Ginzburg Conference
Moscow, June 2, 2012

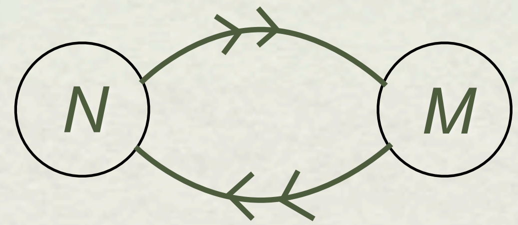
Symmetry Breaking in Higher Spin Holography

Xi Yin

Harvard University

based on work with Chi-Ming Chang, Simone Giombi, Shiraz Minwalla, Shiroman Prakash, Tarun Sharma, Sandip Trivedi, Spenta Wadia, ...

"ABJ Triality"



$U(N)_k \times U(M)_{-k}$ ABJ theory

n=6 **supersymmetric ~~parity~~ Vasiliev theory** with U(M) Chan-Paton factor and $\mathcal{N} = 6$ boundary condition

IIA string theory in $AdS_4 \times CP^3$

bound states of higher spin particles ⇢ strings

The HS/VM Duality

Vasiliev's system describes classical interacting higher spin gauge fields in AdS_4. The interactions are highly constrained (and almost uniquely fixed) by higher spin symmetry, which makes it plausible that the quantum theory is renormalizable and possibly finite, despite the higher derivative (and seemingly nonlocal) interactions. If Vasiliev's theory is a consistent quantum theory of gravity, then it should have a three-dimensional CFT dual.

Conjecture (Klebanov-Polyakov, Sezgin-Sundell '02): Vasiliev's minimal bosonic theory in AdS_4 is holographically dual to the free or critical O(N) vector model.

AdS boundary condition on bulk scalar field

	A-type	B-type
$\Delta=1$	free O(N) boson	critical O(N) fermion (Gross-Neveu)
$\Delta=2$	critical O(N) boson (Wilson-Fisher)	free O(N) fermion

A few basic comments on HS/VM duality

1. They are among the simplest class of examples of AdS/CFT correspondence.

2. Explicit realization of holographic dual of free CFTs (with large N factorization), and their deformations (interacting but typically exactly solvable at infinite N).

3. Bulk interactions involve arbitrarily high order derivative coupling and are generally non-local.

4. Full classical equations of motion known explicitly (in contrast to string theory in AdS where relatively little is known about the full closed string field theory). But the classical action (of the form Fronsdal action + all order nonlinear terms) is not explicitly known and appears to be very complicated (if one tries to recover it order by order in fields).

See Boulanger-Sundell, however, for an alternative approach.

5. In principle tree level correlators can be calculated straightforwardly using Vasiliev's equations. Going to loop order, prescriptions for regularization and gauge fixing/ghosts remain to be understood. Modulo this technicality (presumingly), both sides of the duality are computable order by order in $1/N$.

6. AdS boundary conditions are important, and often break higher spin symmetry (and other symmetries).

Evidence for the duality

Giombi-Yin '09,'10: three-point functions of higher spin currents computed from Vasiliev theory at tree level match exactly with those of free and critical O(N) vector model.

Maldacena-Zhiboedov '11,'12:

exactly conserved higher spin current $\Rightarrow$ CFT is free.

"Approximate" HS symmetry $\Rightarrow$ three-point functions constrained.

Girardello-Porrati-Zaffaroni '02, Hartman-Rastelli '06, Giombi-Yin '10: In A-type Vasiliev theory, higher spin symmetry broken by $\Delta=2$ boundary condition. Duality with free O(N) theory for $\Delta=1$ boundary condition implies the duality with critical O(N) theory for $\Delta=2$ boundary condition, to all order in perturbation theory (1/N).

What remains to be shown: Vasiliev's system can be quantized in a manner in which higher spin symmetry is not anomalous nor broken by boundary conditions.

See Koch-Jevicki-Jin-Rodrigues, Douglas-Mazzucato-Razamat, for alternative approaches toward deriving the duality.

Explicit computation of 4-point function and loop corrections (should be absent for $\Delta=1$ b.c.) in Vasiliev theory yet to be performed.

Generalizations

Gaiotto-Yin '07: Chern-Simons-matter theories provide a large class of 3d CFTs $\Leftrightarrow$ various supersymmetric or non-supersymmetric string theories in AdS_4.

In some examples, there is a semi-classical gravity limit (Aharony-Bergman-Jafferis-Maldacena '08). In many other examples, the dual must always involve higher spin fields (Minwalla-Narayan-Sharma-Umesh-Yin '11).

Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin, Aharony-Gur-Ari-Yacoby '11: Chern-Simons vector models have approximately conserved higher spin currents at large N.

Conjecture: Chern-Simons vector models are dual to ~~parity~~ Vasiliev theories in AdS_4.

Chern-Simons vector model

U(N) or SU(N) Chern-Simons theories coupled to massless scalars (AGY '11) or massless fermions (GMPTWY '11) in the fundamental representation are exactly conformal vector models.

e.g. CS-fermion vector model

$$S = \frac{k}{4\pi}\int \left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) + \int \overline{\psi}\gamma^{\mu}D_{\mu}\psi$$

't Hooft limit: N large, λ= N/k finite.

Note: CS level k is quantized and does not run under RG. For CS-fermion vector model, the critical point is achieved by simply tuning the fermion mass to zero. For CS-scalar vector model, mass and quartic scalar potential terms can be tuned to zero, while the sextic coefficient takes a finite fixed value at critical point.

Higher spin symmetry breaking

Spin-s operator $J^{(s)}{}_{\mu_1 \ldots \mu_s}$ made out of bilinears of fundamental matter fields, conserved at infinite N. Current conservation broken by $1/N$ effects, through mixing with double-trace and triple-trace operators:

$$\partial^\mu J^{(s)}{}_{\mu\ldots} = f(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} + g(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} \partial^{n_3} J^{(s_3)}$$

where the spin-s current $J^{(s)}{}_{\mu\ldots}$ is normalized so that $\langle J^{(s)} J^{(s)} \rangle \sim N$.

Easy generalization to more than one flavor and to the supersymmetric case.

With the same matter fields and Chern-Simons level, $\mathcal{N} = 0,1,2,3$ CS vector models differ merely by double trace and triple trace deformations. With suitable matter content, $\mathcal{N} = 4$ (Gaiotto-Witten) or $\mathcal{N} = 6$ (ABJ) CS vector models are obtained by simply gauging a flavor group with CS coupling.

The higher spin dual

~~Parity~~ Vasiliev theory determine by the function

$f(X) = X \exp(i\,\theta(X)) = e^{i\theta_0} X + b_3 X^3 + b_5 X^5 + \ldots$

Recall equation of motion: $d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K) dz^2 + \bar{f}_*(\mathbb{B} * \bar{K}) d\bar{z}^2$.

Coefficient of X^n determines $n+2$ and higher order coupling. E.g. $e^{i\theta_0}$ controls tree level three-point function, b_3 controls five-point function, etc.

Tree level three-point function

$$\langle J\,J\,J \rangle = \cos^2\theta_0 \,\langle J\,J\,J \rangle_{FB} + \sin^2\theta_0 \,\langle J\,J\,J \rangle_{FF} + \sin\theta_0 \cos\theta_0 \,\langle J\,J\,J \rangle_{odd}$$

free boson   free fermion   parity odd

This structure was also found by perturbative two-loop computation in Chern-Simons-fermion theory (GMPTWY) and shown to follow generally from weakly broken higher spin symmetry by Maldacena-Zhiboedov.

Symmetry breaking by boundary conditions

Global symmetry $\Rightarrow$ Vasiliev's gauge parameter $\varepsilon(x|Y)$ that leaves the AdS vacuum solution invariant: $D_0\varepsilon(x|Y) = 0$.

But, nonlinear "gauge" variation under $\varepsilon(x|Y)$ can lead to violation of boundary conditions.

Example: bulk scalar field $\varphi \subset B(x|Y)$. Gauge variation $\delta_\varepsilon B(x|Y) = -\varepsilon * B + B * \pi(\varepsilon)$ relates fields of different spins. At the presence of a spin-s' boundary source j, the variation of the bulk scalar need not, and generally will not, preserve either Δ=1 or Δ=2 boundary condition.

$\delta_\varepsilon \langle\varphi^{(0)}\rangle_j \sim \underline{\mathscr{D}} \langle\varphi^{(s')}\rangle_j$ + (higher order in fields),
corresponds to anomalous current conservation relation on the boundary:
$\partial^\mu J^{(s)}{}_{\mu\ldots} \sim J^{(0)}\, \underline{\mathscr{D}}\, J^{(s')}$ + (total derivative) + (other dbl trace) + (triple trace)

The higher spin dual to $\mathcal{N} = 6$ ABJ vector model, and more

n-extended supersymmetric ~~parity~~ Vasiliev theory: introduce Grassmannian auxiliary variables $\psi_1, \ldots \psi_n$ in the master fields. They obey Clifford algebra $\{\psi_i, \psi_j\} = \delta_{ij}$. Equation of motion modified to

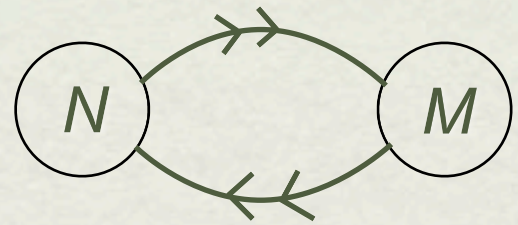
$$d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K)dz^2 + f_*(\mathbb{B} * \overline{K}\Gamma)d\bar{z}^2.$$

The $\mathcal{N} = 0,1,2,3,4,6$ CS vector models differ merely by double trace and triple trace deformations, and gauging a flavor symmetry with Chern-Simons coupling. These correspond to, in the holographic dual, simply changes of boundary condition on the bulk fields.

[Chang-Minwalla-Sharma-Yin, coming soon]

The duality makes a nontrivial prediction on two and three point function coefficients that do not follow from known symmetries.

A Triality



$U(N)_k \times U(M)_{-k}$ ABJ theory

$M \ll N$, $\theta_0 = \pi\lambda/2$

holography

$R_{AdS}/\ell_{string} = \lambda^{1/4}$

$\int_{CP^1} B = (N-M)/k$

n=6 supersymmetric ~~parity~~ Vasiliev theory with U(M) Chan-Paton factor and $\mathcal{N} = 6$ boundary condition

strong **bulk** 't Hooft coupling
$\lambda_{BULK} = M/N$

IIA string theory in $AdS_4 \times CP^3$

bound states of higher spin particles ⇒ strings

Vasiliev theory from string (field) theory?

(Supersymmetric) ~~parity~~ Vasiliev equations can be put in the form

$$d\mathbb{A} + \mathbb{A}*\mathbb{A} = e^{i\theta}\mathcal{B}\, dz^2 + e^{-i\theta}\,\overline{\mathcal{B}}\, d\bar{z}^2$$
$$\partial_{\bar{z}}\mathcal{B} + [\mathbb{A}_{\bar{z}}, \mathcal{B}]_* = 0$$
$$\partial_z\overline{\mathcal{B}} + [\mathbb{A}_z, \overline{\mathcal{B}}]_* = 0$$

$$\Rrightarrow \quad Q_0\mathbb{V} + \mathbb{V}*\mathbb{V} = 0$$

(purely formal at the moment)

$$\mathcal{B} = \overline{\mathcal{B}}*K\bar{K}\Gamma \quad \Rrightarrow \quad (b_0 - \bar{b}_0)\,\mathbb{V} = 0$$

Similar construction based on Poisson sigma model considered previously by Engquist-Sundell

In the "ABJ triality", θ is identified with the B-field flux of type IIA string theory on $AdS_4 \times CP^3$, suggesting that the RHS of Vasiliev's equation come from worldsheet instantons (wrapping CP^1) !

How to derive Vasiliev's system from tensionless limit of the string field theory remains to be seen. A topological open+closed string field theory from D6 brane wrapped on $AdS_4 \times RP^3$ (Jafferis-Gaiotto setup) in the zero radius limit?

Higher spin AdS_3/CFT_2 duality

Pure higher spin gauge theory in AdS_3 can be constructed as $SL(N,R) \times SL(N,R)$ Chern-Simons theory, with appropriate boundary condition [Henneaux-Rey, Campoleoni-Fredenhagen-Pfenninger-Theisen, '10, Gaberdiel-Hartman, '11]. Spin $s=2,\ldots,N$.

Unclear whether the pure HSGT in AdS_3 is non-perturbatively well defined, i.e. whether it has a CFT dual. Even the $N=2$ case, i.e. pure Einstein gravity in AdS_3, the answer is not known (lacking knowledge of irrational CFTs, non-perturbative effects in gravity). [Witten, Yin, Maloney-Witten, Gaberdiel '07]

There are two-dimensional CFTs with exact higher spin symmetry. But are there such large N CFTs with a weakly coupled holographic dual?

Gaberdiel-Gopakumar '11: The 2d version of vector models is the W_N minimal model. They further conjectured its holographic dual to be the AdS_3 Vasiliev system (an ∞ tower of HS fields coupled to massive scalars).

W_N minimal model

(Higher spin generalization of 2d critical Ginzburg-Landau models)

W_N algebra generated by conserved (holomorphic and anti-holomorphic) currents of conformal weights (s,0) and (0,s), s=2,3,…,N.

Can be realized as the coset model $(SU(N)_k \times SU(N)_1)/SU(N)_{k+1}$.
Primaries w.r.t. W_N algebra are labeled by a pair of representations of (affine) $SU(N)$, denoted $(R;R')$.

"Basic" primaries $\phi_1 = (\square;0)$, $\tilde{\phi}_1 = (0;\square)$ generate all primaries via OPE.

't Hooft limit: $N \to \infty$, $\lambda \equiv N/(k+N)$ finite (between 0 and 1).

For representations R and R' that do not grow with N, the primary $(R;R')$ has finite dimension in the 't Hooft limit.

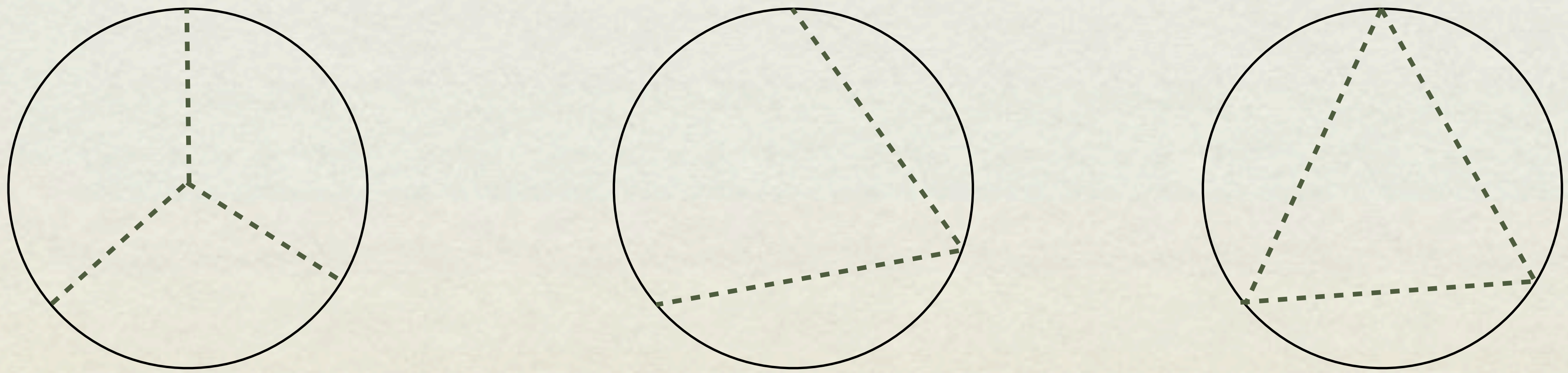
Large N factorization of W_N minimal model

[Chang-Yin '11] In the large N limit, the primaries $(R\,;R')$ are organized into "single-trace" and "multi-trace" operators.

For instance, correlators of a single-trace operator $\mathcal{O}_1$ and a double-trace operator $\mathcal{O}_2$ have different scaling with N :

If we normalize the operators by $\langle\, \mathcal{O}\, \mathcal{O}\, \rangle \sim 1$, then in a large N vector model,

$\langle\, \mathcal{O}_1\, \mathcal{O}_1\, \mathcal{O}_1 \,\rangle \sim N^{-1/2}$ $\quad$ $\langle\, \mathcal{O}_1\, \mathcal{O}_1\, \mathcal{O}_2 \,\rangle \sim 1$ $\quad$ $\langle\, \mathcal{O}_2\, \mathcal{O}_2\, \mathcal{O}_2 \,\rangle \sim 1$



Large N factorization of W_N minimal model

Single-trace operators	Multi-trace operators
$\phi_1 = (\ydiagram{1}\,;0)$	$(\ydiagram{1,1}\,;0) \sim \phi_1^2$
$\phi_2 = (\ydiagram{2}\,;\ydiagram{1}) - (\ydiagram{1,1}\,;\ydiagram{1})$	$(\ydiagram{2}\,;0) \sim \phi_1 \partial\bar{\partial}\phi_1 - \partial\phi_1\bar{\partial}\phi_1$
$\phi_3 = \sqrt{2}\,(\ydiagram{3}\,;\ydiagram{2}) - (\ydiagram{2,1}\,;\ydiagram{2}) - (\ydiagram{2,1}\,;\ydiagram{1,1}) + \sqrt{2}\,(\ydiagram{1,1,1}\,;\ydiagram{1,1})$	$(\mathrm{adj}\,;0) \sim \phi_1\bar{\phi}_1$
....	$(\ydiagram{2}\,;\ydiagram{2}) + (\ydiagram{1,1}\,;\ydiagram{1,1}) \sim \omega_1^2$
$\tilde{\phi}_1 = (0;\ydiagram{1})$, ...	$(\ydiagram{2}\,;\ydiagram{1}) + (\ydiagram{1,1}\,;\ydiagram{1}) \sim \phi_1\omega_1$
	$(\ydiagram{2}\,;\ydiagram{1,1}) + (\ydiagram{1,1}\,;\ydiagram{2}) \sim \omega_1\partial\bar{\partial}\omega_1 - \partial\omega_1\bar{\partial}\omega_1$
$\omega_1 = (\ydiagram{1}\,;\ydiagram{1})$	$\partial\bar{\partial}\omega_1 \sim \frac{\lambda^2}{N}\phi_1\tilde{\phi}_1$
$\omega_2 = (\ydiagram{2}\,;\ydiagram{2}) - (\ydiagram{1,1}\,;\ydiagram{1,1})$	$\partial\bar{\partial}\omega_2 \sim \sqrt{2}\frac{\lambda^2}{N}(\phi_1\tilde{\phi}_2 + \phi_2\tilde{\phi}_1)$
$\omega_3 = (\ydiagram{3}\,;\ydiagram{3}) - (\ydiagram{2,1}\,;\ydiagram{2,1}) + (\ydiagram{1,1,1}\,;\ydiagram{1,1,1})$	$\partial\bar{\partial}\omega_3 \sim \sqrt{3}\frac{\lambda^2}{N}(\phi_1\tilde{\phi}_3 + \phi_2\tilde{\phi}_2 + \phi_3\tilde{\phi}_1)$
....	....

Light states

A primary operator of the form $(R;R)$ has scaling dimension $\Delta = \lambda^2 B(R)/N$ at large N, where $B(R)$ = # boxes in the Young tableaux of R.

$\Rightarrow$ near continuum of low lying states at large N

- does large N factorization really hold?

- yes, correlators do obey large N factorization, provided that one makes the identifications [Raju-Papadodimas, Chang-Yin]

$$N\,\partial\bar{\partial}\omega_1 \sim \lambda^2\, \phi_1\tilde{\phi}_1$$

$$N\,\partial\bar{\partial}\omega_2 \sim \sqrt{2}\,\lambda^2\, (\phi_1\tilde{\phi}_2 + \phi_2\tilde{\phi}_1)$$

$$N\,\partial\bar{\partial}\omega_3 \sim \sqrt{3}\,\lambda^2\, (\phi_1\tilde{\phi}_3 + \phi_2\tilde{\phi}_2 + \phi_3\tilde{\phi}_1)$$

....

$\partial\omega$ becomes null in the large N limit, and $j = \sqrt{N}\,\partial\omega$ effectively becomes a new primary operator.

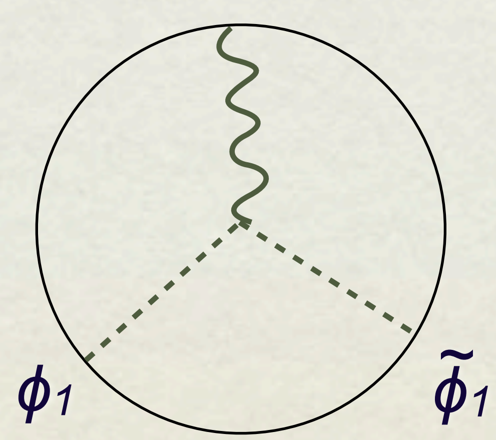
Hidden symmetries

The relation between ω_n and ϕ_n, $\tilde{\phi}_n$ can be recast in terms of non-conservation of the large N currents $j_n = \sqrt{N}\,\partial\omega_n$,

$$\bar{\partial} j_1 \sim \frac{\lambda^2}{\sqrt{N}}\,\phi_1\tilde{\phi}_1, \text{ etc.}$$

These are reminiscent of the non-conservation relation of higher spin currents in 3d vector models.

Interpretation: j_1 is dual to a U(1) Chern-Simons gauge field in AdS_3, under which ϕ_1 is rotated into $\tilde{\phi}_1$. This symmetry is broken by the boundary condition that assigns different scaling dimensions to ϕ_1 and $\tilde{\phi}_1$. The current non-conservation relation is reproduced by the 3-point function in AdS_3.



More Hidden symmetries

Each $j_n = \sqrt{N}\,\partial\omega_n$ is dual to a bulk gauge field whose associated boundary symmetry is broken by the boundary condition assignment on ϕ_m, $\tilde{\phi}_m$.

$\Rightarrow$ A gauge group generator T_n: $\bar{\phi} \rightarrow \tilde{\phi}$, $n=1,2,3,\ldots$

But wait, there is more: $[T_n, T_m] \neq 0$, while $\mathrm{Tr}(T_n [T_m, T_k]) = 0$.

$\Rightarrow$ Bulk Chern-Simon gauge fields with an ∞ dimensional non-Abelian gauge group, under which ϕ_m, $\tilde{\phi}_m$ transform. Gauge group broken completely by boundary conditions.

Conjecture: the holographic dual to the large N W_N minimal model should be Vasiliev theory in AdS$_3$ extended by an ∞ tower of massive matter fields and an ∞ dimensional Chern-Simons gauge group broken by boundary conditions.

A perturbative duality

Vasiliev theory in AdS_3 involves gauge fields of spin $s=2,3,\ldots,\infty$, and a single complex massive scalar field φ. φ should be dual to one of the basic primaries of W_N minimal model, namely $(\square\,;0)$ or $(0;\square)$ depending on the boundary condition.

A modest version of Gaberdiel-Gopakumar conjecture [Chang-Yin '11]: Vasiliev theory in AdS_3 is dual perturbatively to the subsector of W_N minimal model generated by primaries of the form $(R\,;0)$ (or the subsector generated by $(0\,;R)$, with the alternative boundary condition on φ.)

This sector closes perturbatively in $1/N$ on the sphere/plane, but is not modular invariant and non-perturbatively incomplete.

The non-perturbative dual of W_N minimal model ?

…remains illusive. 〔See M.Gaberdiel's talk for a proposal in a different limit〕

How to couple the infinite tower of elementary matter fields (dual to what we identified as single trace primaries) to the higher spin gauge fields?

Role of "conical surplus"? Castro-Gopakumar-Gutperle-Raeymaekers

Does the BTZ black hole dominate the thermodynamics at high temperature? On the CFT side, exact torus two-point function known 〔Chang-Yin〕 but large N behavior not yet understood. Note: despite rational CFT, thermalization behavior possible at large N (though Poincaré recurrence time ~ $N^{\#}$ rather than e^N).