Ginzburg conference 2012

BRST-BV approach to cubic vertices of higher-spin fields

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Self-consistent equations of motion of higher-spin gauge theories

Vasiliev 1990, 2003

1) Arbitrary space-time dimension

2) No restrictions on number of SuSy

Quantum self-consistency of Vasiliev's theories ?

1) **UV behavior ?**

2) **Anomalies ?**

BRST-BV approach

is powerful method for studying

quantum properties of gauge theories

(renormalization, anomalies)

Plan:

Light-cone gauge vertices
(review)

BRST-BV approach

to mixed-symmetry fields (review)

BRST-BV cubic vertices of

higher-spin field fields

Massless spin-1

$$\phi^{\mathbf{i}}$$

so(d-2) algebra vector field

$$\mathbf{i = 1, \ldots, d - 2}$$

Oscillators

$$[\bar{\alpha}^i \, , \, \alpha^j] = \delta^{ij}$$

$$\bar{\alpha}^j |0\rangle = 0$$

ket-vector of massless spin-1 field

$$|\phi\rangle = \alpha^{\mathbf{i}}\phi^{\mathbf{i}}$$

$|\phi\rangle$ degree-1 homogeneous polynomial

in oscillators $\alpha^{\mathbf{i}}$

Massive spin-1

$\phi^{\mathbf{i}}$ $\qquad\qquad$ ϕ

vector field + scalar field

$$so(d-2)$$

Oscillators

$$[\bar{\zeta}, \zeta] = 1$$

$$\bar{\zeta}|0\rangle = 0$$

ket-vector of massive spin-1 field

$$|\phi\rangle = \alpha^{\mathbf{i}}\phi^{\mathbf{i}} + \zeta\phi$$

$|\phi\rangle$ degree-1 homogeneous polynomial

in oscillators $\alpha^{\mathbf{i}}$, ζ

ket-vector of totally symmetric arbitrary spin **massless** field

$$|\phi(\alpha)\rangle = \alpha^{\mathbf{i_1}} \ldots \alpha^{\mathbf{i_s}} \phi^{\mathbf{i_1 \ldots i_s}}$$

$\phi(\alpha)$ degree-s homogeneous polynomial

in oscillators $\alpha^{\mathbf{i}}$

ket-vector of totally symmetric arbitrary spin **massive** field

$$|\phi(\alpha,\zeta)\rangle = \alpha^{\mathbf{i}_1}\ldots\alpha^{\mathbf{i}_s}\phi^{\mathbf{i}_1\ldots\mathbf{i}_s}$$

$$+\ \zeta\alpha^{\mathbf{i}_1}\ldots\alpha^{\mathbf{i}_{s-1}}\phi^{\mathbf{i}_1\ldots\mathbf{i}_{s-1}}$$

$$+\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+\ \zeta^{\mathbf{s}}\phi$$

$\phi(\alpha,\zeta)$ degree-s homogeneous polynomial

in oscillators $\alpha^{\mathbf{i}}$, ζ

$$|\phi(\alpha)\rangle = \phi^{i_1 \ldots i_s} \alpha^{\mathbf{i_1}} \ldots \alpha^{\mathbf{i_s}} |0\rangle$$

$$\alpha^{\mathbf{i}} \Longleftrightarrow \mathrm{U}^{\mathbf{i}}$$

$\phi(\mathrm{U})$ − generating function for arbitrary spin fields

Ginzburg, Tamm 1947

$$\mathcal{L} = \langle\phi|(\Box + \frac{\beta}{2}M^{AB}M^{AB} - \kappa^2)|\phi\rangle$$

$$M^{AB} = U^A\bar{U}^B - U^B\bar{U}^A$$

$$\mathbf{m^2 \sim \beta s^2 + \kappa^2}$$

Ginzburg, Tamm 1947

mixed-symmetry fields

Increase number of oscillators

$$\alpha^i$$

$$\alpha_n^i \qquad n = 1, \ldots, N$$

$$|\phi\rangle = |\phi(\alpha_{\mathbf{n}}^{\mathbf{i}})\rangle$$

$$|\phi\rangle = \alpha_{\mathbf{1}}^{\mathbf{i}} \alpha_{\mathbf{2}}^{\mathbf{j}} \phi^{[ij]}$$

cubic vertices

$$H = H^{(2)} + \mathbf{H^{(3)}} + \ldots$$

$$\mathbf{H^{(3)}} = \int \phi^{i_1\cdots}\phi^{i_2\cdots}\phi^{i_3\cdots}\mathbf{V^{i_1 i_2 i_3 \cdots}}$$

$$\mathbf{V^{i_1 i_2 i_3 \cdots}} = \delta^{i_1 i_2} \ldots \delta^{i_n j_n} p^{i_3} \ldots p^{l_3} + \ldots$$

$$\mathbf{H^{(3)}} = \int \langle\phi_1|\langle\phi_2|\langle\phi_3||\mathbf{V}\rangle$$

$$|\mathbf{V}(p_1, p_2, p_3; \alpha_1, \alpha_2, \alpha_3)\rangle$$

$$= \alpha^{i_1}\alpha^{i_2}\alpha^{i_3} \ldots \mathbf{V^{i_1 i_2 i_3 \cdots}}|0\rangle_1|0\rangle_2|0\rangle_3$$

Problem: find

$$V = |V(p_1, p_2, p_3; \alpha_1, \alpha_2, \alpha_3)\rangle$$

5 types of vertices

$$m_1 = 0 \quad m_2 = 0 \quad m_3 = 0$$

$$m_1 \neq 0 \quad m_2 \neq 0 \quad m_3 \neq 0$$

RRM 1993

$$m_1 = m_2 = 0 \quad m_3 \neq 0$$

$$m_1 = m_2 \neq 0 \quad m_3 = 0$$

$$m_1 \neq 0 \quad m_2 \neq 0 \quad m_3 = 0$$

$$m_1 \neq m_2$$

RRM 2005

Light-cone cubic vertices

General solution

$$\mathbf{V} = \mathbf{V}(\mathbf{L}, \mathbf{Q}, \mathbf{Z})$$

$$\mathbf{L} = \alpha^i p^i + \mathsf{m}\zeta$$

$$\mathbf{Q} = \alpha^i \alpha^i + \frac{1}{\mathsf{m}} p^i \alpha^i \zeta$$

$$\mathbf{Z} = (\alpha\alpha)\alpha^i p^i + \mathsf{m}(\alpha\alpha)\zeta$$

Illustrative example : vertex for massless arbitrary spin s_1, s_2, s_3 fields

$$V = \mathbf{Z}^{\frac{S-k}{2}} \prod_{a=1}^{3} (\mathbf{L}_a)^{s_a + \frac{k-S}{2}}$$

$$S \equiv \sum_a s_a$$

$$\mathbf{S} - \mathbf{2s_{\mathsf{min}}} \leq \mathbf{k} \leq \mathbf{S}$$

k number of derivatives

BRST-BV approach

$$\alpha^i,\ \zeta \quad \rightarrow \quad \alpha^A,\ \zeta,\ \theta,\ \mathbf{b},\ \mathbf{c}$$

$$|\phi(\alpha^i,\zeta)\rangle \quad \rightarrow \quad |\Phi(\alpha^A,\zeta,\theta,\mathbf{b},\mathbf{c})\rangle$$

$\mathbf{b}$ $\mathbf{c}$ ferm. ghost oscillators

θ Grassmann ferm. coordinate

$$\bar{b},\quad \bar{c}: \quad \{\bar{b},c\}=1,\quad \{\bar{c},b\}=1$$

internal ghost number operator N^{int}

external ghost number operator N^{ext}

$$\mathbf{N}^{\mathsf{int}} \equiv \theta \mathbf{p}_\theta + \mathbf{b}\bar{\mathbf{c}} - \mathbf{c}\bar{\mathbf{b}}$$

$$\mathbf{N} \equiv \mathbf{N}^{\mathsf{int}} + N^{\mathsf{ext}}$$

$$\mathbf{N}|\Phi\rangle = 0$$

$$\mathbf{gh}(\theta) = 1 \qquad \mathbf{gh}(c) = 1 \qquad \mathbf{gh}(b) = -1$$

BRST-BV

$$\mathrm{N}|\Phi\rangle = 0$$

$$\mathrm{N}^{\mathrm{int}}|\Phi\rangle \neq 0$$

BRST

$$\mathrm{N}|\Phi\rangle = 0$$

$$\mathrm{N}^{\mathrm{int}}|\Phi\rangle = 0$$

Scalar field

BRST-BV

$$|\Phi\rangle = \phi_0(\mathbf{x}) + \theta\psi_{-1}(\mathbf{x})$$

BV -anti-bracket

$$(\phi_0(\mathbf{x}), \psi_{-1}(\mathbf{x}')) = \delta(x - x')$$

BRST

$$|\Phi\rangle = \phi_0$$

Massless spin-1

$$|\Phi\rangle = |\phi\rangle + \theta|\psi\rangle$$

BRST-BV

$$|\phi\rangle = \alpha^A \phi_{\mathbf{0}}^{\mathbf{A}} + b\phi_{\mathbf{1}} + c\phi_{\mathbf{-1}}$$

$$|\psi\rangle = \alpha^A \psi_{\mathbf{-1}}^{\mathbf{A}} + b\psi_{\mathbf{0}} + c\psi_{\mathbf{-2}}$$

BRST

$$|\phi\rangle = \alpha^A \phi_0^A$$

$$|\psi\rangle = b\psi_0$$

Free BRST-BV action

$$S_2 = \int d^d x d\theta \langle \Phi | \mathbf{Q} | \Phi \rangle$$

$$\delta | \Phi \rangle = \mathbf{Q} | \Lambda \rangle$$

$$\mathbf{Q} = \theta(\Box - \mathsf{m}^2) + S^A p^A + \mathsf{m} S + M p_\theta$$

$$p_A \equiv \partial/\partial x^A , \qquad p_\theta \equiv \partial/\partial\theta$$

$$S^A \equiv c\bar{\alpha}^A - \alpha^A \bar{c}$$

$$S \equiv c\bar{\zeta} + \zeta\bar{c}$$

$$M \equiv c\bar{c}$$

$$\mathcal{L}=\frac{1}{2}\phi_{\mathbf{0}}^{\mathbf{A}}\Box\phi_{\mathbf{0}}^{\mathbf{A}}+\phi_{\mathbf{1}}\Box\phi_{\mathbf{-1}}$$

$$-\psi_{\mathbf{0}}p^{A}\phi_{\mathbf{0}}^{\mathbf{A}}-\phi_{\mathbf{1}}p^{A}\psi_{\mathbf{-1}}^{\mathbf{A}}-\frac{1}{2}\psi_{\mathbf{0}}\psi_{\mathbf{0}}$$

$$\phi_{\mathbf{-1}}=0 \qquad \psi_{\mathbf{-1}}^{\mathbf{A}}=0$$

$$\mathcal{L}=\frac{1}{2}\phi_{\mathbf{0}}^{\mathbf{A}}\Box\phi_{\mathbf{0}}^{\mathbf{A}}-\psi_{\mathbf{0}}p^{A}\phi_{\mathbf{0}}^{\mathbf{A}}-\frac{1}{2}\psi_{\mathbf{0}}\psi_{\mathbf{0}}$$

$$\psi_0=-p^{A}\phi_0$$

$$\mathcal{L}=\frac{1}{2}\phi_{\mathbf{0}}^{\mathbf{A}}\Box\phi_{\mathbf{0}}^{\mathbf{A}}+\frac{1}{2}(p^{A}\phi_{\mathbf{0}}^{\mathbf{A}})^2$$

General setup for cubic vertices

Siegel, West, Neveu (≈ 1985)

$$S = S_2 + S_3$$

$$S_3 = \int d1d2d3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 | | \mathbf{V_{123}} \rangle$$

$$dr = d^d x_r d\theta_r$$

$$|\mathbf{V_{123}}\rangle \equiv V_{123} \delta(x_1 - x_2) \delta(x_2 - x_3) |0\rangle_1 |0\rangle_2 |0\rangle_3$$

$$\delta|\Phi\rangle = Q|\Lambda\rangle - |\Phi\star\Lambda\rangle - |\Lambda\star\Phi\rangle$$

$$|(\Phi\star\Psi)_3\rangle \equiv \int d1d2\langle\Phi_1|\langle\Psi_2||\mathbf{V_{123}}\rangle$$

Equation for cubic vertex

$$\mathbf{Q}^{\text{tot}}|\mathbf{V_{123}}\rangle = 0$$

$$\mathbf{Q}^{\text{tot}} \equiv \sum_{r=1}^{3} Q^{(r)}$$

Field redefinitions

$$|\Phi\rangle \rightarrow |\Phi\rangle + \int \langle\Phi|\langle\Phi||\mathbf{F_{123}}\rangle$$

$$|\mathbf{V_{123}}\rangle \rightarrow |\mathbf{V_{123}}\rangle + \mathbf{Q}^{\mathsf{tot}}|\mathbf{F_{123}}\rangle$$

$$\mathbf{Q}^{\mathrm{tot}}|\mathbf{V_{123}}\rangle = 0$$

$$|\mathbf{V_{123}}\rangle \neq \mathbf{Q}^{\mathrm{tot}}|\mathbf{F_{123}}\rangle$$

Adaptation to higher-spin fields

Buchbinder, Tsulaia, et.al. 2006

totally-symmetric fields

$$\alpha^A, \quad \zeta, \quad \theta, \quad b, \quad c$$

mixed-symmetry fields

$$\alpha_n^A, \quad \zeta_n, \quad \theta, \quad b_n, \quad c_n$$

$$n = 1, \ldots, N$$

$$N \to \infty$$

Sagnotti, Tsulaia

tensionless limit of string

$$N < \infty$$

Alkalaev, Grigoriev, Tipunin

BRST cubic vertices

General solution

$$V = V(\mathbf{L}, \mathbf{Q}, \mathbf{Z})$$

$$\mathbf{L} = \alpha^{\mathbf{A}} \mathbf{p}^{\mathbf{A}} + \mathsf{m}\zeta + \mathbf{p}_\theta \mathbf{c}$$

$$\mathbf{Q} = \alpha^{\mathbf{A}} \alpha^{\mathbf{A}} + \frac{1}{\mathsf{m}} \mathbf{p}^{\mathbf{A}} \alpha^{\mathbf{A}} \zeta + \mathbf{bc}$$

$$\mathbf{Z} = (\alpha\alpha)\alpha^{\mathbf{A}} \mathbf{p}^{\mathbf{A}} + \mathsf{m}(\alpha\alpha\zeta) + (\alpha\alpha)\mathbf{p}_\theta \mathbf{c} + \mathbf{bc}\alpha^{\mathbf{A}} \mathbf{p}^{\mathbf{A}} + \ldots$$

5 types of vertices

$$m_1 = 0 \quad m_2 = 0 \quad m_3 = 0$$

$$m_1 = m_2 = \mathbf{0} \qquad m_3 \neq \mathbf{0}$$

$$m_1 = m_2 \neq \mathbf{0} \qquad m_3 = \mathbf{0}$$

$$m_1 \neq \mathbf{0} \quad m_2 \neq \mathbf{0} \quad m_3 = \mathbf{0}$$

$$m_1 \neq m_2$$

$$m_1 \neq \mathbf{0} \quad m_2 \neq \mathbf{0} \quad m_3 \neq \mathbf{0}$$

$$m_1 = 0 \qquad m_2 = 0 \qquad m_3 = 0$$

off-shell vertices (**metric approach**)

Manvelyan et al (2010)

On-shell vertices (tensionless limit of string amplitudes)

Sagnotti, Taronna (2010)

off-shell vertices (**BRST approach**)

Fotopoulos, Tsulaia (2010)

$$V = V(L^{(1)}, L^{(2)}, L^{(3)}, Z)$$

$$Z = Q^{(12)} L^{(3)} + cycl.perms.$$

$$\delta Q^{(12)} = c^{(1)} L^{(2)} - c^{(2)} L^{(1)}$$