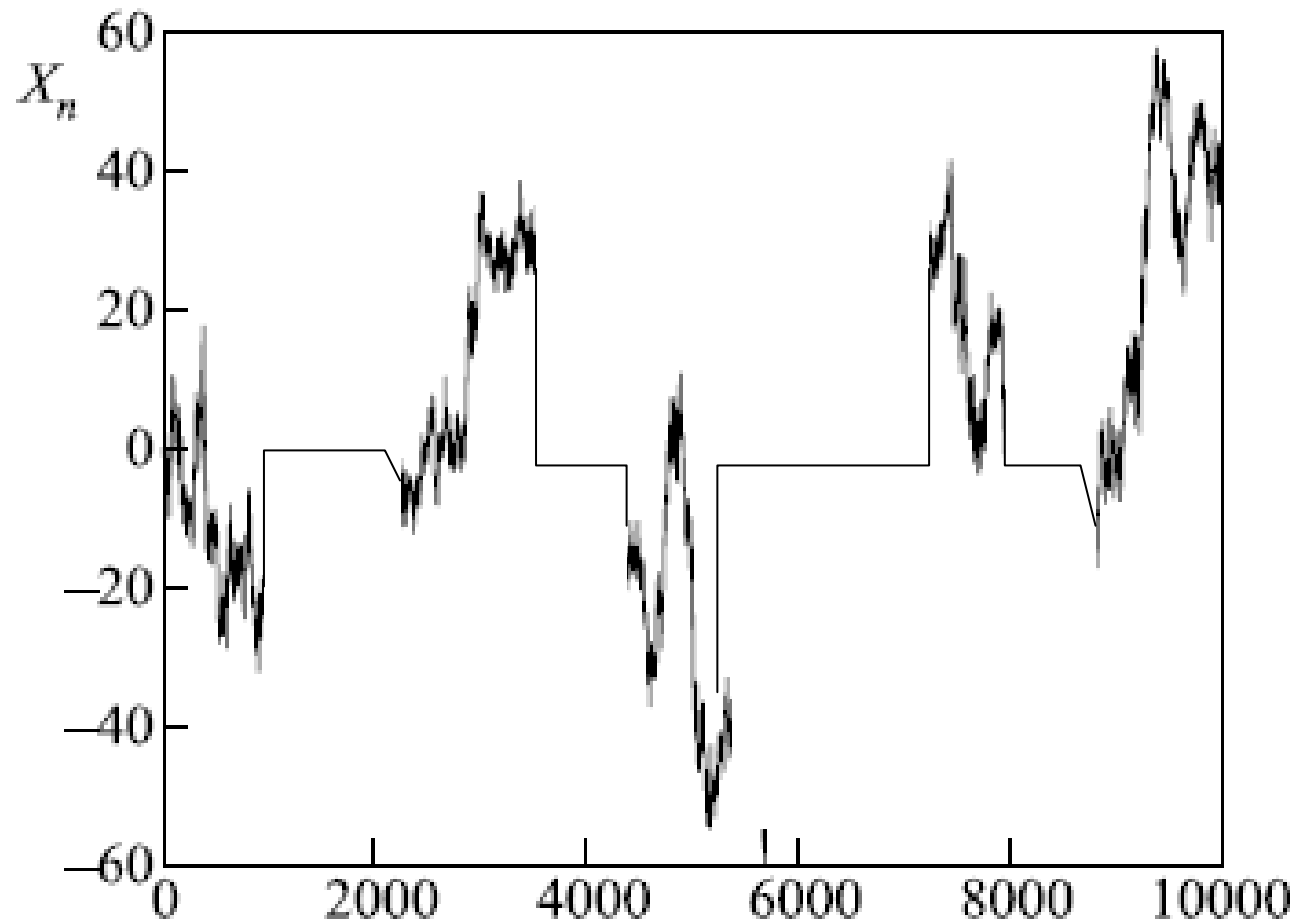


# Monofractal turbulence



Saichev & Filimonov 08

# Monofractal turbulence + intermittency



# Turbulence as monofractal occupying part of volume ( $\beta$ -model)

$$S_p(l) \sim l^{\frac{p}{3} + (3-D)(1-\frac{p}{3})}$$

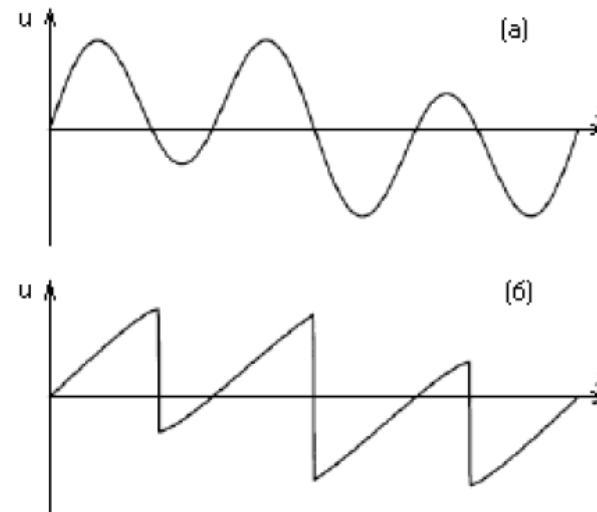
$D \leq 3$  – dimension of a fractal. At  $D=3$   $\zeta_p \rightarrow$  Kolmogorov one.

# Turbulence as bifractal

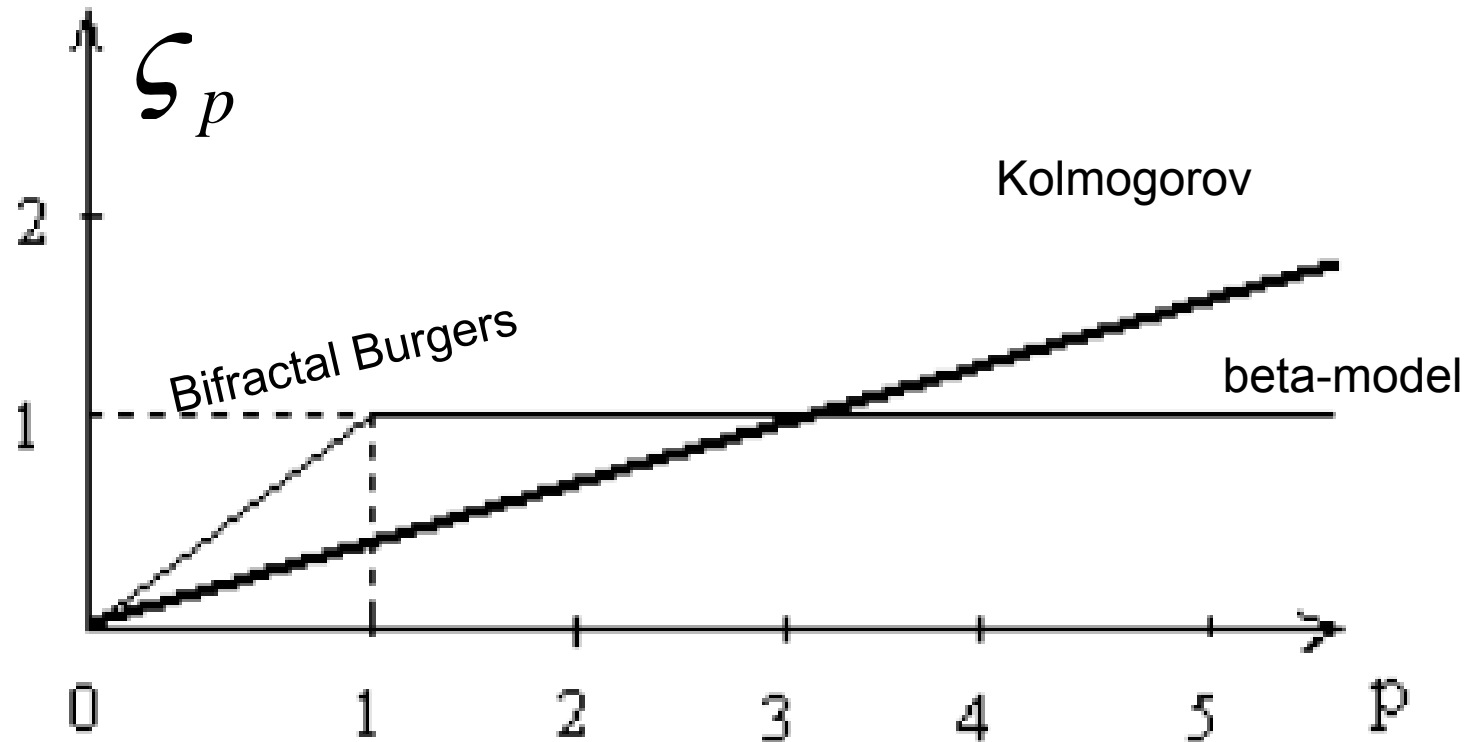
Turbulent motion = 2 modes = 2 physics

Compressible gas: acoustic mode (shocks)  
and vortical mode (vortices)

**Example:**  
Burgers 1D-turbulence

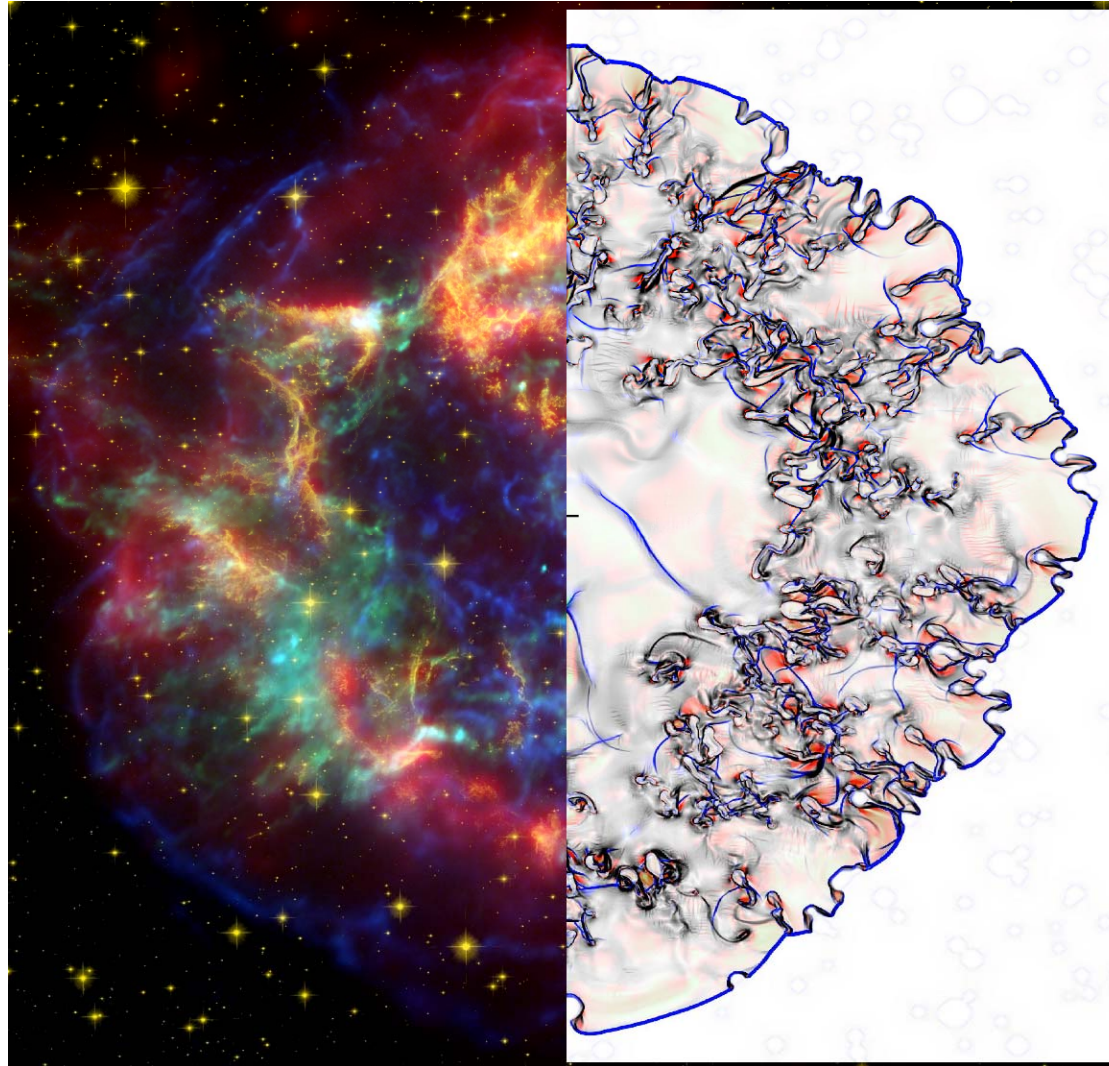


# Turbulence geometry: scaling exponents



# SNR: observations vs numerical modeling

SNR 1680

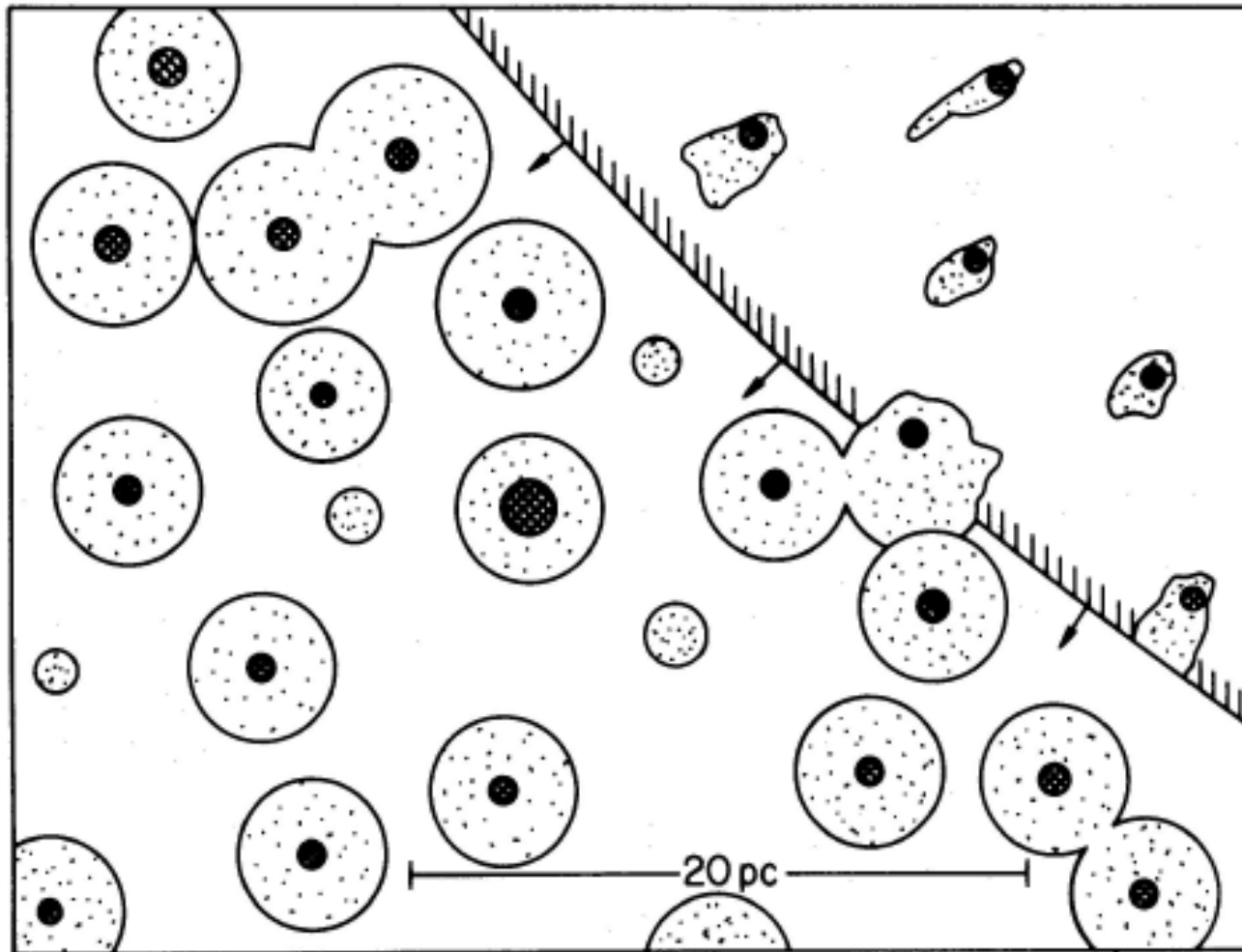


Korolev,  
Kovalenko  
2012

Mesh  
 $512^2 \times 1024$

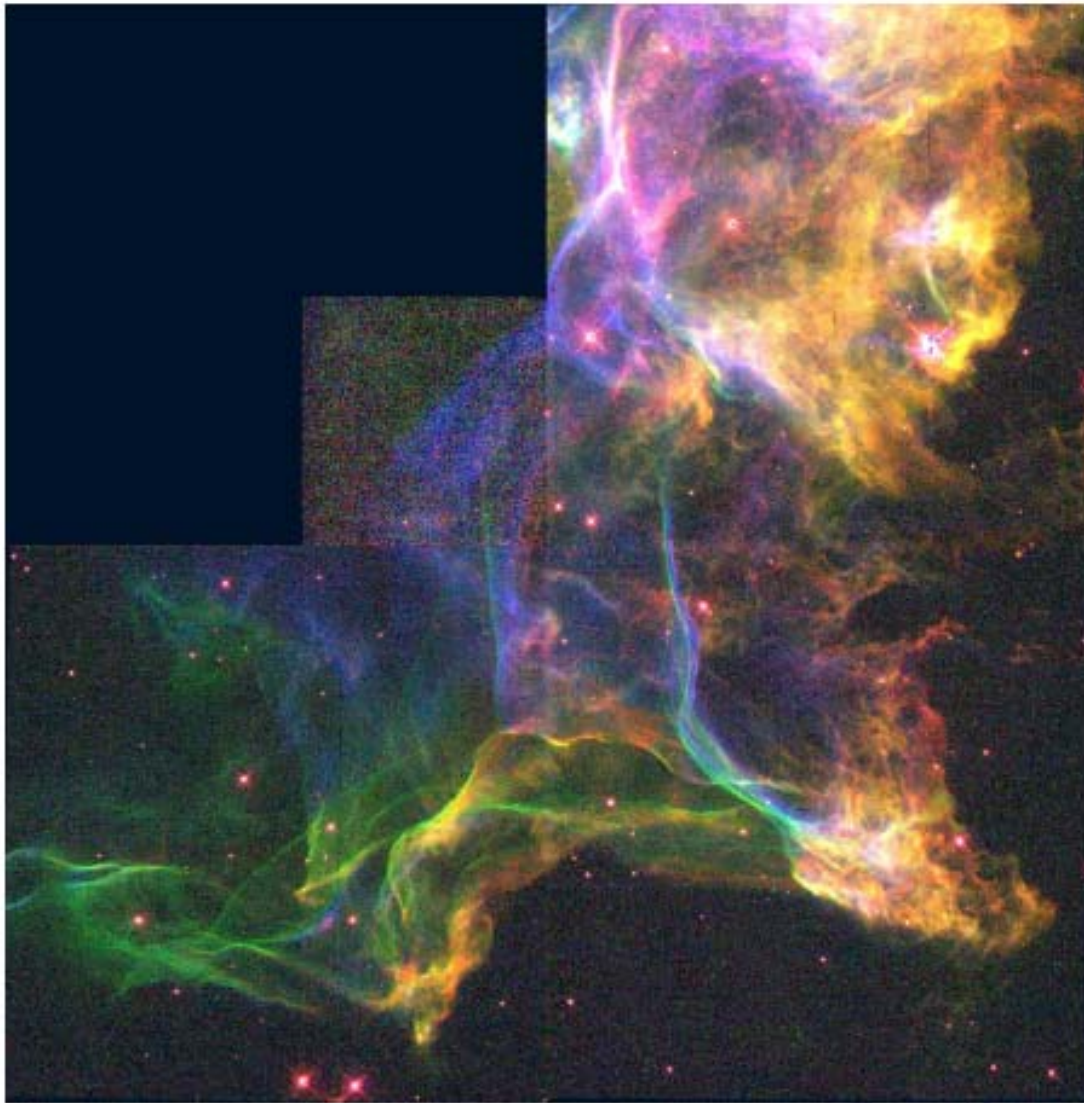


# Interaction of SNRs with clouds: theorist's vision

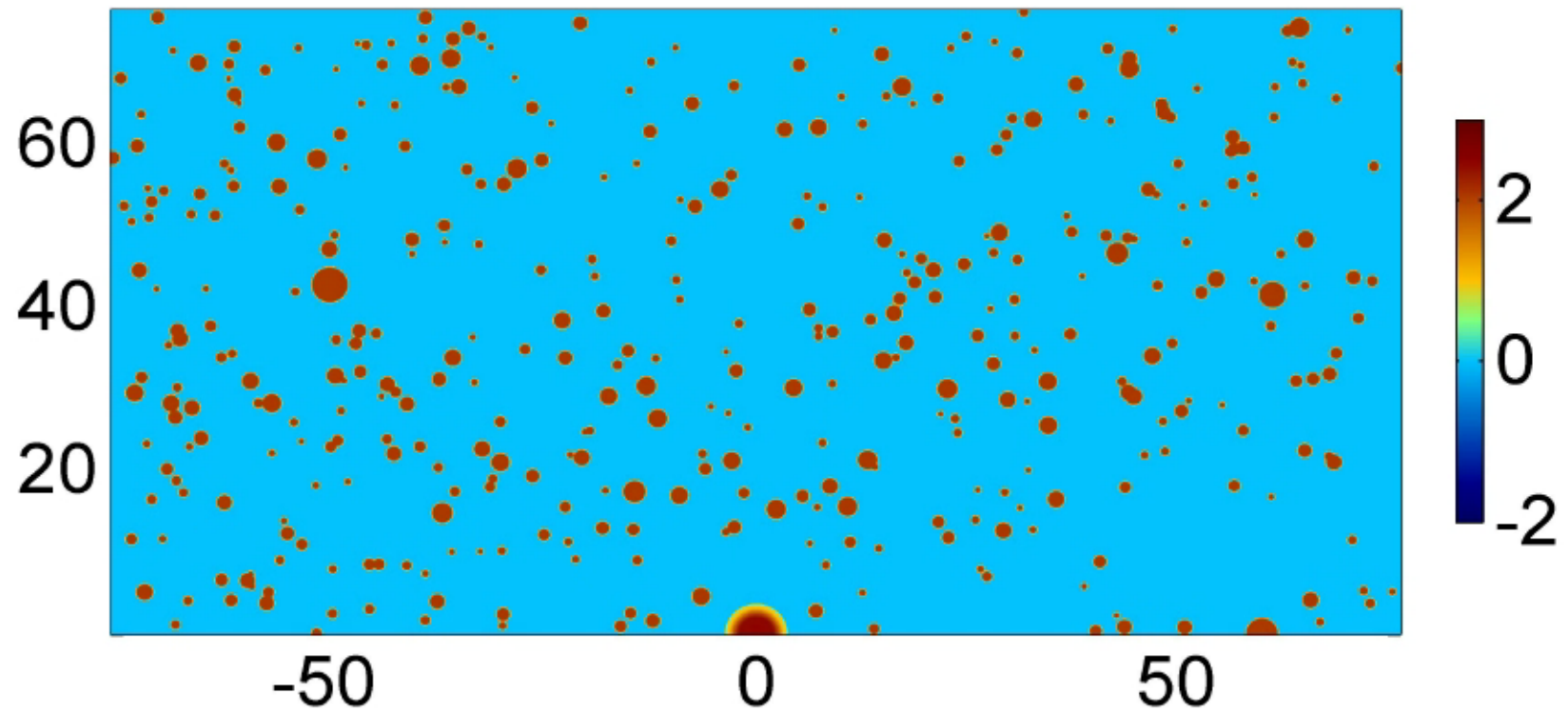


from McKee C. F. & Ostriker J. P., 1977

# Interaction of SNRs with clouds: Cygnus Loop

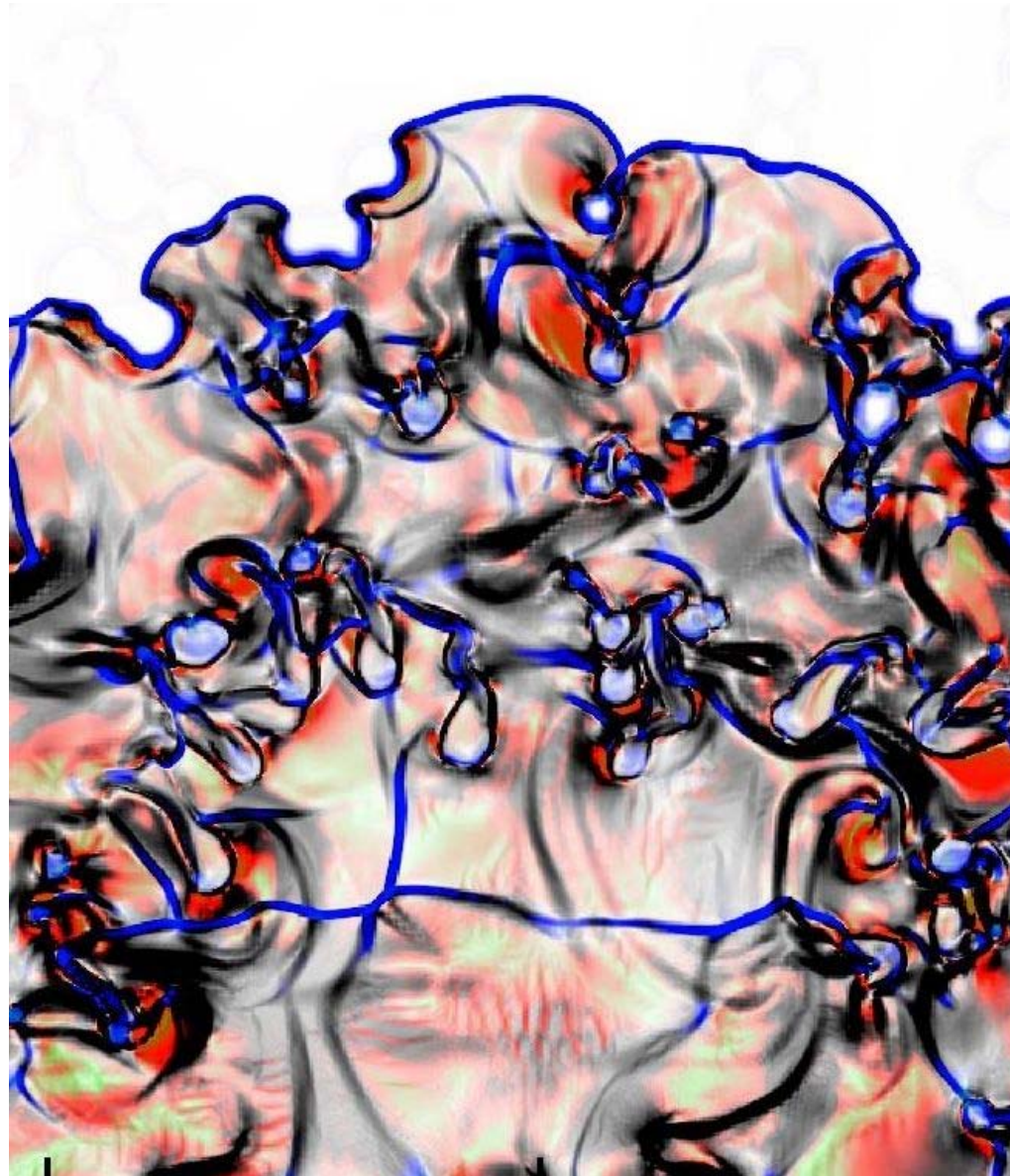


$\lg(\rho(r, z, t = 0.0050))$

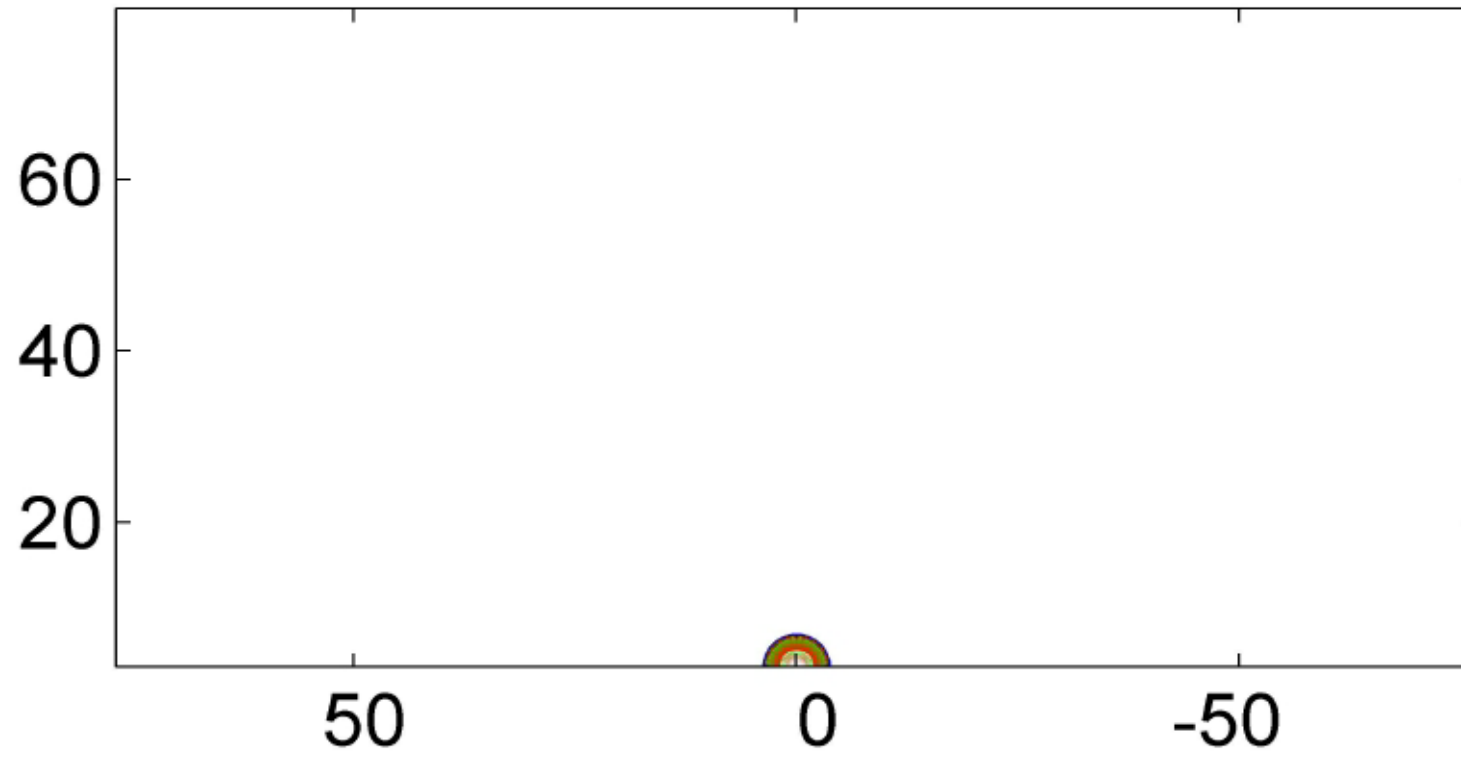




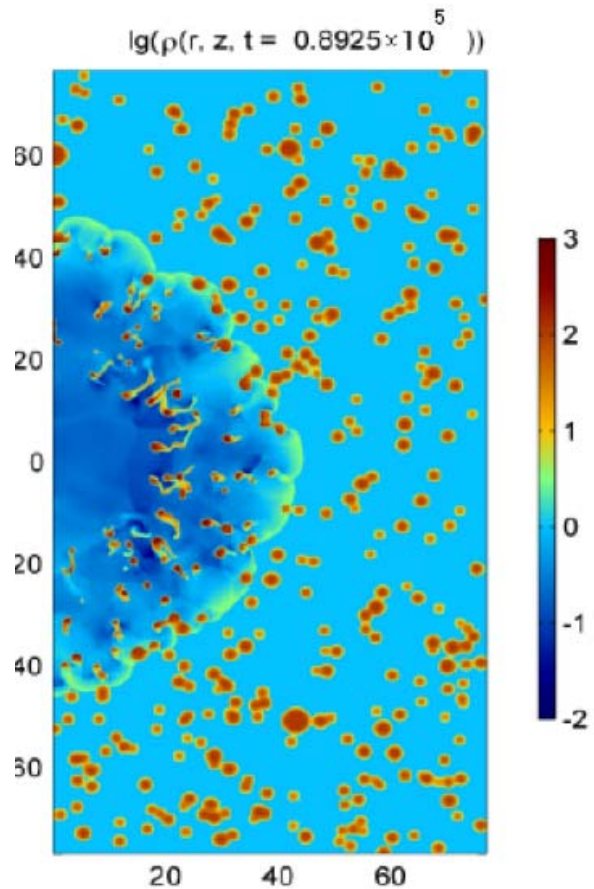
# Rate of strain discontinuity analyzer



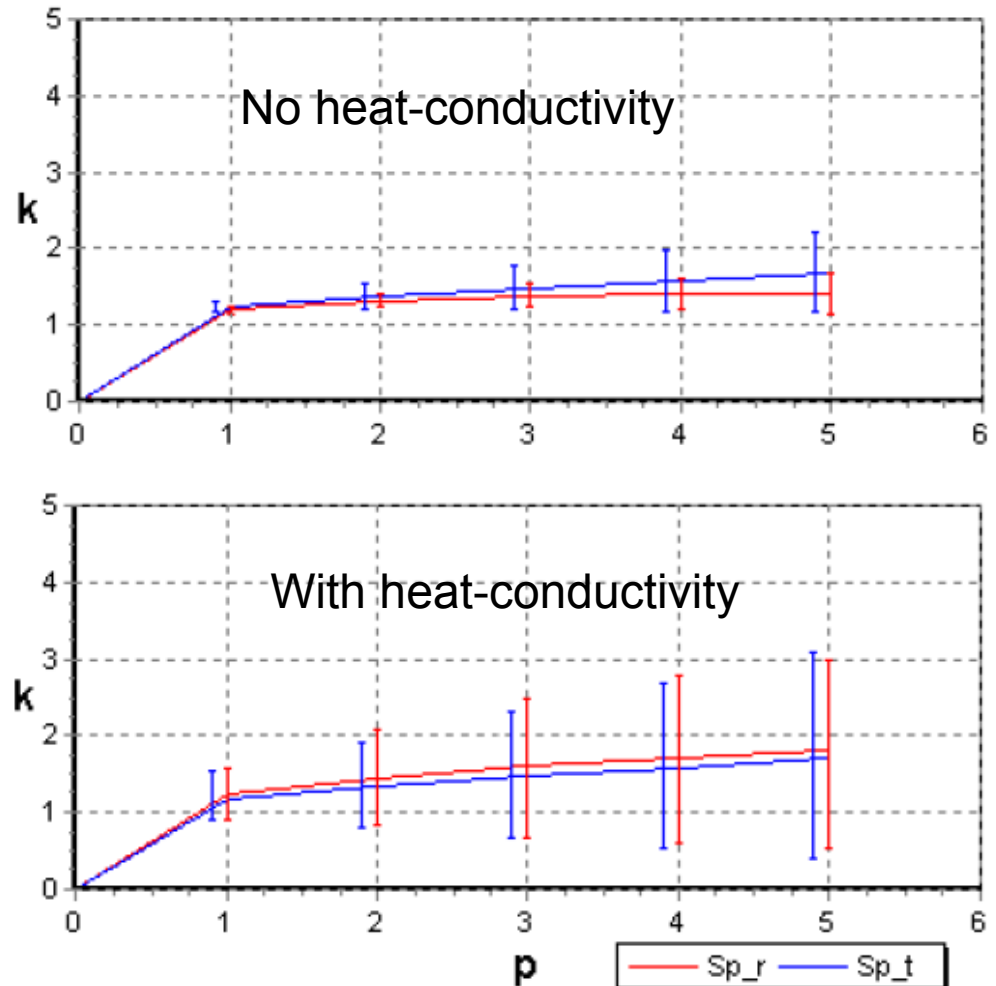
$D(r, z, t = 0.0050)$



# Scaling exponents in the model of ISM turbulized by SNRs (Korolev +IK 2012)



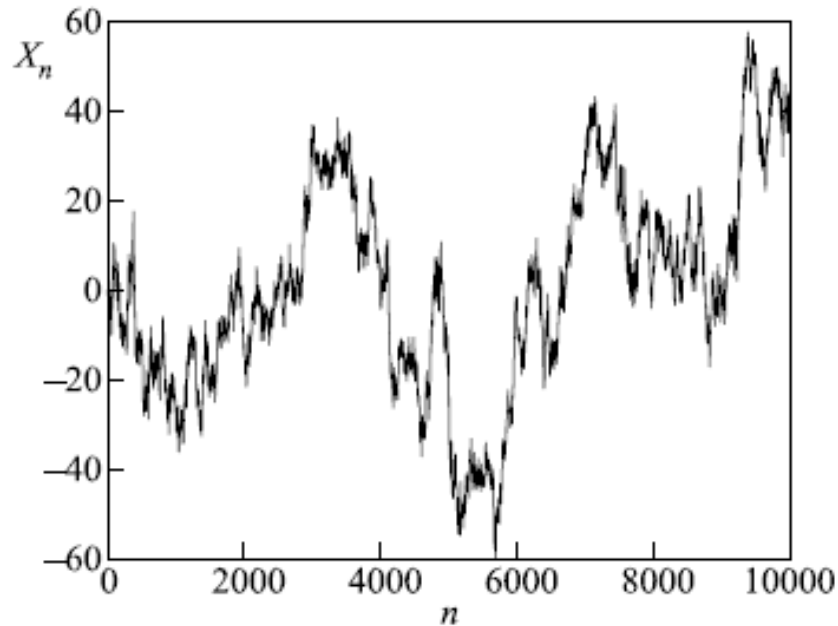
Gin:



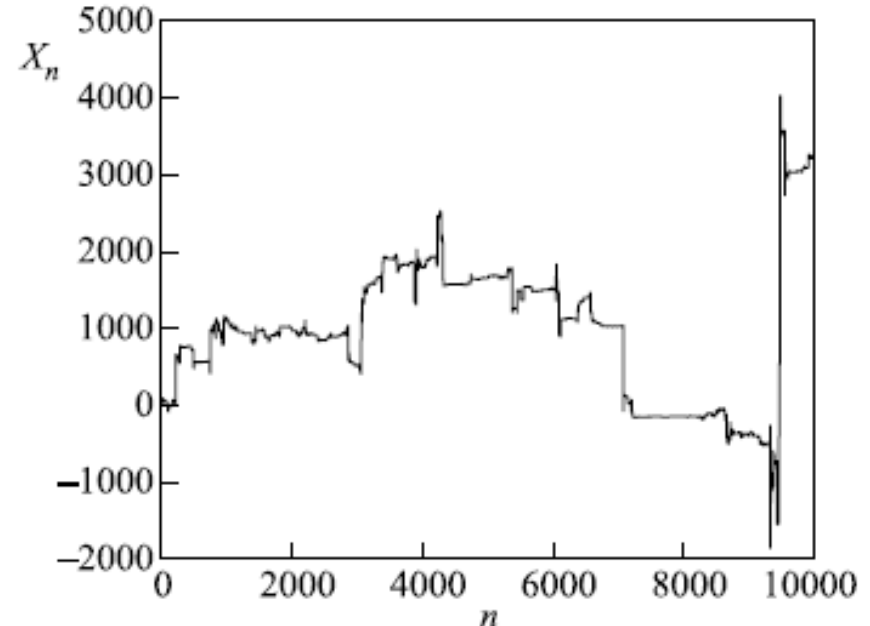


# Multifractal turbulence

(Frisch & Parisi 1985)



Monofractal

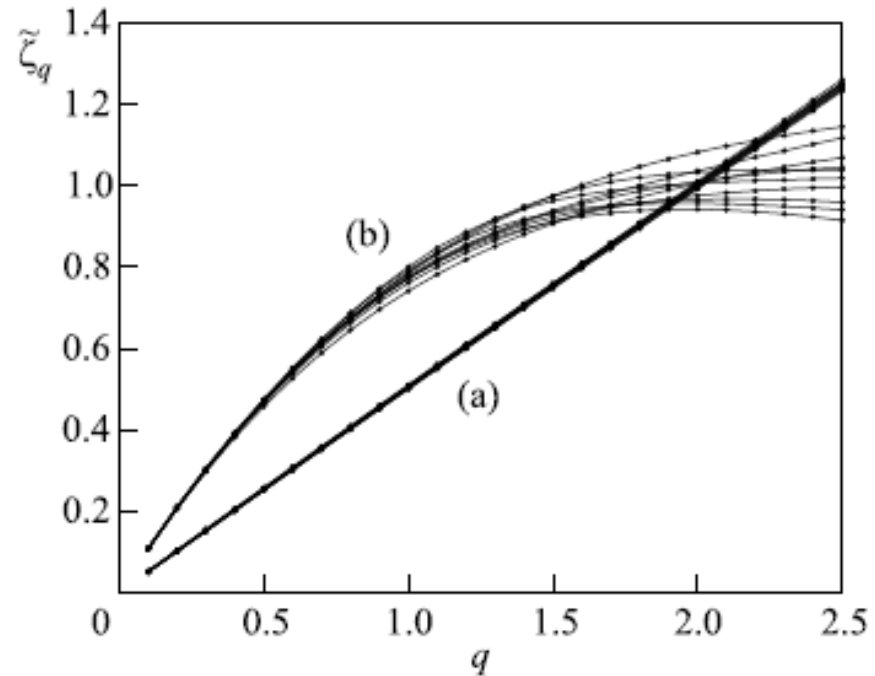


Multifractal

from Saichev & Filimonov 08

# Structural functions and scaling exponents

In astrophysical context:  
Norman & Ferrara 96

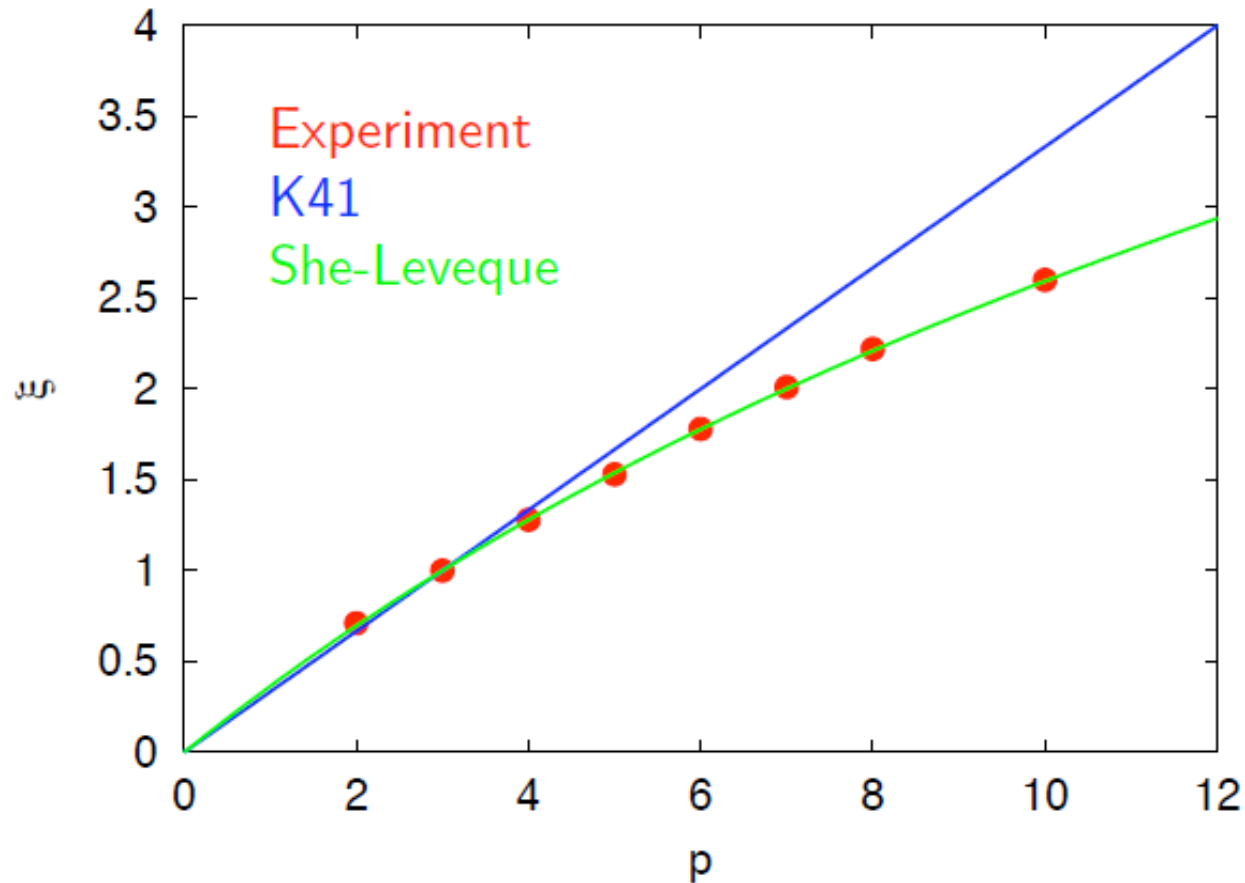


a) Monofractal turbulence  
(Kolmogorov-like)

b) Multifractal turbulence  
(Saichev & Filimonov 08)

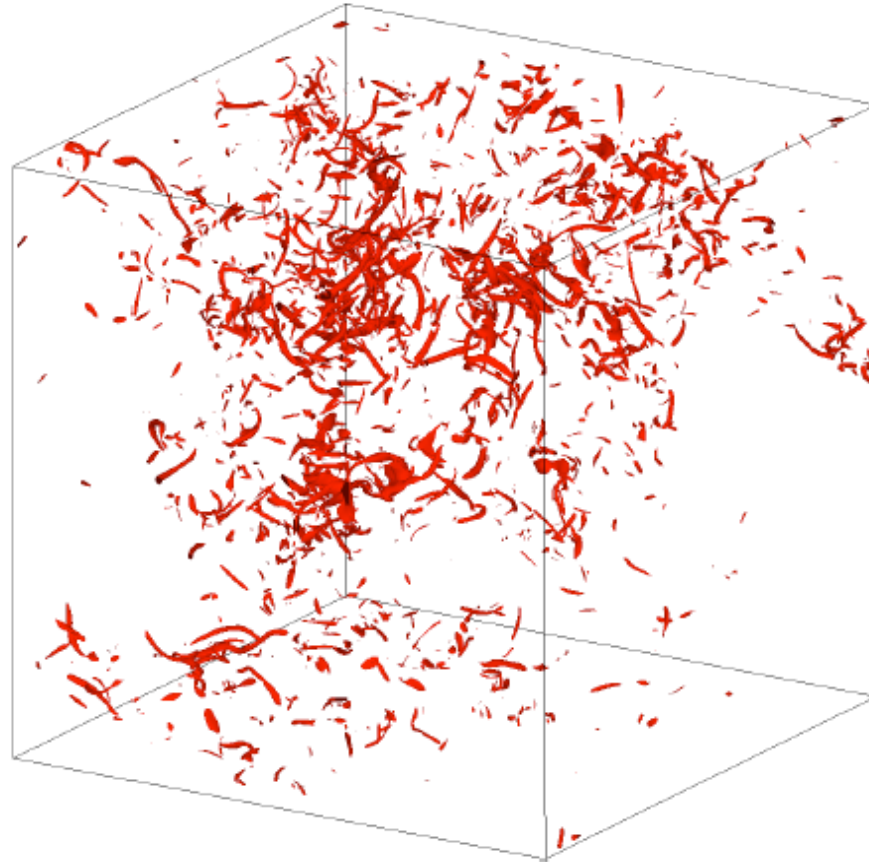
# Multifractal turbulence: role of coherent structures

She-Leveque 1994



From Grauer 09

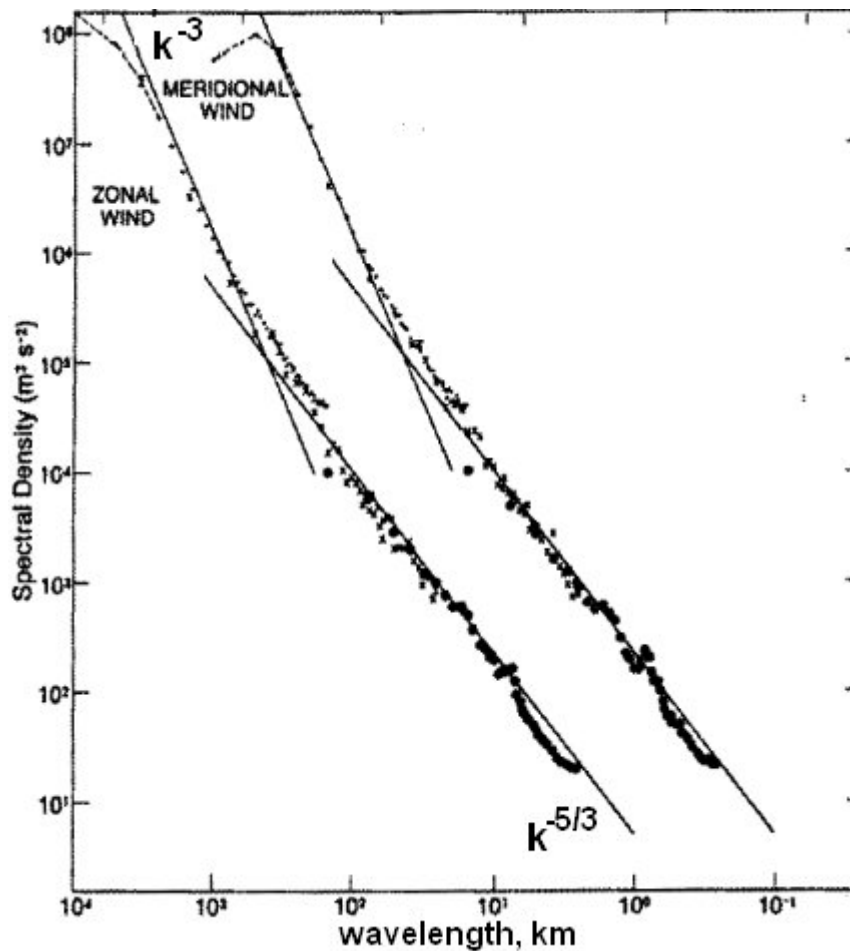
# Multifractal turbulence: role of coherent structures



vortex tubes

From Grauer 09

# Inverse cascade



Kraichnan 1967

Zakharov 1970-80

If there exists 2<sup>nd</sup> invariant  
except energy

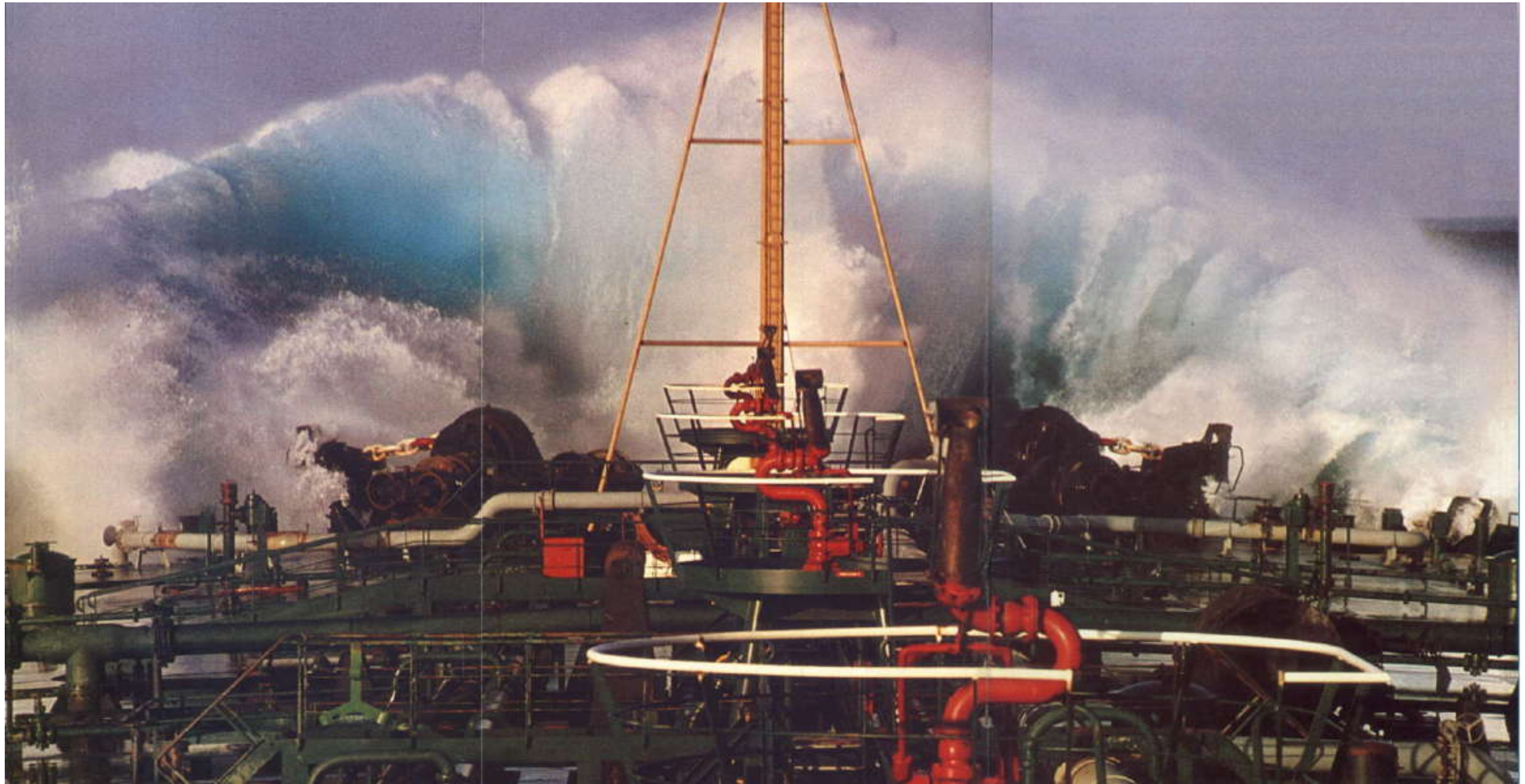
3D: energy and helicity

Self-organization:

Morning Glory (Queensland, Australia)



# Inverse cascade: freak waves



from V.E.Zakharov

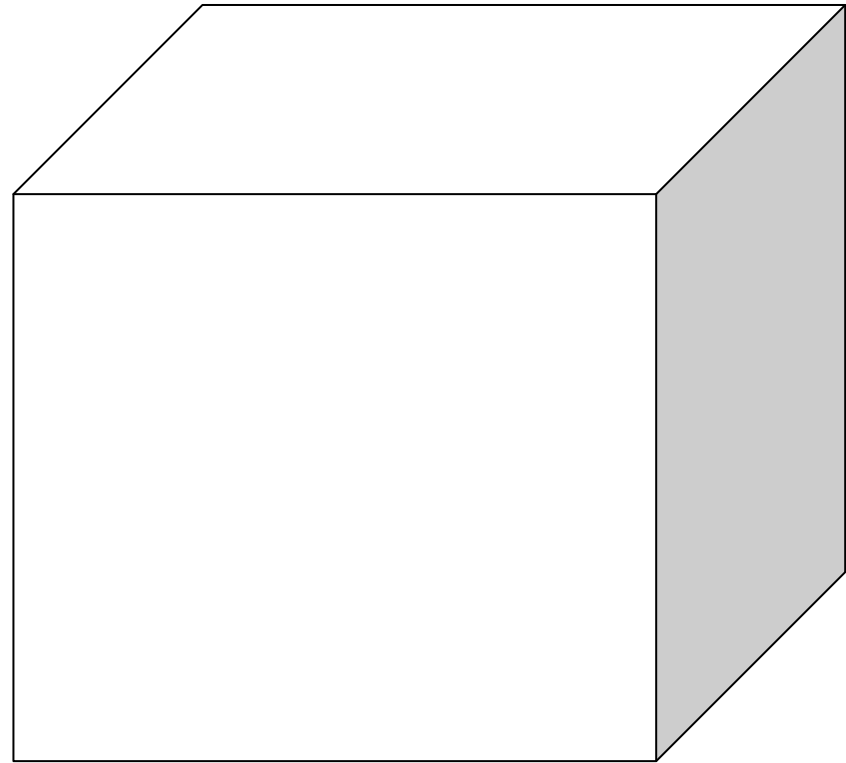
# 'Turbulence in a box': search for universality

$$v_t + (v \cdot \nabla)v = -\nabla p + \frac{1}{\text{Re}} (\Delta + \Delta^{-1})v + f$$

$$\nabla \cdot v = 0$$

or

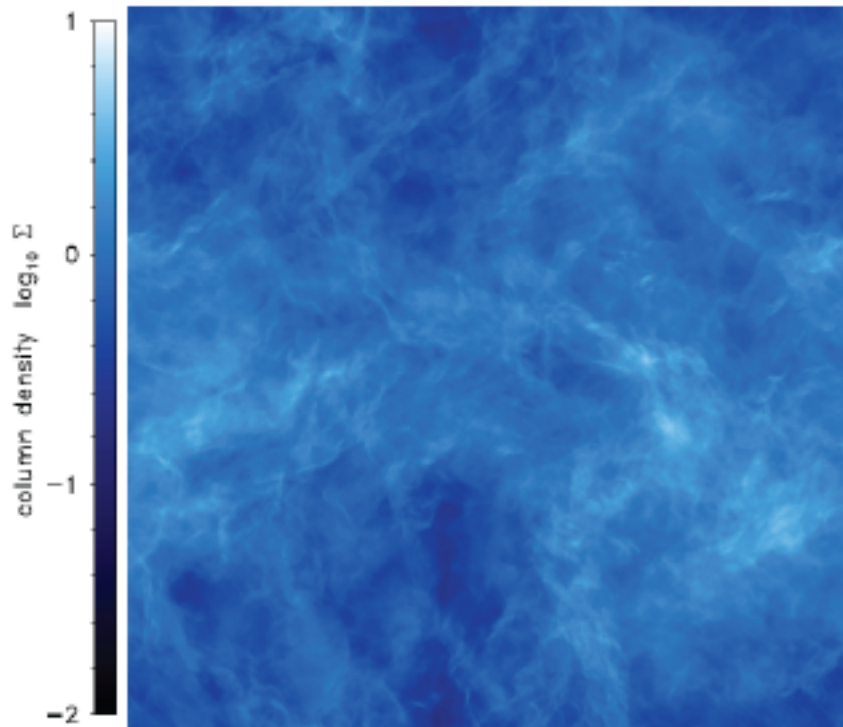
compressible counterpart



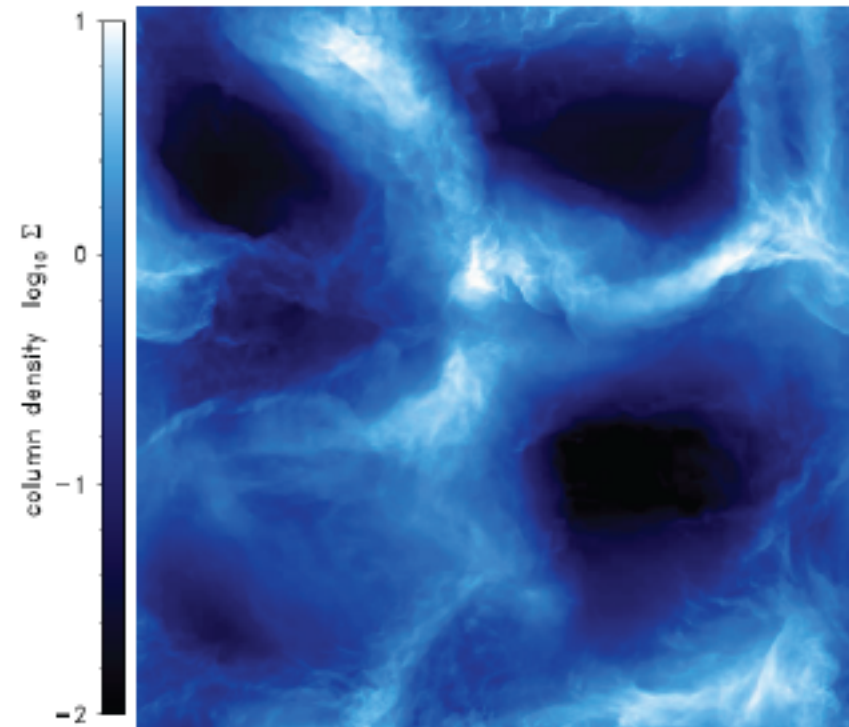


# 'Turbulence in a box': compressible vs solenoidal

Solenoidal forcing



Compressive forcing



**Compressive forcing yields 3 times larger density dispersion for the same Mach number**

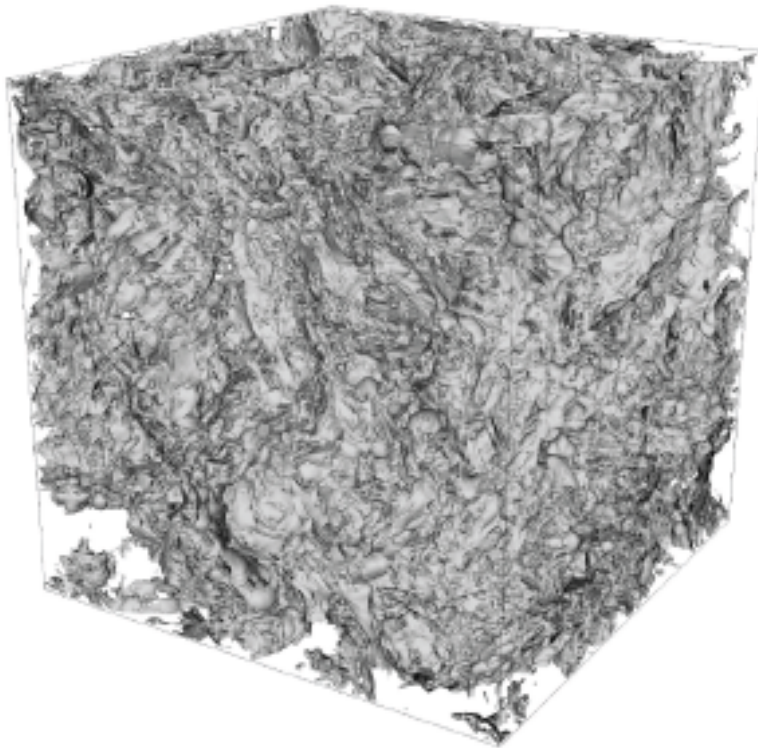
From C.Federrath, 6<sup>th</sup>  
KAW' 2011

Federrath et al. (2008, 2009, 2010)

# 'Turbulence in a box': compressible vs solenoidal

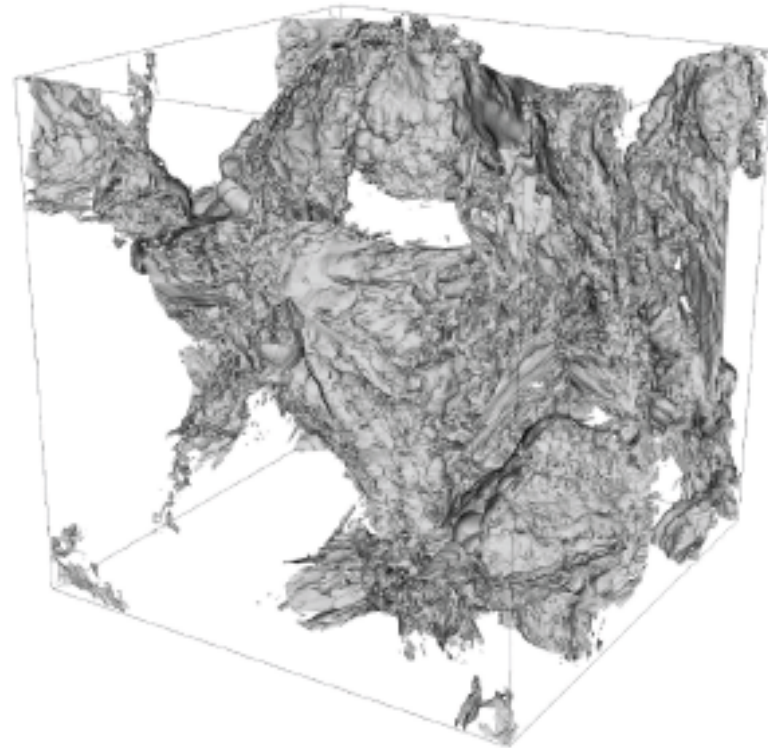
**Fractal structures by box-counting,  $\Delta$ -variance, perimeter-area methods:**

Solenoidal forcing



**$D = 2.6 \pm 0.1$**

Compressive forcing



**$D = 2.3 \pm 0.1$**

From C.Federrath, 6<sup>th</sup>  
KAW' 2011

Federrath, Klessen & Schmidt 2009, ApJ 692, 364

# 'Turbulence in a box': curvelets and contourlets

Computers & Fluids 57 (2012) 76–86



Contents lists available at SciVerse ScienceDirect

Computers & Fluids

journal homepage: [www.elsevier.com/locate/complfluid](http://www.elsevier.com/locate/complfluid)



Extraction of coherent vortices from homogeneous turbulence using curvelets and total variation filtering methods

Tamer Nabil <sup>a,d</sup>, Waleed Abdel Kareem <sup>b,d,\*</sup>, Seiichiro Izawa <sup>c</sup>, Yu Fukunishi <sup>c</sup>

<sup>a</sup> Suez Canal University, Faculty of Computers and Informatics, Basic Science Department, Ismailia, Egypt

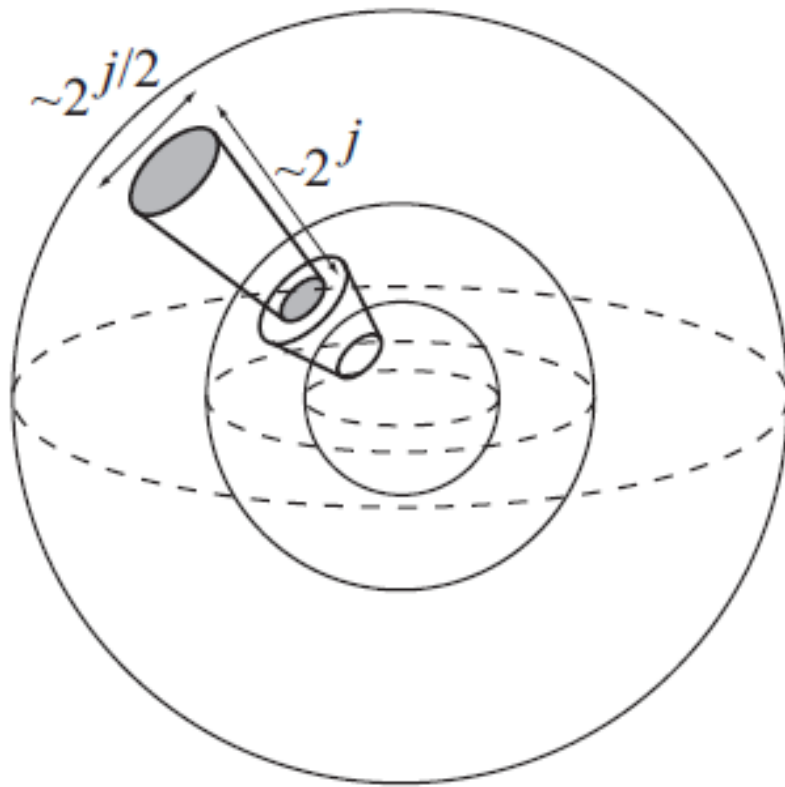
<sup>b</sup> Suez Canal University, Faculty of Science in Suez, Department of Mathematics, Suez, Egypt

<sup>c</sup> Graduate School of Engineering, Department of Mechanical Systems and Design, Tohoku University, Sendai 980-8579, Japan

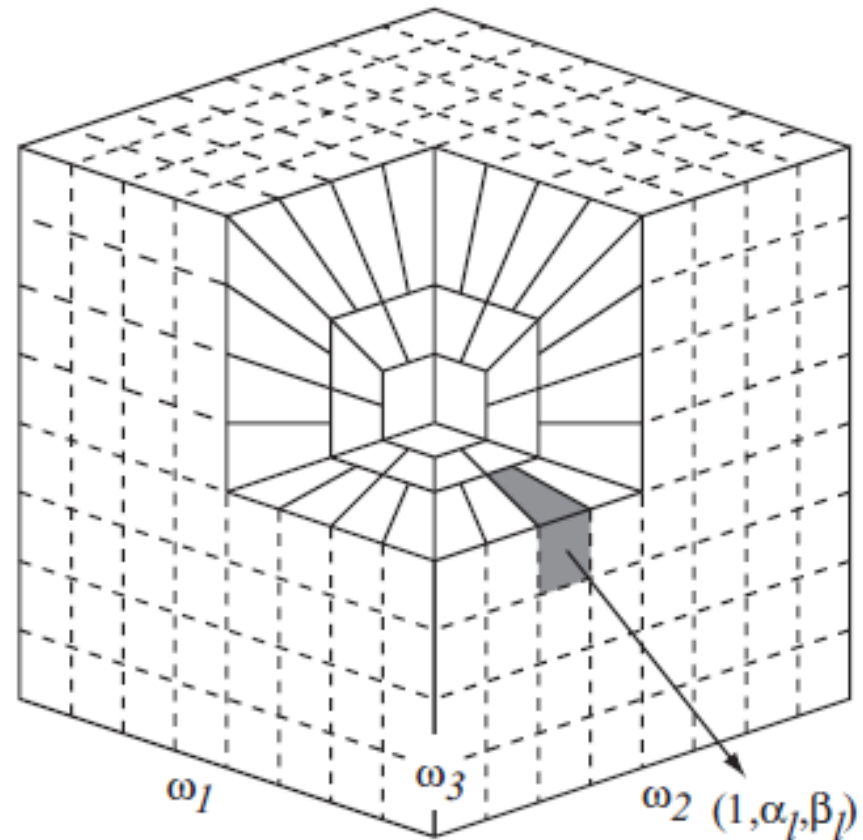
<sup>d</sup> King Khalid University, Faculty of Science, Department of Mathematics, Abha, Saudi Arabia

# Multiscale methodology of study of the nonlocal geometry of turbulence: curvelets

Ying et al 2005; Bermejo-Moreno & Pullin 2008



(a)



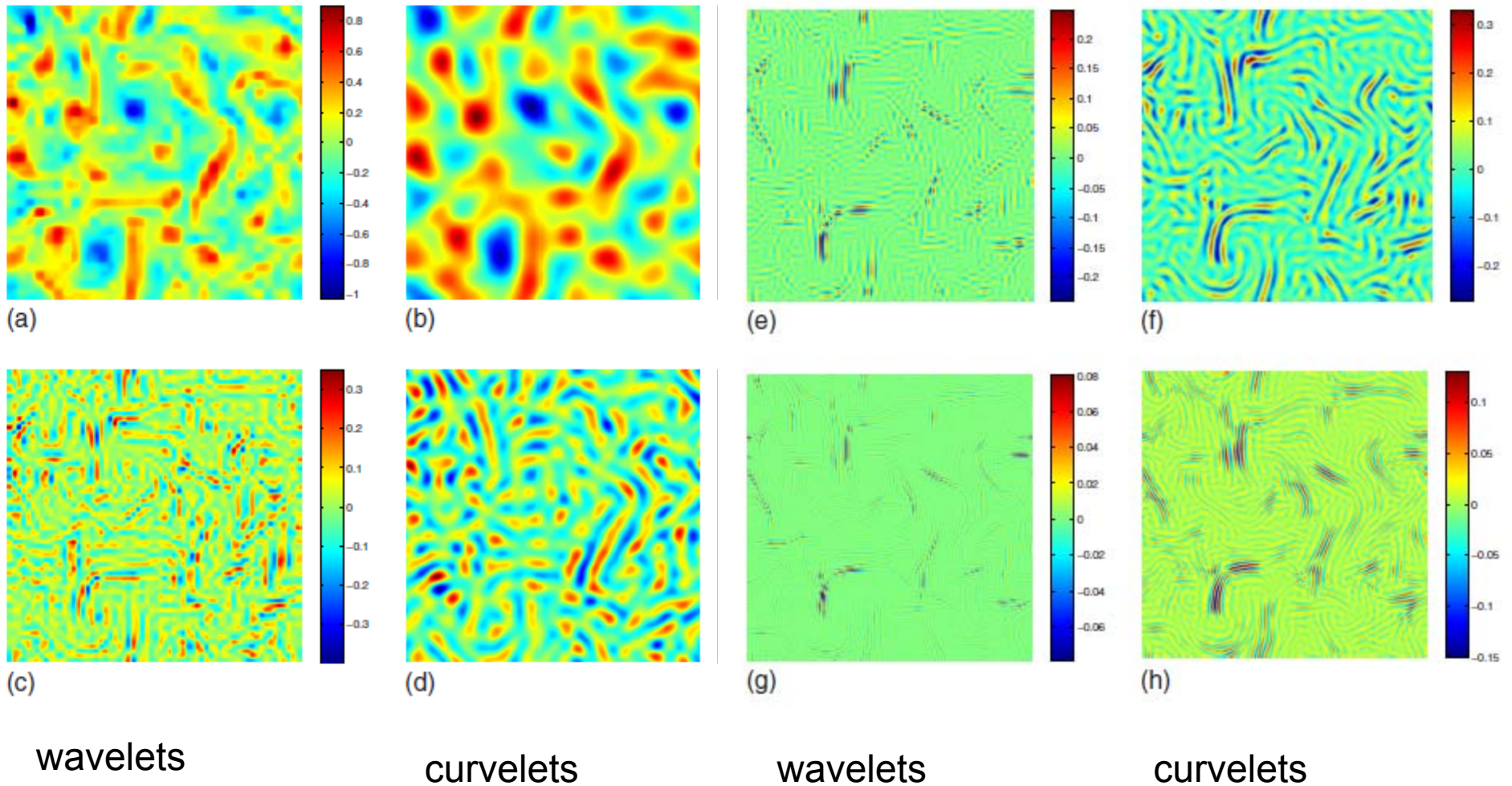
(b)



# Reconstruction of vorticity: wavelets vs curvelets

075104-13 Multiscale geometric analysis of turbulence

Ma et al. 2009



# Classification of structures by using curvelets

Bermejo-Moreno & Pullin 2008

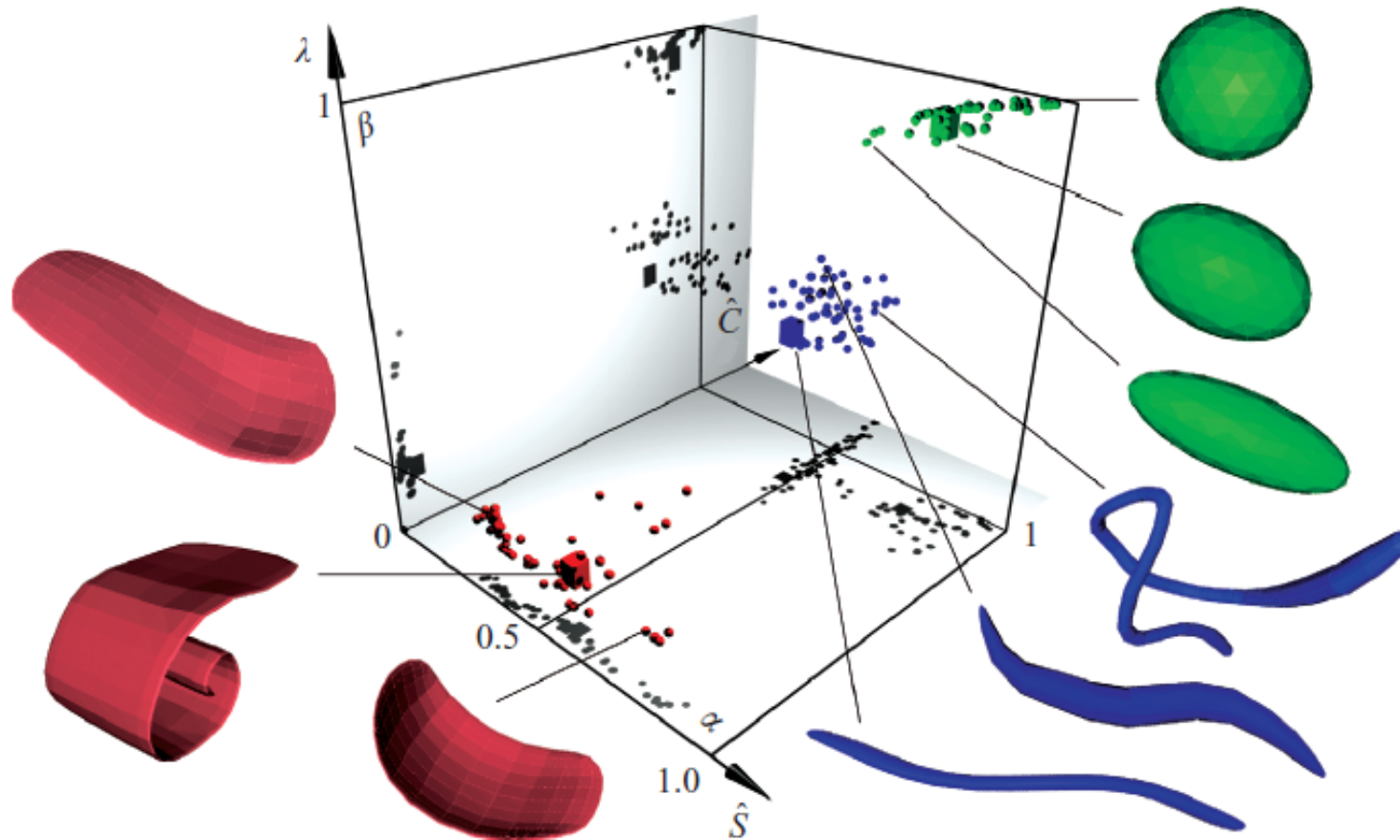


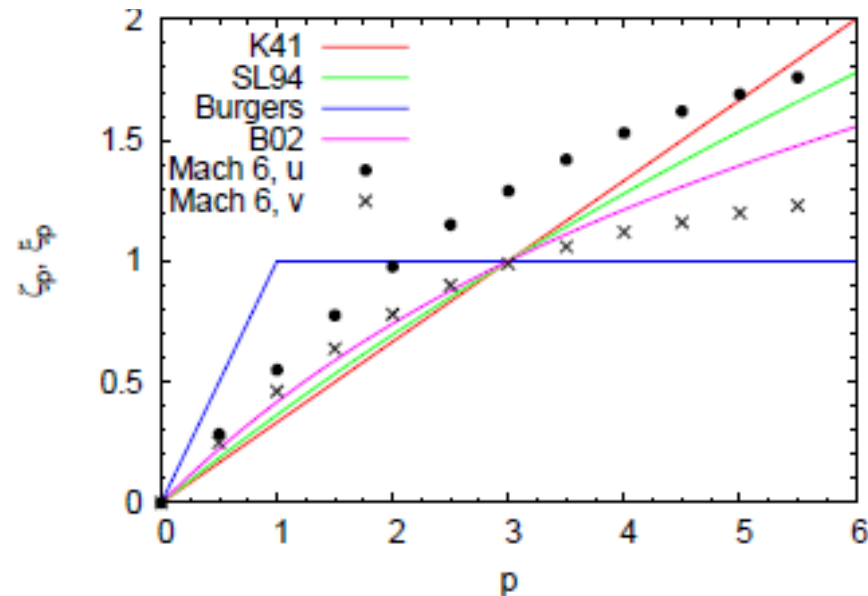
FIGURE 7. Visualization space with clustering results for the virtual set of modelled structures, with representative examples shown at the sides.

from

Ginzburg Conference' 2012

40

# 'Turbulence in a box': compressible vs solenoidal



- $1024^3$  model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2007].
- Density-weighted velocity:  $v \equiv \rho^{1/3} u$ . Total energy is conserved:  $E = \rho u^2/2 + c_s^2 \rho \ln \rho$ .
- Linear scaling:  $S_3(v, \ell) \propto \ell^1$  independent of the Mach number.

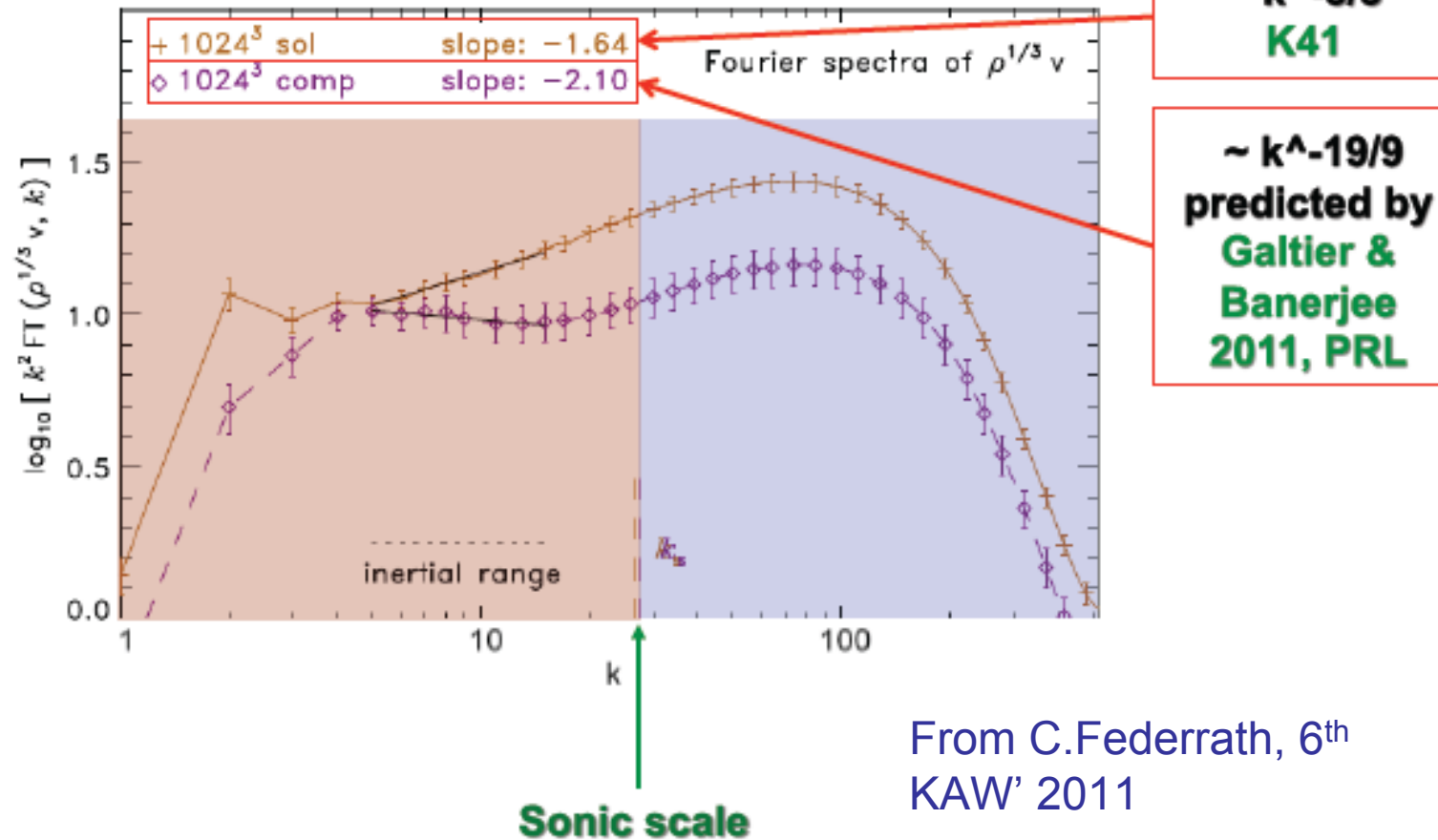
**$v \equiv \rho^{1/3} u$  is a good candidate for universal behavior**

from A. Kritsuk, Tempe 2012

# 'Turbulence in a box': compressible vs solenoidal

$\rho^{1/3} v$  - spectrum

(suggested by Kritsuk et al. 2007;  
see also Kowal et al. 2008; Schmidt et al. 2008; Aluie 2011)



Federrath, Roman-Duval, Klessen, Schmidt, Mac Low (2010, A&A 512, A81)



# Two-way 'Direct-Inverse' cascade

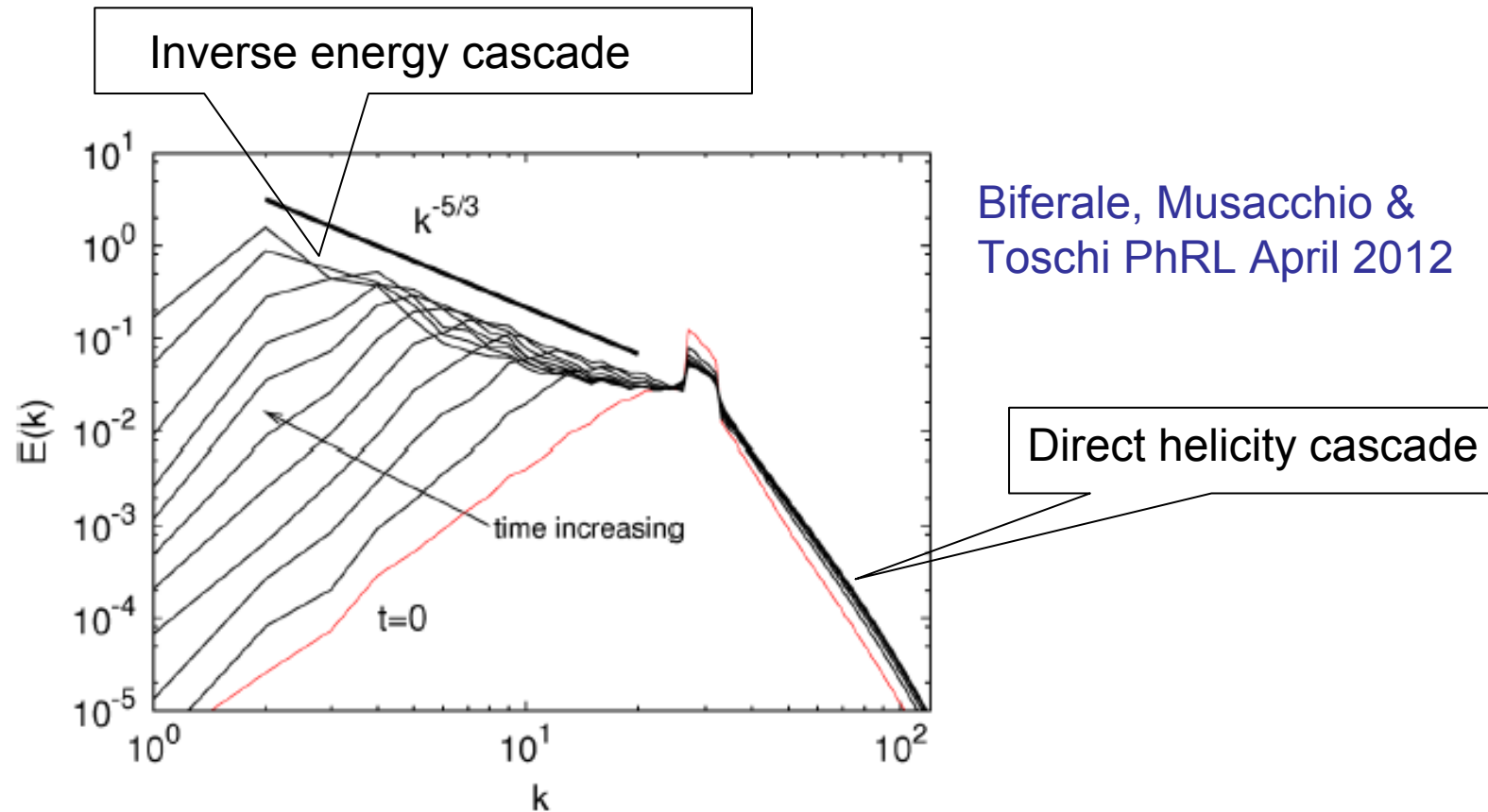


FIG. 2 (color online). Nonstationary spectrum in the inverse energy cascade regime. The straight dashed line represents the  $k^{-5/3}$  slope.

# Two-way 'Direct-Inverse' cascade

Galtier & Banerjee PhRL  
November 2011

Source term (S) modifies  $\epsilon \rightarrow \epsilon_{\text{eff}}$  from pressure

$$-2\epsilon = \langle (\nabla' \cdot \mathbf{u}') (R - E) \rangle + \langle (\nabla \cdot \mathbf{u}) (\bar{R} - E') \rangle + \nabla_r \cdot \left\langle \left[ \frac{\delta(\rho \mathbf{u}) \cdot \delta \mathbf{u}}{2} + \delta \rho \delta e - C_s^2 \bar{\delta} \rho \right] \delta \mathbf{u} + \bar{\delta} e \delta(\rho \mathbf{u}) \right\rangle,$$

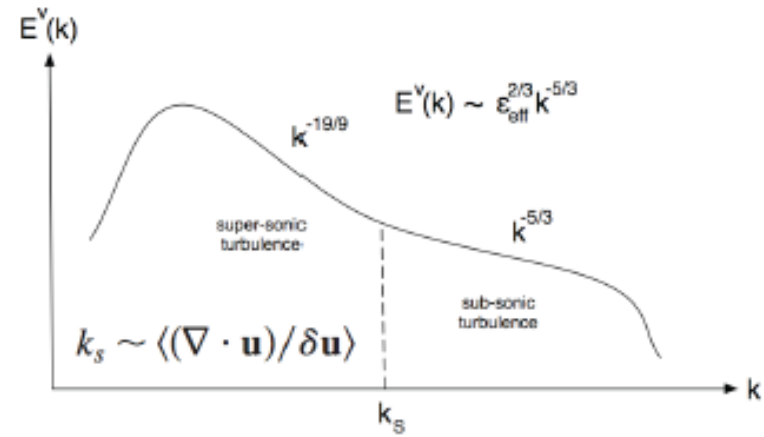
## ISOTROPIC TURBULENCE

$R - E$  is mainly negative.

$$S(r) \simeq -\langle \bar{\delta}(\nabla \cdot \mathbf{u}) \left[ \frac{1}{4} \delta(\rho \mathbf{u}) \cdot \delta \mathbf{u} + \frac{1}{2} \delta \rho \delta e \right] \rangle$$

$\sim r^{2/3}$

$\mathbf{v} \equiv \mathbf{u} \rho^{1/3}$



: direct cascade

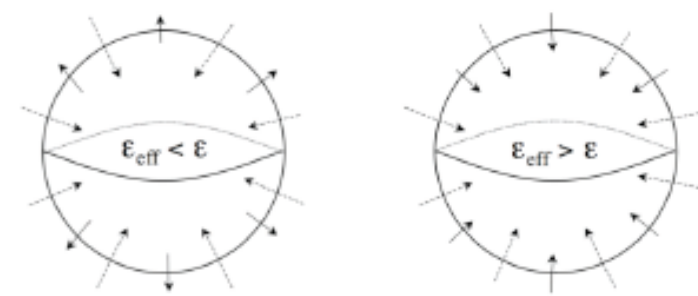
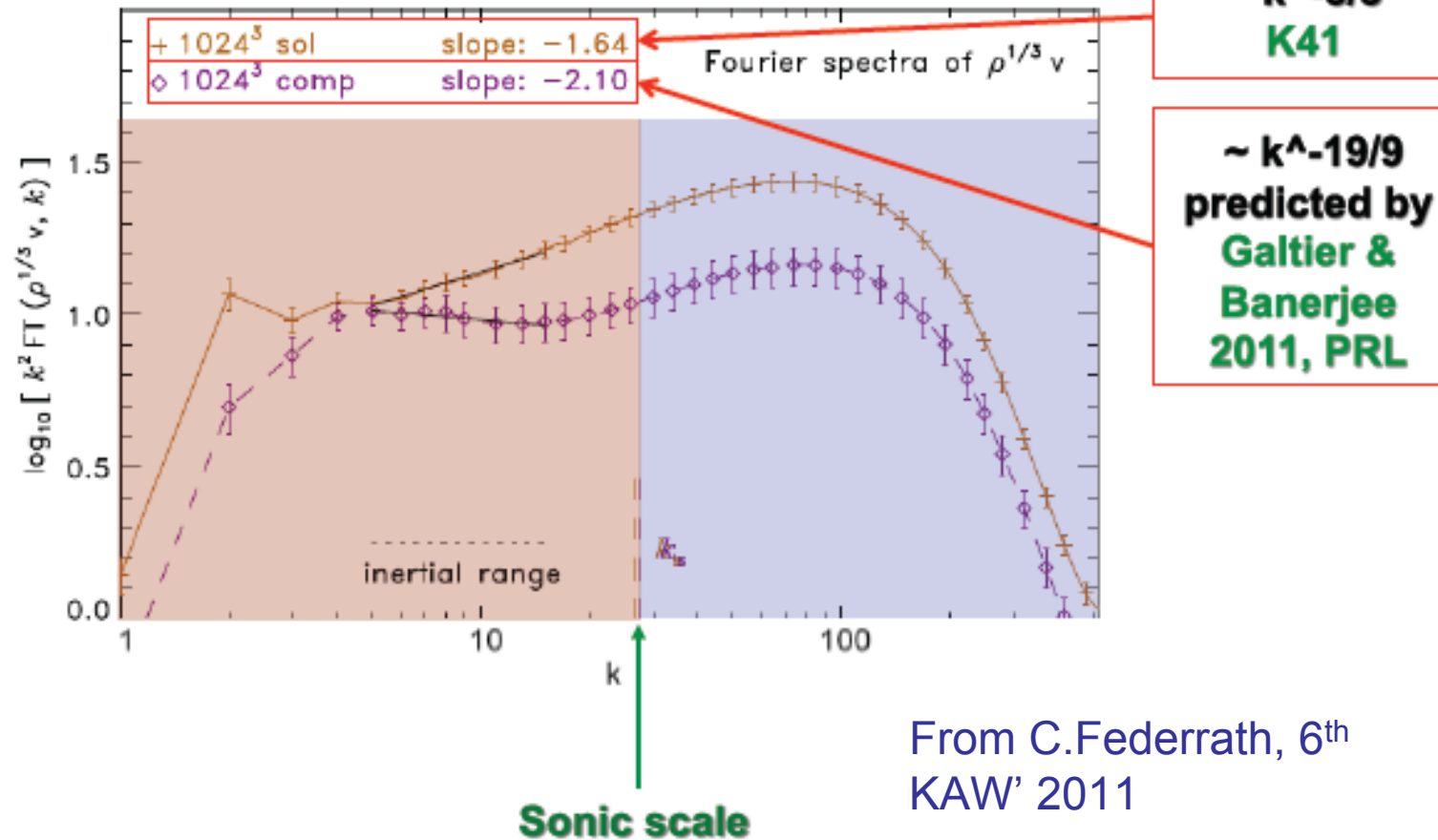


FIG. 1: Dilatation (left) and compression (right) phases in space correlation for isotropic turbulence. In a direct cascade scenario the flux vectors  $\mathcal{F}$  (dashed arrows) are oriented towards the center of the sphere. Dilatation and compression (solid arrows) are additional effects which act respectively in the opposite or in the same direction as the flux vectors.

# 'Turbulence in a box': compressible vs solenoidal

$\rho^{1/3} v$  - spectrum

(suggested by Kritsuk et al. 2007;  
see also Kowal et al. 2008; Schmidt et al. 2008; Aluie 2011)



Federrath, Roman-Duval, Klessen, Schmidt, Mac Low (2010, A&A 512, A81)

## Conclusion

- There is a hope that all key universalities in compressible Hydro turbulence are found.
- Many unclear questions remain in MHD turbulence.