DID THE UNIVERSE HAVE A BEGINNING?

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Singularity theorems

Penrose, Hawking (1960's)

Assume strong energy condition: $R_{\mu\nu}u^{\mu}u^{\nu} \geq 0, \qquad u^{\mu}u_{\mu} = 1.$

+ make assumptions about the global structure of spacetime.
Show that such a spacetime is past geodesically incomplete.

Hawking & Ellis (1973):

"The results we have obtained support the idea that the universe began a finite time ago."

"Beginning" means that spacetime has a past boundary, different from the past infinity.

Assume semiclassical spacetime.

Is it possible to avoid this conclusion?

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- Cyclic evolution
- Static seed ("emergent universe")

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- Eternal inflation
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Geodesically incomplete

- Static seed (“emergent universe”) – Quantum instability

Eternal inflation

Guth (1981)
A.V. (1983)
Linde (1986)

Inflation is generically eternal to the future.
Could it have no beginning in the past?

Strong energy condition is violated during inflation.

⇨ Penrose-Hawking singularity theorems do not apply.

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Extension to inflationary models

Borde & A.V. (1990's)

Assume weak energy condition (WEC): $R_{\mu\nu} n^{\mu} n^{\nu} \geq 0, \qquad n^{\mu} n_{\mu} = 0.$

(+ global assumptions)

However, even the WEC is violated by quantum fluctuations during inflation.

A kinematic incompleteness theorem *(loose formulation):*

A spacetime that is on average expanding, $H_{av} > 0$, is past geodesically incomplete.

Borde, Guth & A.V. (2003)

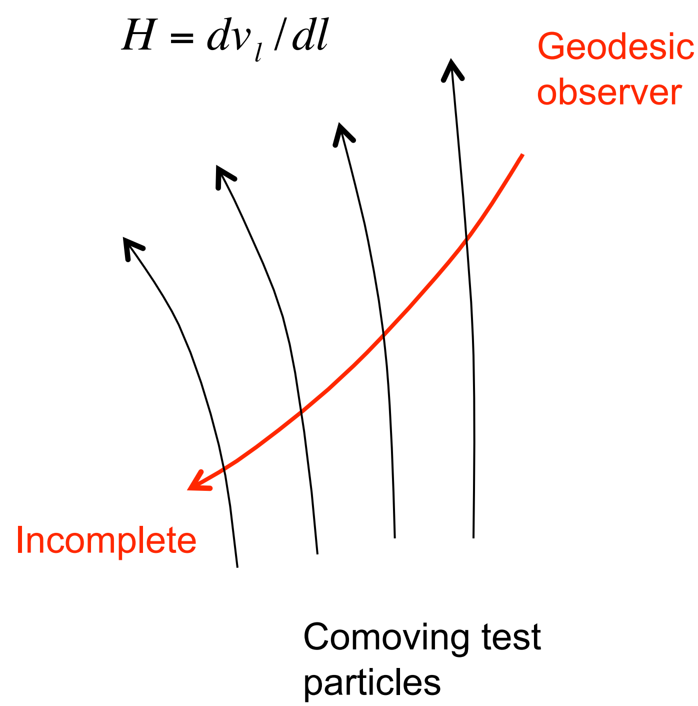
Does not rely on Einstein's equations or energy conditions.

A more precise statement of the theorem:

We say a spacetime is expanding if it can be filled with an expanding congruence of "comoving test particles".

Let H be the Hubble expansion rate of the congruence along the worldline of some geodesic observer.

If $H_{av} > 0$ along the worldline, this worldline must be past-incomplete.



Corollary:

Inflating spacetimes are past geodesically incomplete

⇨ have a beginning.

Cyclic universe

Steinhardt & Turok (2001)

The solutions where the universe successively expands and contracts … have an incontestable poetic charm and bring to mind the Phoenix of the legend.

Lemaitre (1933)

The problem with periodic cycles: entropy S will increase in every cycle.

Tolman (1934): The volume should grow in every cycle.

$\Rightarrow$ S grows, while S/V is bounded.

This is what happens in the cyclic model of Steinhardt & Turok.

$H_{av} > 0$ $\Rightarrow$ the spacetime is past-incomplete.

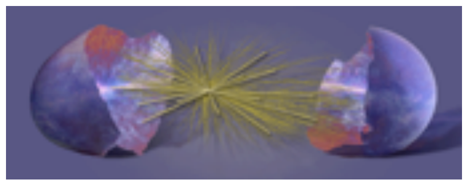
Emergent universe scenario

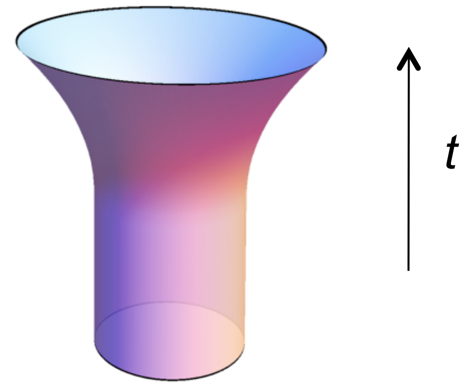
Eddington, Lemaitre (1930's)
Ellis et al (2004,2005)
del Campo et al (2011)
Graham et al (2011)

- Assumes that the universe is closed & static in the asymptotic past.

"Cosmic egg"

(Rig Veda)





Note: $H_{av} = 0$ ⟹ the theorem does not apply.

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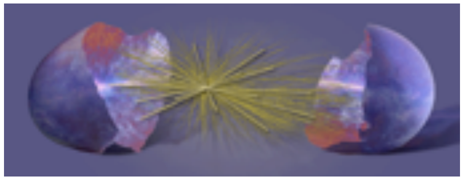
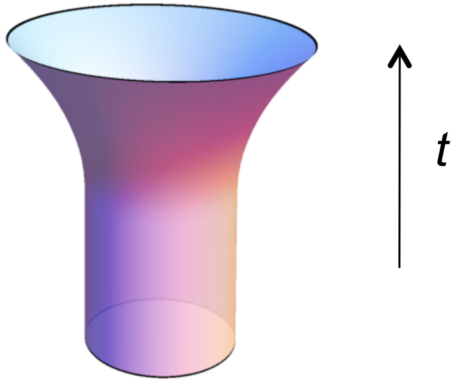
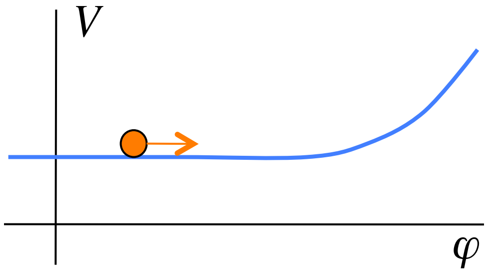




- Needs a mechanism to end the static phase.

e.g., a rolling scalar field

Ellis et al (2004)



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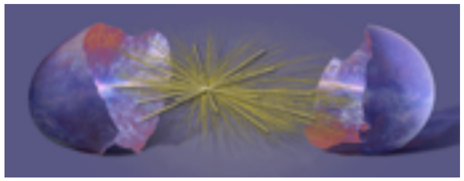
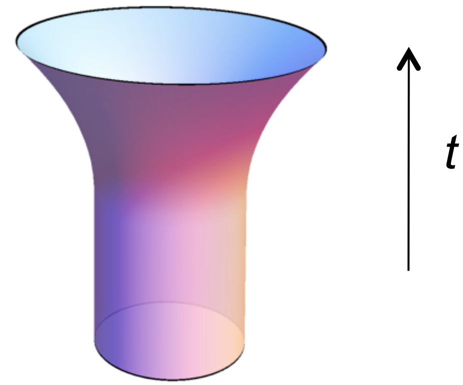




- To ensure stability, need 'exotic' matter or modified gravity.

- Assume matter with $P = w\rho$ plus a cosmological constant Λ.

Stable static solutions exist if

$\Lambda < 0$ and $-1 < w < -1/3$.

Gravity of matter is repulsive, while gravity of Λ is attractive – opposite to Einstein universe.

“Simple harmonic universe”

Graham, Horn, Kachru, Rajendran & Torroba (2011)

$$w = -2/3\ , \qquad \rho(a) = \Lambda + \rho_0 a^{-1}$$

$$\Lambda < 0, \qquad \rho_0 > 0.$$

Domain wall network

The Friedmann equation

$$\dot{a}^2 + 1 = \frac{8\pi G}{3} a^2 \rho(a)$$

has a solution

$$a(t) = \omega^{-1}\left(\gamma - \sqrt{\gamma^2 - 1}\cos(\omega t)\right)$$

$$\omega = \sqrt{\frac{8\pi}{3} G\,|\Lambda|}$$

$$\gamma = \sqrt{\frac{2\pi G \rho_0^2}{3\,|\Lambda|}}$$

For $\gamma = 1$, static solution: $a(t) = \omega^{-1}$.

We will check for quantum-mechanical instabilities.

Audrey Mithani & A.V. (2011)

Hamiltonian dynamics

$$\mathcal{H} = -\frac{G}{3\pi a}\left(p_a^2 + U(a)\right)$$

$$p_a = -\frac{3\pi}{2G} a\dot{a}$$

$$U(a) = \left(\frac{3\pi}{2G}\right)^2 a^2\left(1 - \frac{8\pi G}{3} a^2 \rho(a)\right)$$

Hamiltonian constraint: $\mathcal{H} = 0$.

$$\rho(a) = \Lambda + \rho_0 a^{-1}$$

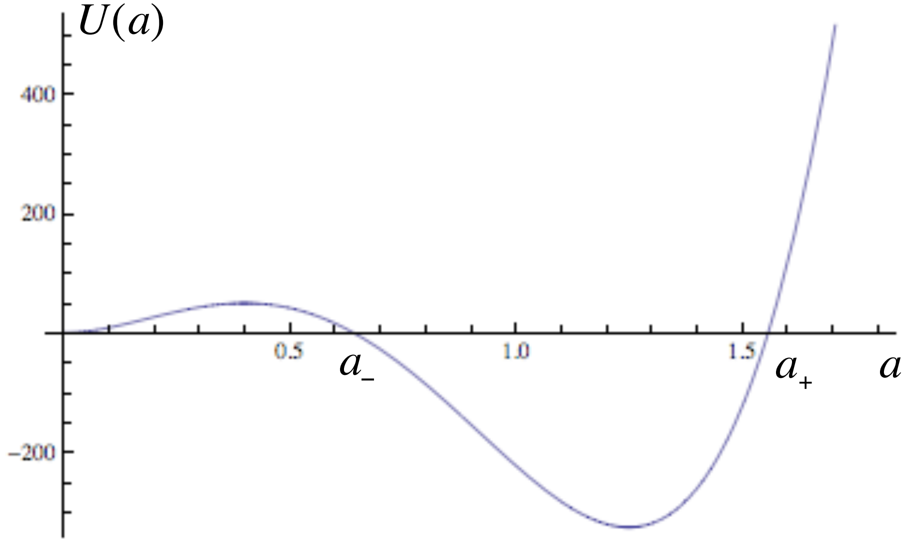
Minisuperspace quantization:

$$\psi = \psi(a)$$

$$p_a \to -i\frac{d}{da} \,, \qquad \mathcal{H}\,\psi = 0 \,.$$

WDW equation:

$$\left(-\frac{d^2}{da^2} + U(a)\right)\psi(a) = 0 \,.$$



Note: the potential is *not* that of a harmonic oscillator.

$$\omega = \sqrt{\frac{8\pi}{3} G \,|\Lambda|}$$

$$\gamma = \sqrt{\frac{2\pi G \rho_0^2}{3\,|\Lambda|}}$$

Turning points: .

$$a_{\pm} = \omega^{-1}\left(\gamma \pm \sqrt{\gamma^2 - 1}\right)$$

The universe can tunnel from a_- to $a = 0$.

Semiclassical tunneling: $P \sim e^{-2S}$

$$S = \int_0^{a_-} \sqrt{U(a)}\, da$$

For a static universe $(\gamma = 1)$: $S = \dfrac{3M_P^4}{32\,|\Lambda|}$

⇨ Simple harmonic universe cannot last forever.

Early work on oscillating universe models
& semiclassical tunneling:

Dabrowski & Larsen (1995)
Dabrowski (1996)

Solving the WDW equation

Compare with harmonic oscillator: $$\frac{1}{2}\left(-\frac{d^2}{dx^2}+\omega^2 x^2\right)\psi(x)=E\psi(x)\ .$$

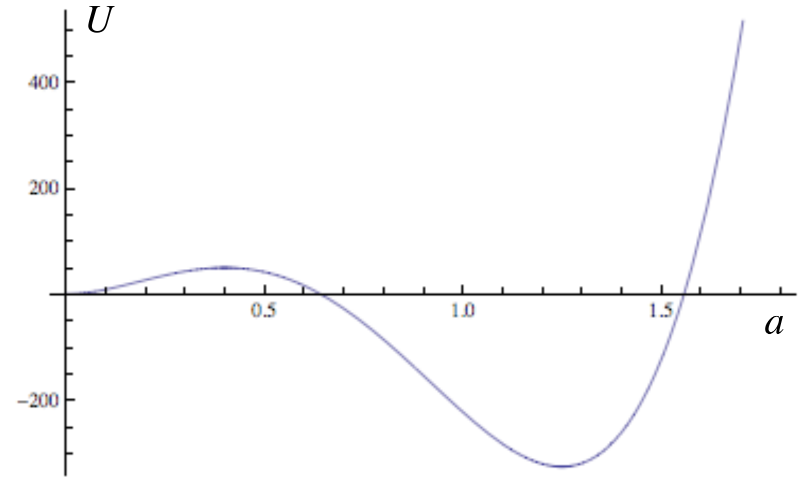
Boundary conditions: $\psi(x\to\pm\infty)\ =0$ $\Rightarrow$ $E=\left(n+\dfrac{1}{2}\right)\omega$

In our case, the energy eigenvalue is fixed at $E=0$.

$\Rightarrow$ We have freedom to impose only one boundary condition.

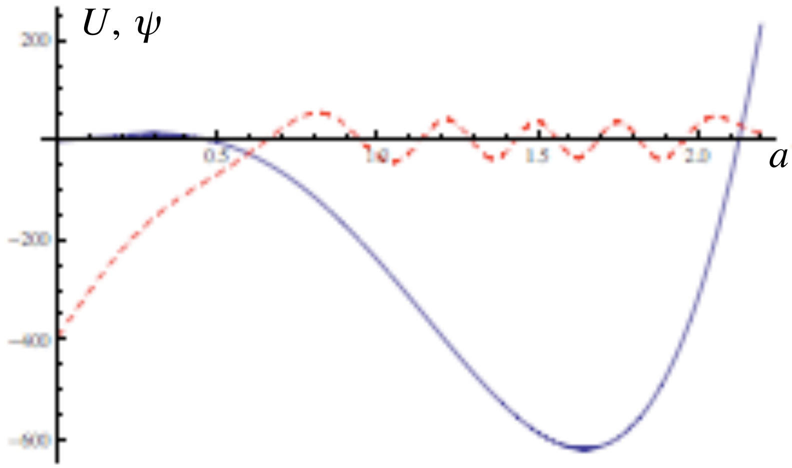
We choose $\psi(a\to\infty)\ =0.$

This fully specifies the solution.

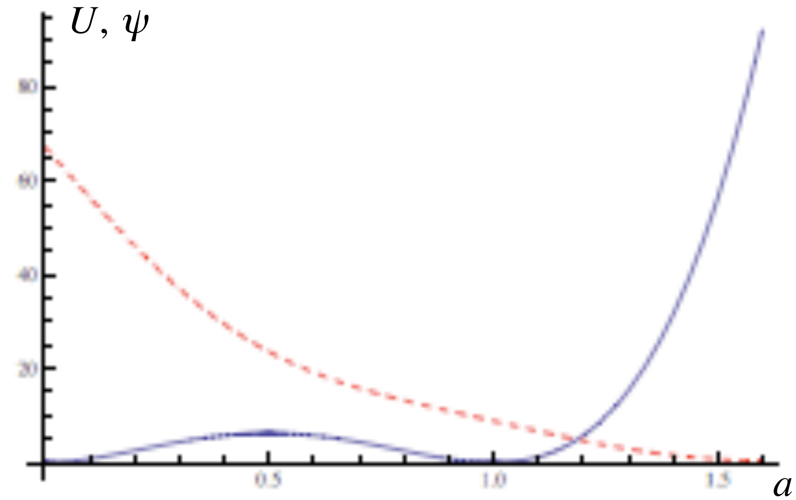


Solving the WDW equation

Numerical solutions for oscillating and static universe:



$$|\Lambda|/M_P^4 = 0.028, \quad \gamma = 1.3$$



$$|\Lambda|/M_P^4 = 0.056, \quad \gamma = 1$$

The wave function is nonzero at $a = 0$,
indicating a nonzero probability for collapse.

Adding dust, radiation, etc., does not change this conclusion.

Did the universe have a beginning?   Probably yes.

- Inflationary spacetimes are past incomplete.
- Cyclic spacetimes are either incomplete or lead to thermal death.
- Asymptotically static & oscillating universes suffer from quantum instability.

Incompleteness theorems do not tell us anything about the nature of the beginning.

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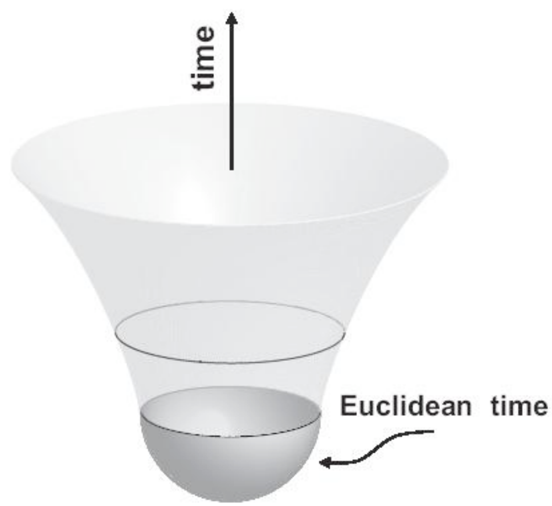
Quantum nucleation from nothing?



Grishchuk & Zeldovich (1981)
A.V. (1982,84)
Hartle & Hawking (1983)
Linde (1984)
Rubakov (1984)
Zeldovich & Starobinsky (1984)