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Hard double parton scattering in hadronic collisions

A.M. Snigirev
D.V.Skobeltsyn Institute of Nuclear Physics
M.V.Lomonosov Moscow State University
Moscow, Russia, 119991

PARTON MODEL

Elastic scattering : electron — proton

——> proton (hadron) NOT point-like

Deep inelastic scattering : electron — proton

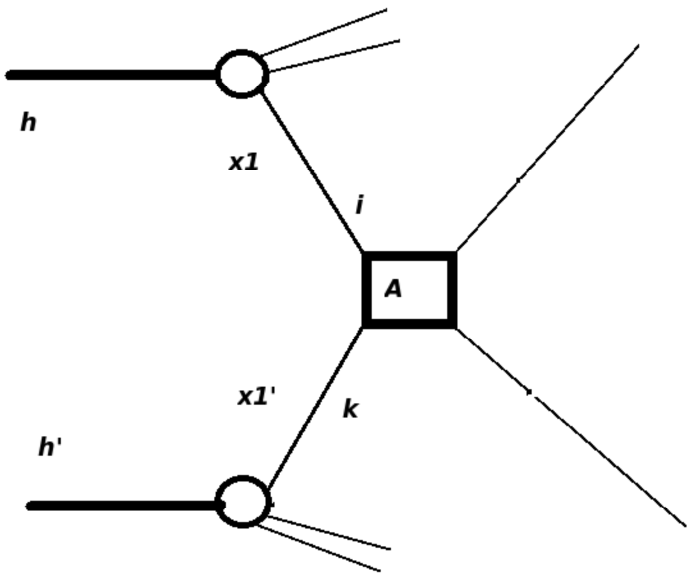
——> proton (hadron) consists of point-like particles-partons

$$\text{Cross section (hadron)} = \Sigma \text{ cross section (parton)} \times \text{weights}$$

Weights — probabilities in the system of infinite momentum

Bjorken, Feynman

In QCD weights depend on scale Q of hard processes
(SCALING VIOLATION)



$$\sigma^{A}_{\text{SPS}} = \sum_{i,k} \int D^{i}_{h}(x_1; Q_1^2)\hat{\sigma}^{A}_{ik}(x_1, x_1^{'})D^{k}_{h'}(x_1^{'}; Q_1^2)dx_1 dx_1^{'}$$

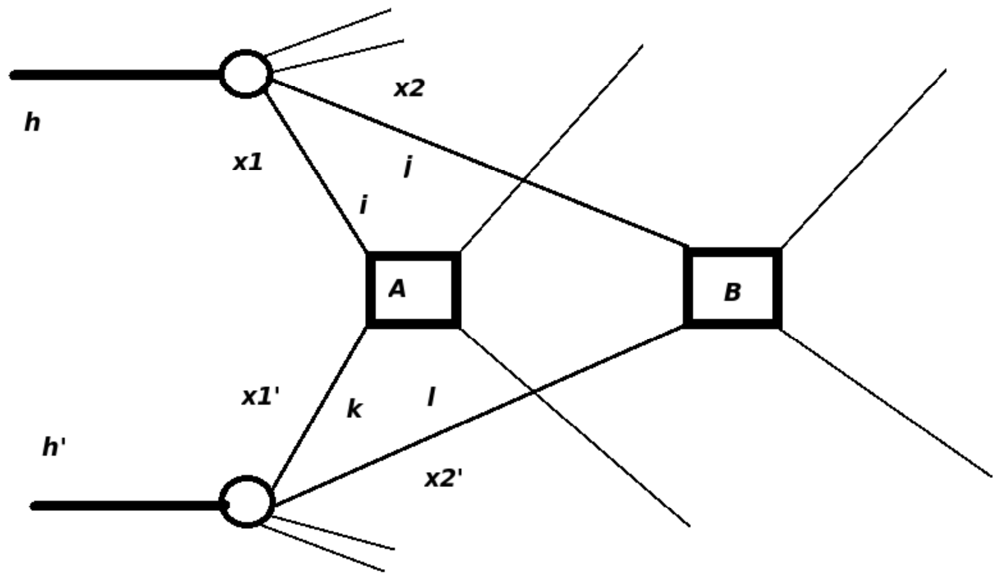
Scaling violation (dependence on Q) from
DGLAP (*Dokshitzer-Gribov-Lipatov-Altarelli-Parisi*) equations:

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x',t) P_{j' \to j}\left(\frac{x}{x'}\right)$$

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}{\ln\left(\frac{\mu^2}{\Lambda^2_{QCD}}\right)}\right], \quad b = \frac{33 - 2n_f}{12\pi},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2,
n_f is the number of active flavours,
Λ_{QCD} is the dimensional QCD parameter.

It is possible (BUT very rarely): hard double parton scattering (subprocesses A and B)



The inclusive cross section of a double parton scattering process in a hadron collision is written in the following form (with the assumption of factorization of the two hard parton subprocesses A and B)

$$\sigma_{DPS}^{AB} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) \hat{\sigma}_{ik}^{A}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}_{jl}^{B}(x_2, x_2^{'}, Q_2^2)$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2 b_1 d^2 b_2 d^2 b,$$

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two parton undergoing hard processes A and B at the scales Q_1 and Q_2.

$\hat{\sigma}_{ik}^{A}$ and $\hat{\sigma}_{jl}^{B}$ are the parton-level subprocess cross sections.

The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain the information when probing the hadron in two different points simultaneously, through the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\boldsymbol{\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2)},$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\boldsymbol{\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1},$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (NOT calculated in pQCD)

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2),$$

the cross section of double parton scattering can be expressed in the simple form

$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2} \frac{\sigma_{\rm SPS}^A \sigma_{\rm SPS}^B}{\sigma_{\rm eff}},$$

$$\pi R_{\rm eff}^2 = \sigma_{\rm eff} = [\int d^2 b (T({\rm b}))^2]^{-1}$$

is the effective interaction transverse area (effective cross section)

$R_{\rm eff}$ is an estimate of the size of the hadron

The momentum (*instead of the mixed (momentum and coordinate)*) representation is more convenient:

$$\sigma^{D}_{(A,B)} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_{ij}(x_1, x_2; \mathrm{q}; Q_1^2, Q_2^2)\hat{\sigma}^{A}_{ik}(x_1, x_1^{'})\hat{\sigma}^{B}_{jl}(x_2, x_2^{'})$$

$$\times\Gamma_{kl}(x_1^{'}, x_2^{'}; -\mathrm{q}; Q_1^2, Q_2^2)dx_1 dx_2 dx_1^{'} dx_2^{'}\frac{d^2 q}{(2\pi)^2}.$$

Here the transverse vector q is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such dependence arises because the difference of parton transverse momenta within the parton pair is not conserved.

The main problem is to make the correct calculation of the two-parton functions $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ **WITHOUT simplifying factorization assumptions** (*which are not sufficiently justified and should be revised:*
Blok, Dokshitzer, Frankfurt, Strikman;
Diehl, Schafer;
Gaunt, Stirling;
Ryskin, Snigirev)

These functions were available in the current literature only for $\mathbf{q} = 0$ in the collinear approximation. In this approximation the two-parton distribution functions, $\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ with the two hard scales set equal, satisfy the generalized Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations
(*Kirshner; Shelest, Snigirev, Zinovjev*)

The functions in question have a **specific interpretation** in the leading logarithm approximation of perturbative QCD: they are the inclusive **probabilities** that in a hadron h one finds two bare partons of types i and j with the given longitudinal momentum fractions x_1 and x_2.

$$\frac{dD_h^{j_1 j_2}(x_1, x_2, t)}{dt} = \sum_{j_1{}'} \int_{x_1}^{1-x_2} \frac{dx_1{}'}{x_1{}'} D_h^{j_1{}' j_2}(x_1{}', x_2, t) P_{j_1{}' \to j_1}\left(\frac{x_1}{x_1{}'}\right)$$

$$+ \sum_{j_2{}'} \int_{x_2}^{1-x_1} \frac{dx_2{}'}{x_2{}'} D_h^{j_1 j_2{}'}(x_1, x_2{}', t) P_{j_2{}' \to j_2}\left(\frac{x_2}{x_2{}'}\right)$$

$$+ \sum_{j'} D_h^{j'}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right)$$

Solution of generalized DGLAP equations $\neq$
Product of single distribution functions
(factorization component)

The solution of generalized DGLAP evolution equations with a given initial condition at the reference scale μ^2 can be presented (*Snigirev (2003)*):

$$D_h^{j_1j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= D_{h1}^{j_1j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) + D_{h2}^{j_1j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2),$$

$$D_{h1}^{j_1j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= \sum_{j_1{'}j_2{'}} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2}\, D_h^{j_1{'}j_2{'}}(z_1, z_2; \mu^2) D_{j_1{'}}^{j_1}(\frac{x_1}{z_1}; \mu^2, Q_1^2) D_{j_2{'}}^{j_2}(\frac{x_2}{z_2}, \mu^2, Q_2^2),$$

$$D_{h2}^{j_1j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) = \sum_{j'j_1{'}j_2{'}} \int_{\mu^2}^{\min(Q_1^2, Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \times$$

$$D_h^{j'}(z_1 + z_2; \mu^2, k^2) \frac{1}{z_1 + z_2} P_{j' \to j_1{'}j_2{'}}(\frac{z_1}{z_1 + z_2}) D_{j_1{'}}^{j_1}(\frac{x_1}{z_1}; k^2, Q_1^2) D_{j_2{'}}^{j_2}(\frac{x_2}{z_2}; k^2, Q_2^2)$$

The pQCD correlations versus factorization componet was calculated (*Korotkikh, Snigirev, (2004)*):

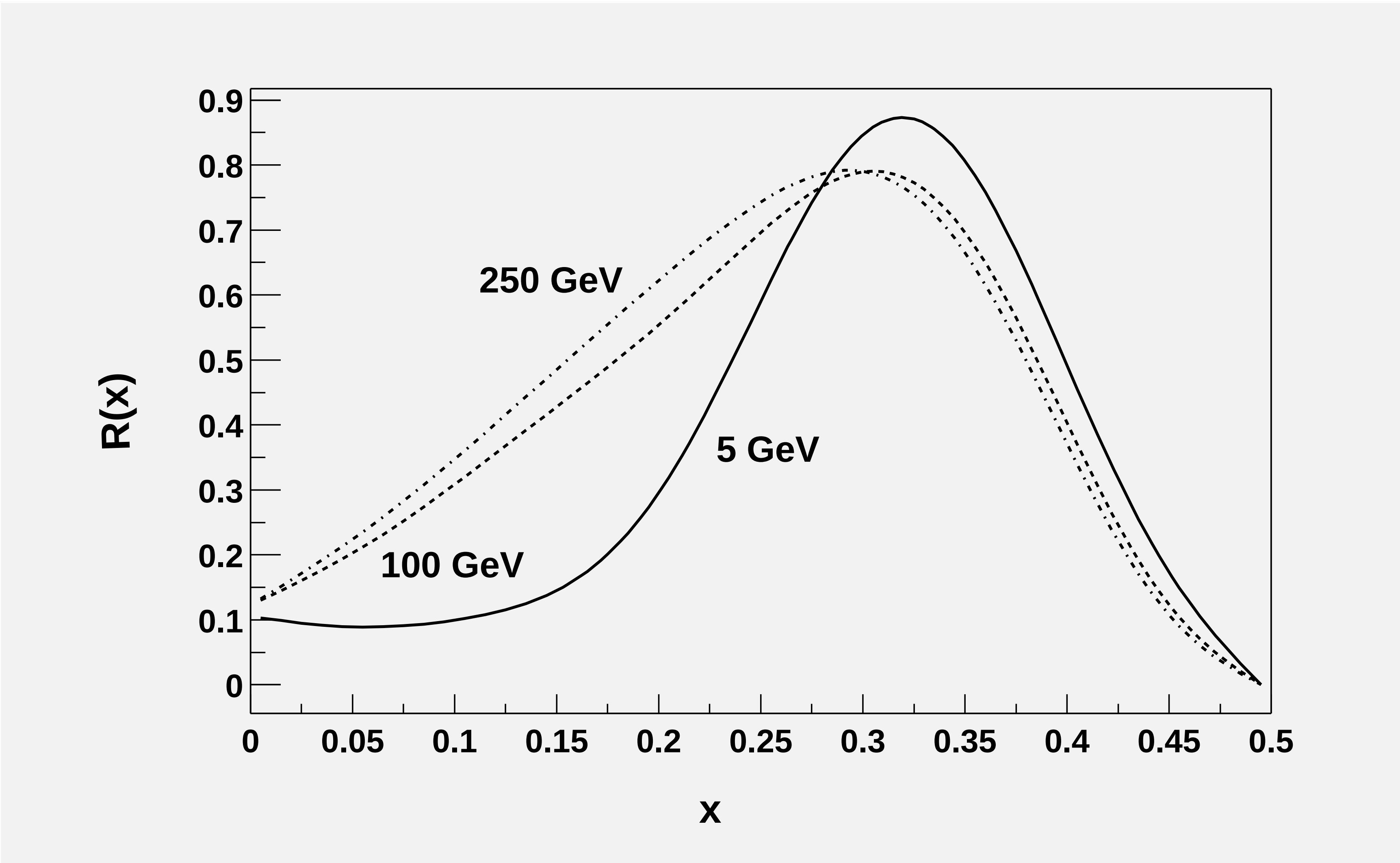
$$R(x,t) = (D^{gg}_{p(QCD)}(x_1,x_2,t)/D^g_p(x_1,t)D^g_p(x_2,t)(1-x_1-x_2)^2)|_{x_1=x_2=x}.$$

Then generalized DGLAP equations have been numerically integrated (*Gaunt, Stirling (2010)*), and a set of publicly available grids covering the ranges:

$$10^{-6} < x_1 < 1, \quad 10^{-6} < x_2 < 1, \quad 1 < Q^2 < 10^9 GeV$$

is given.

Possible manifestation of QCD evolution at the LHC was calculated for a number of processes



R(x)
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.05
0.15
0.25
0.35
0.45
250 GeV
100 GeV
5 GeV

Promising candidate processes to probe double parton scattering at the LHC:

- same-sign W production ("pure", BUT very rare)
- $\gamma + 3$ jets (Tevatron also: D0, CDF)
- $W(Z) + 2$ jets (ATLAS — first measurement σ_{eff} at LHC)
- 4 jets (Tevatron also: CDF)
- $b\bar{b}$ пара $+ 2$ jets
- $b\bar{b}$ pair $+ W$ boson
- pairs of heavy mesons (in particular, double J/ψ production)
  (*Baranov, Snigirev, Zotov (2011)* also)
  (LHCb — first measurement of double J/ψ production)

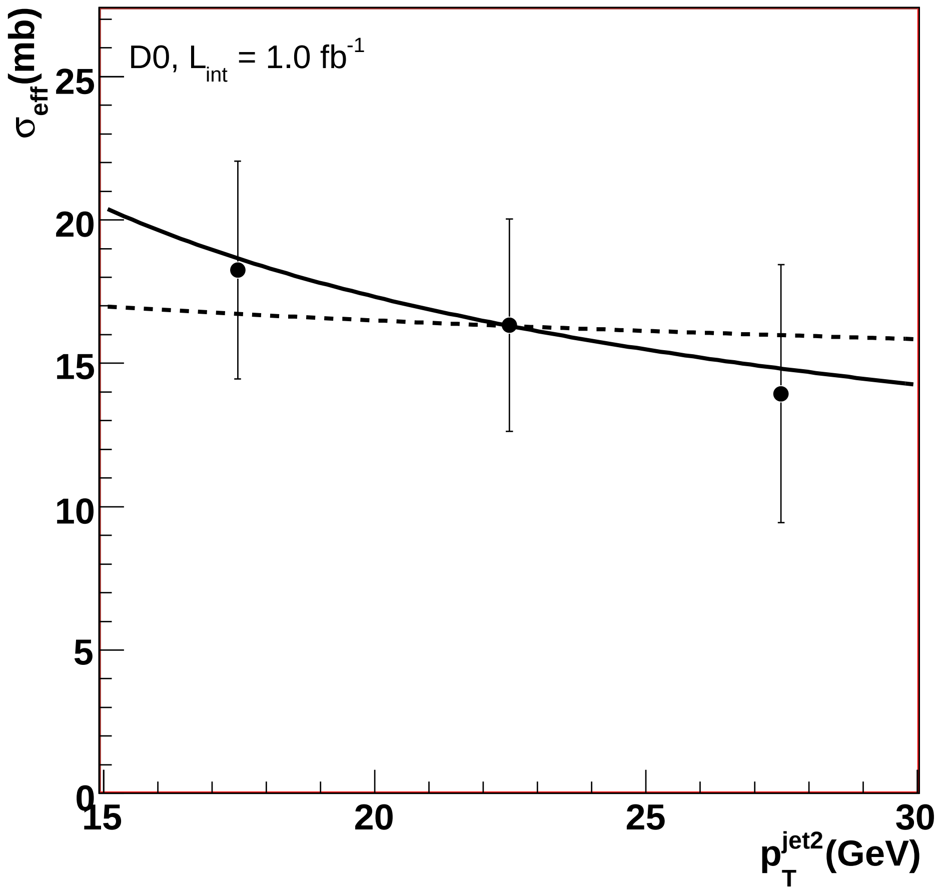
D0 Collaboration (Tevatron) has measured σ_{eff} at 3 different scales (*Phys. Rev. D 81, 052012 (2010)*)

in process with γ +3 jets in final state.

These results can be interpreted as a first inderect observation of the QCD evolution of double parton distributions

Snigirev (2010)

Flensburg, Gustafson, Lonnblad, Ster (2011)



Experimental extraction:

$$\frac{\sigma_{DPS}^{\gamma+3j}}{\sigma^{\gamma j}\sigma^{jj}} = [\sigma_{\rm eff}^{\rm exp}]^{-1}$$

Theoretical "prediction":

$$\sigma_{\rm eff}^{\rm exp} = \sigma_{\rm eff}^{0}[1 + k\ln(p_T^{\rm jet2}/p_{T0}^{\rm jet2}]^{-1}$$

inspired by the explicit expression for the correlation term and the evolution variable t ($k = 0.1$ (dashed line) and $k = 0.5$ (solid line))

MAIN RESULTS

- Generalized DGLAP equations
- Solution = Factorization component + Correlations
- Ratio: (Correlations)/(Factorization component)
  — NOT small, observable
- New correct formulas for cross section calculation taking into account QCD evolution of double parton distributions (Generalization for $q \neq 0$)
- Deviation from factorization for effective cross section (dependence on scale of hard process)
- First estimations of cross section of heavy meson pair production in double parton scattering

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