New (1,0) superconformal models in six dimensions

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[HS, E Sezgin, R Wimmer, arXiv:1108.4060]

[HS, E Sezgin, R Wimmer, L. Wulff, arXiv:1204.0542]

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motivation: superconformal models in six dimensions

- **(2,0) chiral tensor multiplet in six dimensions**

  $$\{B_{\mu\nu}, \chi^i, \phi^{ij}\} \qquad\qquad (dB)^- = 0$$

  - dynamics of a single M5 brane

- **goal : non-abelian extension of the (2,0) field equations**
  - multiple branes: non-abelian deformation
  - various no-go theorems

- **inspiration from M2 branes** [Bagger, Lambert, Gustavsson '07] [Aharony, Bergman, Jafferis, Maldacena, '08]
  - non-dynamical fields may be crucial (BLG)
  - full supersymmetry may not be manifest (ABJM)

$\longrightarrow$ **study (1,0) non-abelian superconformal models in six dimensions**

plan

- **nonabelian tensor fields in six dimensions**
  - non-abelian tensor hierarchy
  - parameters and constraints
- **supersymmetry: superconformal field equations**
  - (1,0) supersymmetry and dynamics
  - action (?!)
- **examples**
  - solving the constraints
  - gauge groups and representation content
- **conclusions / outlook**

nonabelian tensor fields in six dimensions

nonabelian tensor fields in six dimensions

- **field content** $\{A_\mu^r,\, B_{\mu\nu}^I,\, C_{\mu\nu\rho\, r}\}$

nonabelian tensor fields in six dimensions

- **field content** $\{A_\mu^r,\, B_{\mu\nu}^I,\, C_{\mu\nu\rho\, r}\}$

- **covariant field strengths (Yang-Mills)**

$$\mathcal{F}_{\mu\nu}^r \;\equiv\; 2\partial_{[\mu}A_{\nu]}^r - f_{st}{}^r A_\mu^s A_\nu^t$$

- **non-abelian gauge transformations**

$$\delta A_\mu^r \;=\; D_\mu \Lambda^r$$

with structure constants $f_{rs}{}^t$ and gauge generators X_r

nonabelian tensor fields in six dimensions

- **field content** $\{A_\mu^r,\, B_{\mu\nu}^I,\, C_{\mu\nu\rho\, r}\}$

tensor hierarchies
[B de Wit, HS, '05]
[B de Wit, H Nicolai, HS, '08]
[J. Hartong, T. Ortin, '09]

- **covariant field strengths**

$$\begin{aligned}
\mathcal{F}_{\mu\nu}^r &\equiv 2\partial_{[\mu} A_{\nu]}^r - f_{st}{}^r A_\mu^s A_\nu^t + h_I^r\, B_{\mu\nu}^I \\
\mathcal{H}_{\mu\nu\rho}^I &\equiv 3D_{[\mu} B_{\nu\rho]}^I + 6\, d_{rs}^I A_{[\mu}^r \partial_\nu A_{\rho]}^s - 2 f_{pq}{}^s d_{rs}^I A_{[\mu}^r A_\nu^p A_{\rho]}^q + g^{Ir} C_{\mu\nu\rho\, r}
\end{aligned}$$

- **non-abelian gauge transformations**

$$\begin{aligned}
\delta A_\mu^r &= D_\mu \Lambda^r - h_I^r \Lambda_\mu^I \\
\Delta B_{\mu\nu}^I &= 2D_{[\mu} \Lambda_{\nu]}^I - 2\, d_{rs}^I \Lambda^r \mathcal{F}_{\mu\nu}^s - g^{Ir} \Lambda_{\mu\nu\, r} \\
\Delta C_{\mu\nu\rho\, r} &= 3D_{[\mu} \Lambda_{\nu\rho]\, r} + 3\, b_{Irs}\, \mathcal{F}_{[\mu\nu}^s\, \Lambda_{\rho]}^I + b_{Irs}\, \mathcal{H}_{\mu\nu\rho}^I\, \Lambda^s + \ldots
\end{aligned}$$

$$\begin{aligned}
\Delta B_{\mu\nu}^I &\equiv \delta B_{\mu\nu}^I - 2 d_{rs}^I\, A_{[\mu}^r\, \delta A_{\nu]}^s \\
\Delta C_{\mu\nu\rho\, r} &\equiv \delta C_{\mu\nu\rho\, r} - 3\, b_{Irs}\, B_{[\mu\nu}^I\, \delta A_{\rho]}^s - 2\, b_{Irs}\, d_{pq}^I\, A_{[\mu}^s\, A_\nu^p\, \delta A_{\rho]}^q
\end{aligned}$$

in terms of constant tensors $d_{rs}^I,\, b_{Irs},\, f_{rs}{}^t,\, g^{Ir},\, h_I^r$ and gauge generators X_r

nonabelian tensor fields in six dimensions

covariant field strengths

$$\mathcal{F}^r_{\mu\nu} \equiv 2\partial_{[\mu}A^r_{\nu]} - f_{st}{}^r A^s_\mu A^t_\nu + h^r_I\, B^I_{\mu\nu}$$

$$\mathcal{H}^I_{\mu\nu\rho} \equiv 3D_{[\mu}B^I_{\nu\rho]} + 6\, d^I_{rs} A^r_{[\mu}\partial_\nu A^s_{\rho]} - 2 f_{pq}{}^s d^I_{rs} A^r_{[\mu}A^p_\nu A^q_{\rho]} + g^{Ir} C_{\mu\nu\rho\, r}$$

consistency requires several constraints on the parameters $d^I_{rs}, b_{Irs}, f_{rs}{}^t, g^{Ir}, h^r_I$

gauge group generators

$$D_\mu = \partial_\mu - A^r_\mu\, X_r$$

$$(X_r)_s{}^t = - f_{rs}{}^t + d^I_{rs}\, h^t_I$$

$$(X_r)_I{}^J = 2\, h^s_I d^J_{rs} - g^{Js} b_{Isr}$$

- charged tensor fields require Stückelberg-type coupling
- generalized Bianchi identities $D\mathcal{F}^r = h^r_I\, \mathcal{H}^I$, etc.
- continues to 4-forms, 5-forms, ...

must close into the algebra $[X_r, X_s] = (X_r)_s{}^t\, X_t$

nonabelian tensor fields in six dimensions

covariant field strengths

$$
\begin{aligned}
\mathcal{F}^{r}_{\mu\nu} &\equiv 2\partial_{[\mu}A^{r}_{\nu]} - f_{st}{}^{r}A^{s}_{\mu}A^{t}_{\nu} + h^{r}_{I}\,B^{I}_{\mu\nu} \\
\mathcal{H}^{I}_{\mu\nu\rho} &\equiv 3D_{[\mu}B^{I}_{\nu\rho]} + 6\,d^{I}_{rs}A^{r}_{[\mu}\partial_{\nu}A^{s}_{\rho]} - 2f_{pq}{}^{s}d^{I}_{rs}A^{r}_{[\mu}A^{p}_{\nu}A^{q}_{\rho]} + g^{Ir}C_{\mu\nu\rho\,r}
\end{aligned}
$$

consistency requires several constraints on the parameters $d^{I}_{rs}, b_{Irs}, f_{rs}{}^{t}, g^{Ir}, h^{r}_{I}$

$$
\begin{aligned}
2\,(d^{J}_{r(u}d^{I}_{v)s} - d^{I}_{rs}d^{J}_{uv})h^{s}_{J} &= 2f_{r(u}{}^{s}d^{I}_{v)s} - b_{Jsr}d^{J}_{uv}\,g^{Is} \\
(d^{J}_{rs}\,b_{Iut} + d^{J}_{rt}\,b_{Isu} + 2\,d^{K}_{ru}b_{Kst}\delta^{J}_{I})\,h^{u}_{J} &= f_{rs}{}^{u}b_{Iut} + f_{rt}{}^{u}b_{Isu} + g^{Ju}b_{Iur}b_{Jst} \\
f_{[pq}{}^{u}f_{r]u}{}^{s} - \tfrac{1}{3}h^{s}_{I}\,d^{I}_{u[p}f_{qr]}{}^{u} &= 0 \\
h^{r}_{I}g^{Is} &= 0 \\
f_{rs}{}^{t}h^{r}_{I} - d^{J}_{rs}\,h^{t}_{J}h^{r}_{I} &= 0 \\
g^{Js}h^{r}_{K}b_{Isr} - 2h^{s}_{I}h^{r}_{K}\,d^{J}_{rs} &= 0 \\
-f_{rt}{}^{s}g^{It} + d^{J}_{rt}\,h^{s}_{J}g^{It} - g^{It}g^{Js}\,b_{Jtr} &= 0
\end{aligned}
$$

nonabelian tensor fields in six dimensions

covariant field strengths

$$
\begin{aligned}
\mathcal{F}^r_{\mu\nu} &\equiv 2\partial_{[\mu} A^r_{\nu]} - f_{st}{}^r A^s_\mu A^t_\nu + h^r_I\, B^I_{\mu\nu} \\
\mathcal{H}^I_{\mu\nu\rho} &\equiv 3D_{[\mu} B^I_{\nu\rho]} + 6\, d^I_{rs} A^r_{[\mu}\partial_\nu A^s_{\rho]} - 2 f_{pq}{}^s d^I_{rs} A^r_{[\mu} A^p_\nu A^q_{\rho]} + g^{Ir} C_{\mu\nu\rho\, r}
\end{aligned}
$$

consistency requires several constraints on the parameters $d^I_{rs}, b_{Irs}, f_{rs}{}^t, g^{Ir},\ h^r_I$

$$
\begin{aligned}
2\,(d^J_{r(u} d^I_{v)s} - d^I_{rs} d^J_{uv}) h^s_J &= 2 f_{r(u}{}^s d^I_{v)s} - b_{Jsr} d^J_{uv}\, g^{Is} \\
(d^J_{rs}\, b_{Iut} + d^J_{rt}\, b_{Isu} + 2\, d^K_{ru} b_{Kst} \delta^J_I)\, h^u_J &= f_{rs}{}^u b_{Iut} + f_{rt}{}^u b_{Isu} + g^{Ju} b_{Iur} b_{Jst} \\
f_{[pq}{}^u f_{r]u}{}^s - \tfrac{1}{3} h^s_I\, d^I_{u[p} f_{qr]}{}^u &= 0 \\
h^r_I g^{Is} &= 0 \\
f_{rs}{}^t h^r_I - d^J_{rs}\, h^t_J h^r_I &= 0 \\
g^{Js} h^r_K b_{Isr} - 2 h^s_I h^r_K\, d^J_{rs} &= 0 \\
-f_{rt}{}^s g^{It} + d^J_{rt}\, h^s_J g^{It} - g^{It} g^{Js}\, b_{Jtr} &= 0
\end{aligned}
$$

violation of Jacobi identities

orthogonality

too many constraints ? are there solutions ?

nonabelian tensor fields in six dimensions

covariant field strengths

$$\mathcal{F}^r_{\mu\nu} \equiv 2\partial_{[\mu}A^r_{\nu]} - f_{st}{}^r A^s_\mu A^t_\nu + h^r_I\, B^I_{\mu\nu}$$
$$\mathcal{H}^I_{\mu\nu\rho} \equiv 3D_{[\mu}B^I_{\nu\rho]} + 6\, d^I_{rs}A^r_{[\mu}\partial_\nu A^s_{\rho]} - 2f_{pq}{}^s d^I_{rs}A^r_{[\mu}A^p_\nu A^q_{\rho]} + g^{Ir}C_{\mu\nu\rho\, r}$$

consistency requires several constraints on the parameters $d^I_{rs}, b_{Irs}, f_{rs}{}^t, g^{Ir}, h^r_I$

$$\begin{aligned}
2\,(d^J_{r(u}d^I_{v)s} - d^I_{rs}d^J_{uv})h^s_J &= 2f_{r(u}{}^s d^I_{v)s} - b_{Jsr}d^J_{uv}\, g^{Is}\\
(d^J_{rs}\, b_{Iut} + d^J_{rt}\, b_{Isu} + 2\, d^K_{ru}b_{Kst}\delta^J_I)\, h^u_J &= f_{rs}{}^u b_{Iut} + f_{rt}{}^u b_{Isu} + g^{Ju}b_{Iur}b_{Jst}\\
f_{[pq}{}^u f_{r]u}{}^s - \tfrac{1}{3}h^s_I\, d^I_{u[p}f_{qr]}{}^u &= 0\\
h^r_I g^{Is} &= 0\\
f_{rs}{}^t h^r_I - d^J_{rs}\, h^t_J h^r_I &= 0\\
g^{Js}h^r_K b_{Isr} - 2h^s_I h^r_K\, d^J_{rs} &= 0\\
-f_{rt}{}^s g^{It} + d^J_{rt}\, h^s_J g^{It} - g^{It}g^{Js}\, b_{Jtr} &= 0
\end{aligned}$$

example : Yang-Mills with neutral tensor fields

$$g^{Ir} = 0,\ h^r_I = 0,\ d^I_{rs} = d^I\eta_{rs}$$

no Stückelberg-type couplings

[E. Bergshoeff, E. Sezgin, E. Sokatchev, '96]

nonabelian tensor fields in six dimensions

covariant field strengths

$$
\begin{aligned}
\mathcal{F}^r_{\mu\nu} &\equiv 2\partial_{[\mu}A^r_{\nu]} - f_{st}{}^r A^s_\mu A^t_\nu + h^r_I\, B^I_{\mu\nu} \\
\mathcal{H}^I_{\mu\nu\rho} &\equiv 3D_{[\mu}B^I_{\nu\rho]} + 6\, d^I_{rs} A^r_{[\mu}\partial_\nu A^s_{\rho]} - 2 f_{pq}{}^s d^I_{rs} A^r_{[\mu}A^p_\nu A^q_{\rho]} + g^{Ir} C_{\mu\nu\rho\, r}
\end{aligned}
$$

consistency requires several constraints on the parameters $d^I_{rs}, b_{Irs}, f_{rs}{}^t, g^{Ir}, h^r_I$

$$
\begin{aligned}
2\,(d^J_{r(u}d^I_{v)s} - d^I_{rs}d^J_{uv})h^s_J &= 2f_{r(u}{}^s d^I_{v)s} - b_{Jsr}d^J_{uv}\, g^{Is} \\
(d^J_{rs}\, b_{Iut} + d^J_{rt}\, b_{Isu} + 2\, d^K_{ru} b_{Kst}\delta^J_I)\, h^u_J &= f_{rs}{}^u b_{Iut} + f_{rt}{}^u b_{Isu} + g^{Ju} b_{Iur} b_{Jst} \\
f_{[pq}{}^u f_{r]u}{}^s - \tfrac{1}{3} h^s_I\, d^I_{u[p} f_{qr]}{}^u &= 0 \\
h^r_I g^{Is} &= 0 \\
f_{rs}{}^t h^r_I - d^J_{rs}\, h^t_J h^r_I &= 0 \\
g^{Js} h^r_K b_{Isr} - 2 h^s_I h^r_K\, d^J_{rs} &= 0 \\
-f_{rt}{}^s g^{It} + d^J_{rt}\, h^s_J g^{It} - g^{It} g^{Js}\, b_{Jtr} &= 0
\end{aligned}
$$

example : Yang-Mills with adjoint tensor fields

$h^r_s = 0,\ g^{rs} = \eta^{rs},\ b_{t\,rs} = f_{rst}$ coupling of three-forms, charged tensors

supersymmetry: superconformal field equations

- **result : every consistent bosonic system can be supersymmetrized !**

superconformal field equations

(1,0) supermultiplets

vector $\{A^r_\mu,\, \lambda^r_i,\, Y^r_{ij}\}$ tensor $\{B^I_{\mu\nu}, \chi^I_i, \phi^I\}$ three-form $\{C_{\mu\nu\rho\, r}\}$

off-shell on-shell ??

superconformal field equations

(1,0) supermultiplets

vector $\{A^r_\mu,\ \lambda^r_i,\ Y^r_{ij}\}$ tensor $\{B^I_{\mu\nu}, \chi^I_i, \phi^I\}$ three-form $\{C_{\mu\nu\rho\, r}\}$

- **closure of the supersymmetry algebra on the tensor multiplet implies**

$$
\begin{aligned}
\mathcal{H}^{I\,-}_{\mu\nu\rho} &= -d^I_{rs}\bar{\lambda}^r\gamma_{\mu\nu\rho}\lambda^s \\
\gamma^\sigma D_\sigma \chi^{iI} &= \tfrac{1}{2}d^I_{rs}\mathcal{F}^r_{\sigma\tau}\,\gamma^{\sigma\tau}\lambda^{is} + 2d^I_{rs}Y^{ij\,r}\,\lambda^s_j + \left(d^I_{rs}h^s_J - 2b_{Jsr}g^{Is}\right)\phi^J\lambda^{ir} \\
D^\mu D_\mu\,\phi^I &= -\tfrac{1}{2}d^I_{rs}\left(\mathcal{F}^r_{\mu\nu}\mathcal{F}^{\mu\nu\,s} - 4\,Y^r_{ij}Y^{ij\,s} + 8\bar{\lambda}^r\gamma^\mu D_\mu\lambda^s\right) \\
&\quad -2\left(b_{Jsr}g^{Is} - 8d^I_{rs}h^s_J\right)\bar{\lambda}^r\chi^J - 3\,d^I_{rs}h^r_J h^s_K\,\phi^J\phi^K
\end{aligned}
$$

tensor multiplet is on-shell: Yukawa couplings, (cubic) scalar potential

- **supersymmetry of these equations implies**

$$
\begin{aligned}
b_{Irs}\left(Y^s_{ij}\,\phi^I - 2\bar{\lambda}^s_{(i}\chi^I_{j)}\right) &= 0 \\
b_{Irs}\left(\mathcal{F}^s_{\mu\nu}\phi^I - 2\,\bar{\lambda}^s\gamma_{\mu\nu}\chi^I\right) &= \tfrac{1}{4!}\,\varepsilon_{\mu\nu\lambda\rho\sigma\tau}\,\mathcal{H}^{(4)\,\lambda\rho\sigma\tau}_r
\end{aligned}
$$

vector multiplet (partially) on-shell: three-forms are dual to vectors $K_{rs} \equiv b_{I\,rs}\,\phi^I$

superconformal field equations

- **can these equations be lifted to an action ?**
  modulo the standard subtleties for self-dual forms (HT, PST, "democratic")

yes! provided that there is a constant metric η_{IJ}
and the parameters are related as $h_I^r = \eta_{IJ} g^{Jr}$, $2\, d^I_{rs} = \eta^{IJ} b_{Jrs}$

$$
\begin{aligned}
\mathcal{L} \;=\; & -\tfrac{1}{8} D^\mu \phi_I\, D_\mu \phi^I - \tfrac{1}{2} \bar{\chi}_I\, \gamma^\mu D_\mu \chi^I + \tfrac{1}{16} b_{Irs} \phi^I \left(\mathcal{F}^r_{\mu\nu} \mathcal{F}^{\mu\nu\, s} - 4 Y^r_{ij} Y^{ij\, s} + 8 \bar{\lambda}^r \gamma^\mu D_\mu \lambda^s \right) \\
& - \tfrac{1}{96} \mathcal{H}^I_{\mu\nu\rho}\, \mathcal{H}_I^{\mu\nu\rho} - \tfrac{1}{48} b_{Irs} \mathcal{H}^I_{\mu\nu\rho}\, \bar{\lambda}^r \gamma^{\mu\nu\rho} \lambda^s - \tfrac{1}{4} b_{Irs} \mathcal{F}^r_{\mu\nu}\, \bar{\lambda}^s \gamma^{\mu\nu} \chi^I + b_{Irs} Y^r_{ij}\, \bar{\lambda}^{i\, s} \chi^{j\, I} \\
& + \tfrac{1}{2} \left(b_{Jsr} g^s_I - 4 b_{Isr} g^s_J \right) \phi^I \bar{\lambda}^r \chi^J + \tfrac{1}{8} b_{Irs} g^r_J g^s_K\, \phi^I \phi^J \phi^K - \tfrac{1}{48} \mathcal{L}_{\text{top}} \\
& - \tfrac{1}{24} b_{Irs} b^I_{uv}\, \bar{\lambda}^r \gamma^\mu \lambda^u \bar{\lambda}^s \gamma_\mu \lambda^v \;,
\end{aligned}
$$

Yukawa couplings, topological term : $\displaystyle\int_{\partial M_7} \mathcal{L}_{\text{top}} \;\propto\; \int_{M_7} \left(b_{Irs}\, \mathcal{F}^r \wedge \mathcal{F}^s \wedge \mathcal{H}^I - \mathcal{H}^I \wedge D\mathcal{H}_I \right)$

cubic scalar potential (superconformal), indefinite

indefinite metrics (ghosts) : $g^{Ir} \eta_{IJ}\, g^{Js} \;\equiv\; 0$ $\qquad K_{rs} \;\equiv\; b_{I\,rs}\, \phi^I$

missing gauge symmetry..? (cf. Lorentzian 3-algebras)
missing constraints..?

examples

examples

solutions of

$$
\begin{aligned}
2\,(d_{r(u}^{J} d_{v)s}^{I} - d_{rs}^{I} d_{uv}^{J}) h_J^s &= 2 f_{r(u}{}^{s} d_{v)s}^{I} - b_{Jsr} d_{uv}^{J}\, g^{Is} \\
(d_{rs}^{J}\, b_{Iut} + d_{rt}^{J}\, b_{Isu} + 2\, d_{ru}^{K} b_{Kst} \delta_I^J)\, h_J^u &= f_{rs}{}^{u} b_{Iut} + f_{rt}{}^{u} b_{Isu} + g^{Ju} b_{Iur} b_{Jst} \\
f_{[pq}{}^{u} f_{r]u}{}^{s} - \tfrac{1}{3} h_I^s\, d_{u[p}^{I} f_{qr]}{}^{u} &= 0 \\
h_I^r g^{Is} &= 0 \\
f_{rs}{}^{t} h_I^r - d_{rs}^{J}\, h_J^t h_I^r &= 0 \\
g^{Js} h_K^r b_{Isr} - 2 h_I^s h_K^r\, d_{rs}^{J} &= 0 \\
-f_{rt}{}^{s} g^{It} + d_{rt}^{J}\, h_J^s g^{It} - g^{It} g^{Js}\, b_{Jtr} &= 0
\end{aligned}
$$

examples

- **models with G adjoint tensor multiplets** $\{B^r_{\mu\nu}\}$

  for any compact group G

- **vectors in G** $\{A^r_\mu\}$

  $$h^r_s = 0,\ g^{rs} = \eta^{rs},\ b_{t\,rs} = f_{rst}$$

  YM field strength, Stückelberg-type coupling of tensors to three-forms, potential vanishes, no action

examples

- **models with G adjoint tensor multiplets** $\{B^r_{\mu\nu}\}$

  for any compact group G

- **vectors in G** $\{A^r_\mu\}$

$$h^r_s = 0,\ g^{rs} = \eta^{rs},\ b_{t\,rs} = f_{rst}$$

YM field strength, Stückelberg-type coupling of tensors to three-forms,

potential vanishes, no action

- **vectors in G x G** $\left\{A^r_\mu, A^{r'}_\mu\right\}$ [Chu, arXiv:1108.5131]

$$f_{rs}{}^t = \mathsf{f}_{rs}{}^t, \quad f_{rs'}{}^{t'} = -f_{s'r}{}^{t'} = \frac{1}{2}\mathsf{f}_{rs'}{}^{t'}$$
$$d^t_{rs'} = d^t_{s'r} = -\frac{1}{2}\mathsf{f}_{rs'}{}^t, \quad h^{r'}_s = \delta^{r'}_s$$

$$g^{rs} = 0 = g^{rs'}$$

no three-forms, Stückelberg-type coupling of vectors to tensors,

potential vanishes, no action

examples

- **more general examples** [HS, Sezgin, Wimmer, Wulff]

choose a basis $A^r_\mu \longrightarrow \{A^\alpha_\mu, A^a_\mu\}$ $\quad B_{\mu\nu}{}^I \longrightarrow \{B_{\mu\nu a'}, B_{\mu\nu}{}^a\}$

such that

$$h^r{}_I = \begin{pmatrix} h^{\alpha b'} & h^\alpha{}_b \\ h^{ab'} & h^a{}_b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \delta^a_b \end{pmatrix} \qquad g^{Ir} = \begin{pmatrix} g_{a'}{}^\alpha & g_{a'}{}^b \\ g^{a\alpha} & g^{ab} \end{pmatrix} = \begin{pmatrix} g_{a'}{}^\alpha & g_{a'}{}^b \\ 0 & 0 \end{pmatrix}$$

then the system of constraints implies that

- $f_{\alpha\beta}{}^\gamma$ are the structure constants of a Lie algebra $\mathfrak{g}$
- $d^b{}_{\alpha a} = f_{a\alpha}{}^b = \frac{1}{2}(T_\alpha)_a{}^b$ form a representation $\mathcal{R}$ of $\mathfrak{g}$
- all remaining tensors are $\mathfrak{g}$-invariant

which leaves three classes of models: according to representation $\mathcal{R}'$ of a'

- $\mathcal{R}'$ trivial : Lie algebra $\mathfrak{g}$ with representation $\mathcal{R}$, no action
- $\mathcal{R}'$ adjoint : Lie algebra $\mathfrak{g}$ with representation $\mathcal{R}$, no action
- $\mathcal{R}'$ contragredient to $\mathcal{R}$: action

examples

- **more general examples** [HS, Sezgin, Wimmer, Wulff]

  choose a basis $A^r_\mu \longrightarrow \{A^\alpha_\mu, A^a_\mu\}$ $\quad B_{\mu\nu}{}^I \longrightarrow \{B_{\mu\nu a'}, B_{\mu\nu}{}^a\}$

  - $\mathcal{R}'$ contragredient to $\mathcal{R}$: action

  p-form field content : $\{A^\alpha, A^a, B^a, B_a, C_a\}$

  defined by choice of $\mathfrak{g}$ and $\mathcal{R}$ and invariant tensors $d_{abc}, d_{a(\alpha\beta)}, d_{(ab)\alpha}$

  e.g. scalar potential $\mathcal{L}_{\rm pot} \;=\; d_{abc}\,\phi^a\phi^b\phi^c$ (can be vanishing)

  with all *d*-tensors set to zero:

$$
\begin{aligned}
\mathscr{L} \;=\; & -\frac{1}{4}D^\mu\phi_a D_\mu\phi^a - \bar\chi_a\gamma^\mu D_\mu\chi^a + \frac{1}{8}(T_\alpha)_b{}^a\phi_a\Big(\mathscr{B}^b_{\mu\nu}F^{\mu\nu\,\alpha} - 4Y^b_{ij}Y^{ij\,\alpha} + 8\bar\lambda^{(b}\gamma^\mu D_\mu\lambda^{\alpha)}\Big) \\
& -\frac{1}{8}\mathscr{C}_a^{\mu\nu\rho\,(+)}\Big(D_\mu\mathscr{B}^a_{\nu\rho} + \frac{1}{3}(T_\alpha)_b{}^a\bar\lambda^\alpha\gamma_{\mu\nu\rho}\lambda^b\Big) - \frac{1}{4}(T_\alpha)_b{}^a\mathscr{B}^b_{\mu\nu}\bar\lambda^\alpha\gamma^{\mu\nu}\chi_a \\
& -\frac{1}{4}(T_\alpha)_b{}^aF^\alpha_{\mu\nu}\bar\lambda^b\gamma^{\mu\nu}\chi_a + 2(T_\alpha)_b{}^aY^{(b}_{ij}\bar\lambda^{i\,\alpha)}\chi^j_a + \frac{1}{2}(T_\alpha)_a{}^b\bar\lambda^\alpha(\phi^a\chi_b - 4\phi_b\chi^a)\;.
\end{aligned}
$$

  (classical) dynamics remains complicated...

conclusions / outlook

- **non-abelian tensor fields**
  - non-abelian two-forms
  - coupling to three-form gauge potentials
- **supersymmetry implies equations of motion**
  - (1,0) superconformal system
  - action
- **consistency constraints and classes of examples**

conclusions / outlook

- **non-abelian tensor fields**
  - non-abelian two-forms
  - coupling to three-form gauge potentials
- **supersymmetry implies equations of motion**
  - (1,0) superconformal system
  - action
- **consistency constraints and classes of examples**
- **understand their structure / quantization**
  - ghosts: gauge fixing, imposing further constraints ...
  - cubic potential, vector field dynamics
- **classification: solutions to the consistency constraints**
  - Jacobi identities, fundamental identities, ...
  - Killing spinor equations, BPS solutions [Akyol, Papadopoulos]
- **extension to (2,0) theories, relation to D=5 SYM**
  - include hypermultiplets, non-propagating vector multiplets