Exact Results for Wilson Loops in Supersymmetric Theories

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AdS/CFT correspondence

Yang-Mills theory with N=4 supersymmetry

⇕

Maldacena'97

String theory on $AdS_5 x S^5$ background

AdS/CFT correspondence

$\mathcal{N}=4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators Scaling dimension: Δ	String states Energy: E

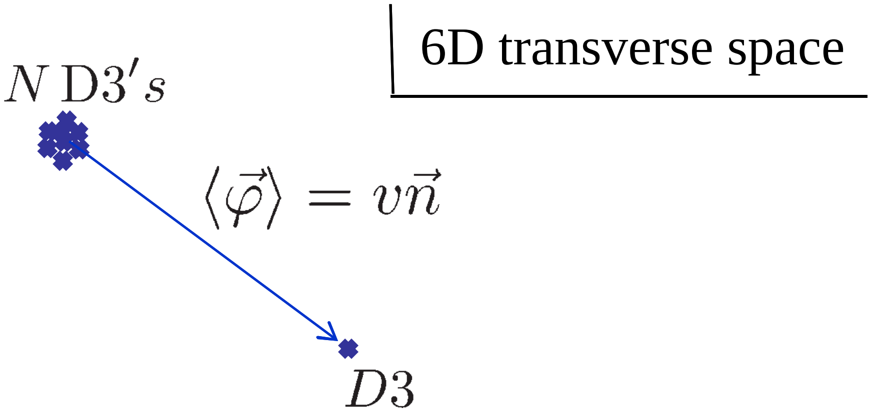
N=∞
in this talk

Gubser,Klebanov,Polyakov'98
Witten'98

Wilson loops

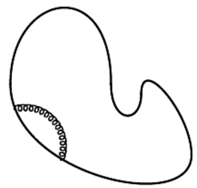
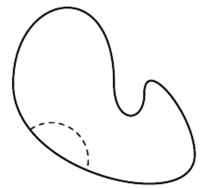
$$W(C) = \left\langle \mathrm{P}\ \exp\left[\int_C ds\,\left(A_\mu(x)\dot{x}^\mu + i\Phi_I n^I\,|\dot{x}|\right)\right]\right\rangle$$

- describes heavy W-boson of $U(N+1) \to U(N) \times U(1)$
- (the only way to couple N=4 SYM to matter)
- hence the scalar term in Wilson loop



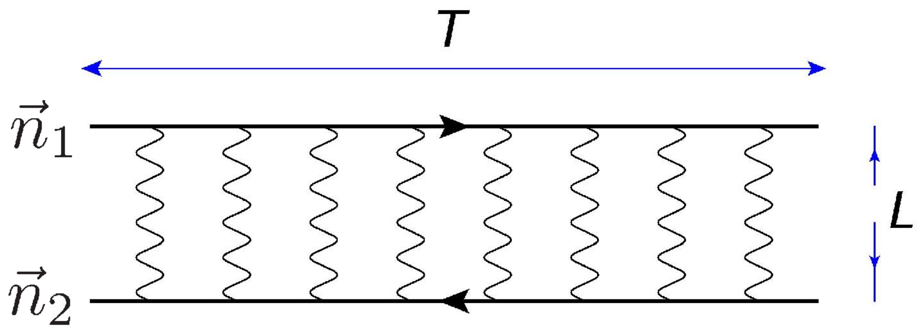
Wilson loops: weak coupling

$$W(C) = \left\langle \mathrm{P}\,\exp\left[\int_C ds\,\left(A_\mu(x)\dot{x}^\mu + i\Phi_I n^I\,|\dot{x}|\right)\right]\right\rangle$$

$$1 \;+$$  $$+$$  $$+\;\ldots$$

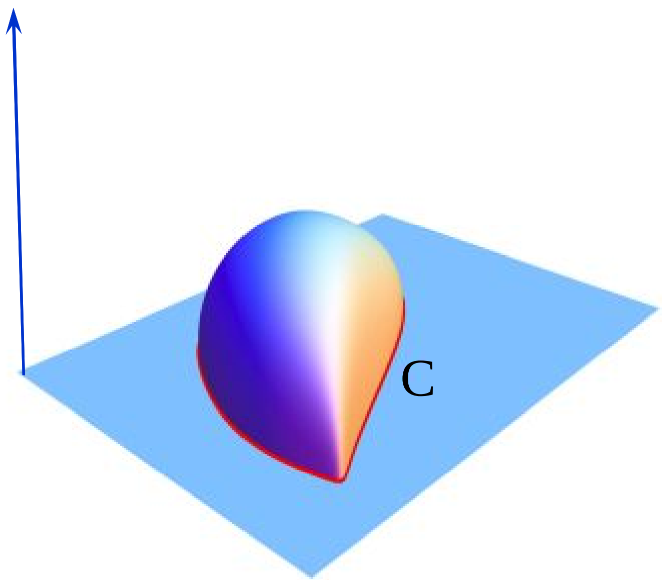
Effective Coulomb charge:

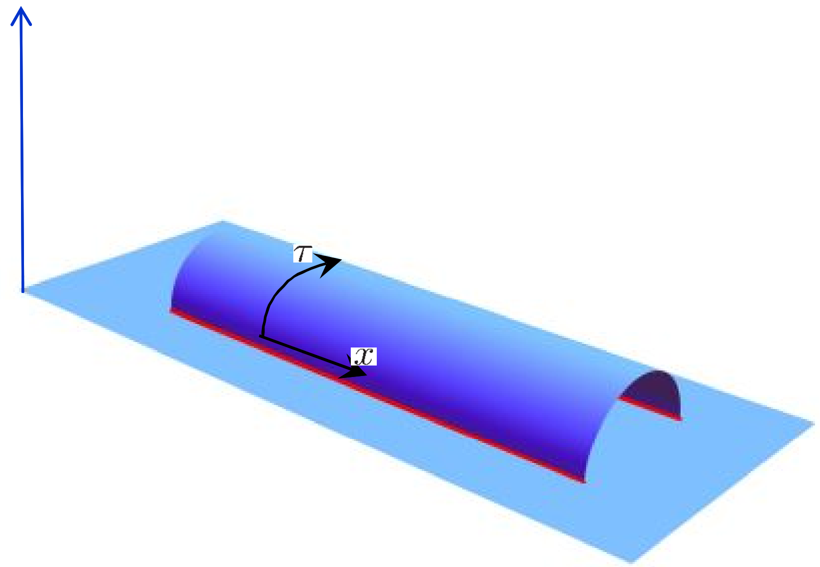
$$\vec{n}_1 \cdot \vec{n}_2 = \cos\theta$$



$$W(C_{T\times L}) \simeq \mathrm{e}^{-\alpha(\theta,\lambda)T/L} \qquad (T \gg L) \qquad V(L) = -\frac{\alpha(\theta,\lambda)}{L}$$

Wilson loops: strong coupling



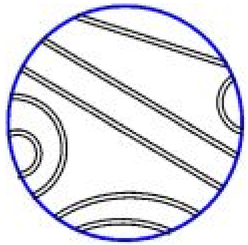


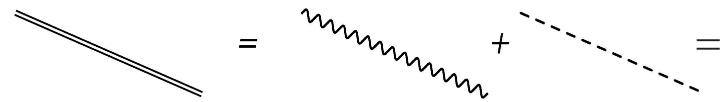
Area law:

$$\langle W(C) \rangle \overset{\lambda \to \infty}{=} \text{const}\, \lambda^{-\frac{3}{4}}\, \mathrm{e}^{-\frac{\sqrt{\lambda}}{2\pi}\, \mathrm{Area}(C)}$$

Maldacena'98
Rey,Yee'98

Circular Wilson loop

$$W(C) = \sum$$ 



$$= \frac{1}{16\pi^2}$$

$\Longrightarrow$ Combinatorial problem (matrix model)

$$W(C) = \left\langle \mathrm{tr}\, \mathrm{e}^{2\pi M} \right\rangle_{m.m.} \qquad Z = \int d^{N^2}M \, \exp\left(-\frac{8\pi^2}{\lambda} N \mathrm{tr} M^2 \right)$$

Large-N solution (Wigner distribution):

$$\rho(x) = \frac{1}{N}\, \mathrm{tr}\, \delta(x - M) = \frac{4}{\lambda}\sqrt{\lambda - 4\pi^2 x^2}$$

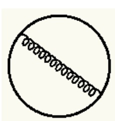
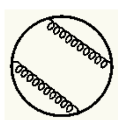
Circular Wilson loop: exact result

$$W(\text{circle}) = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)$$

Erickson,Semenoff,Zarembo'00
Drukker,Gross'00
Pestun'07

$\lambda \to 0$

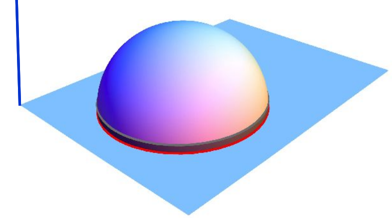
$$W(\text{circle}) = 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \ldots$$

1 +  +  + …

$\lambda \to \infty$

$$W(\text{circle}) = \sqrt{\frac{2}{\pi}}\lambda^{-3/4}\, \mathrm{e}^{\sqrt{\lambda}}$$

Minimal area law in AdS_5

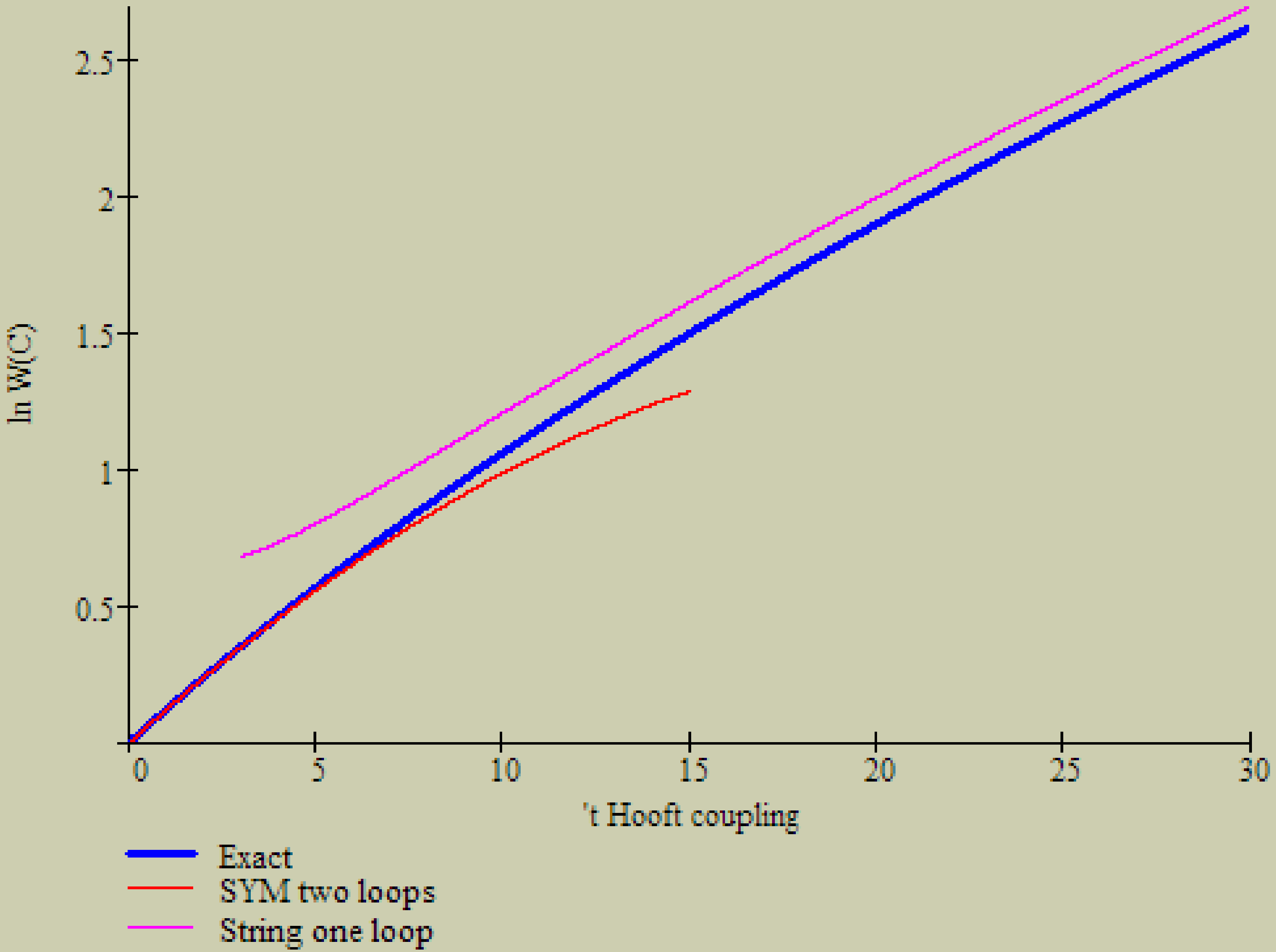


$$\text{Area} = 2\pi R \int_{\varepsilon}^{R} \frac{dz}{z^2} = -2\pi + \frac{2\pi R}{\epsilon}$$

regularized area

- used to describe Schwinger pair production

talk by Yu.Makeenko



2.5
2
1.5
1
0.5
ln W(C)
0
5
10
15
20
25
30
't Hooft coupling
Exact
SYM two loops
String one loop

Wilson loops in N=2 SYM

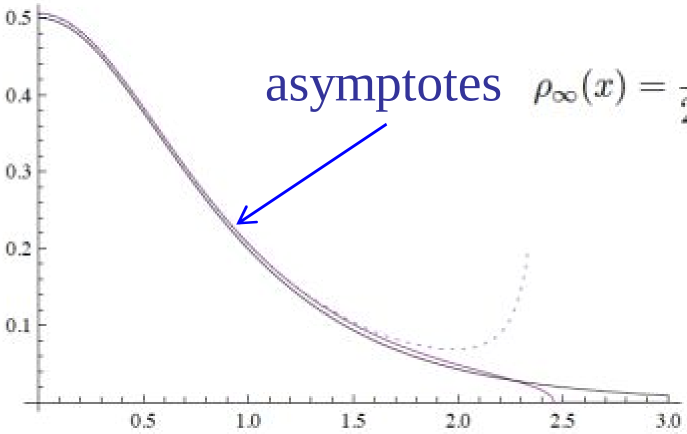
$$Z=\int d^{N-1}a\prod_{i<j}(a_i-a_j)^2\,\mathrm{e}^{-\frac{8\pi^2}{g^2}\sum_i a_i^2}\mathcal{Z}_{\text{1-loop}}(a)\left|\mathcal{Z}_{\text{inst}}(a;g^2)\right|^2$$

Pestun'07

In N=2, $N_f=2N_c$ SCFT:

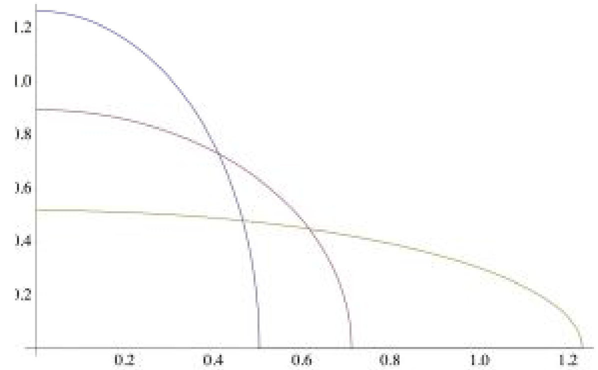
$$\mathcal{Z}_{\text{1-loop}}=\frac{\prod_{i<j}H^2(a_i-a_j)}{\prod_i H^{2N}(a_i)}$$

$$H(x)=\prod_{n=1}^{\infty}\left[\left(1+\frac{x^2}{n^2}\right)^n\mathrm{e}^{-\frac{x^2}{n}}\right]$$



$$\rho_\infty(x) = \frac{1}{2\cosh\frac{\pi x}{2}}$$

N=2



N=4

At $\lambda \to \infty$:

$$W(C_{\mathrm{circle}}) = \mathrm{const}\,\frac{\lambda^3}{(\ln\lambda)^{3/2}}$$

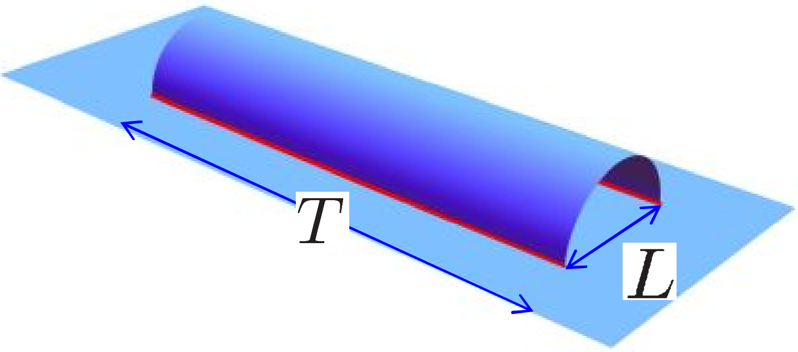
Passerini, Z.'11

- quantum-fluctuation prefactor for tensionless string?
- string with effective tension

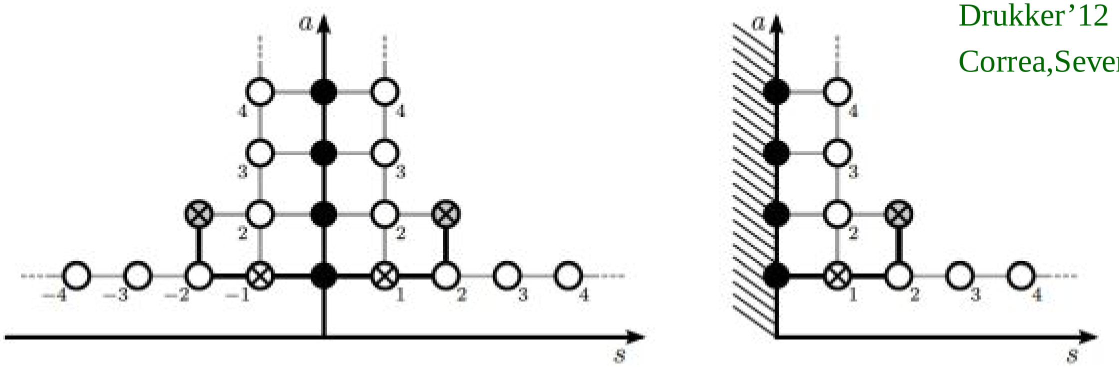
$$T = \frac{3}{2\pi}\ln\lambda \quad ?$$

Coulomb charge in N=4 SYM

$$W(C_{T\times L}) \simeq \mathrm{e}^{-\alpha(\theta,\lambda)T/L}$$

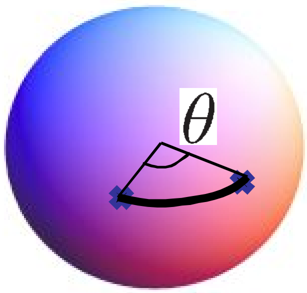


Known, in principle, from TBA equations



Drukker'12

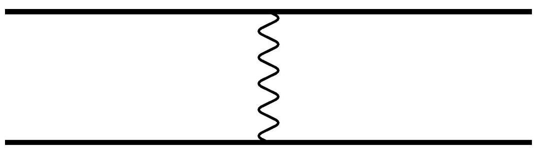
Correa,Sever,Maldacena'12



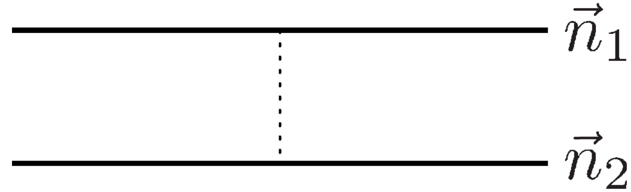
talk by V. Kazakov

- complicated…
- goal: describe a limiting case by simple QFT methods

Ladder diagrams



λ



$$\lambda \vec{n}_1 \cdot \vec{n}_2 = \lambda \cos\theta$$

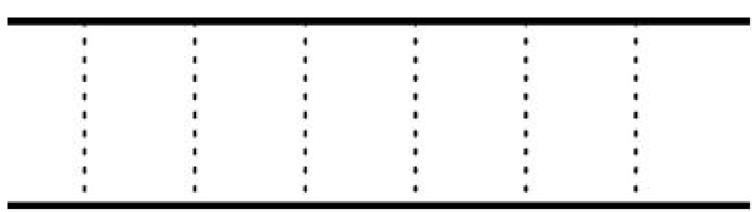
Limit:

$$\theta = i\vartheta \qquad \hat{\lambda} = \frac{\lambda}{2}(1+\cos\theta)$$

$$\vartheta \to \infty \qquad \lambda\, \mathrm{e}^{\vartheta} - \text{fixed}$$

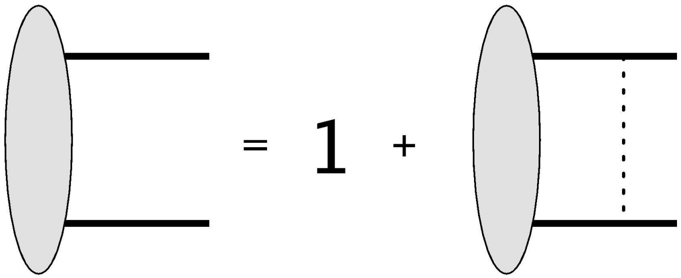
Correa,Henn,Maldacena,Sever'12

$$\sum$$



Summing ladders

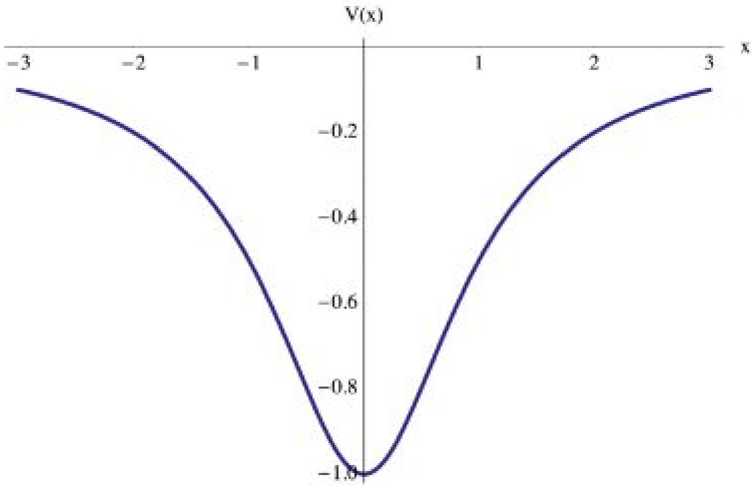
Bethe-Salpeter:



$$\frac{d^2\psi}{dx^2} + \frac{\hat{\lambda}}{4\pi^2\left(x^2+1\right)}\,\psi = \frac{\alpha^2}{4}\,\psi$$

Erickson,Semenoff,Szabo,Z.’99

Ladders: Strong Coupling

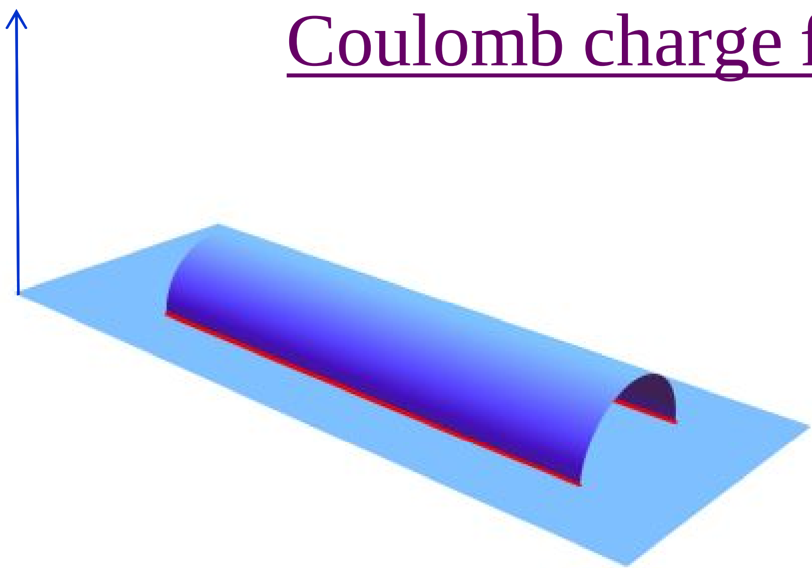


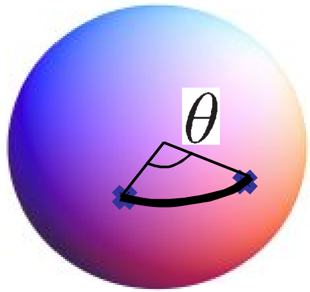
$$-\frac{\alpha^2}{4} \simeq \min_x \left(-\frac{\hat{\lambda}}{4\pi^2(x^2+1)} \right)$$

$$\alpha \simeq \frac{\sqrt{\hat{\lambda}}}{\pi}$$

Erickson,Semenoff,Szabo,Z.'99

Coulomb charge from string theory





$$\alpha = \frac{2\sqrt{\lambda}\,[E-(1-k^2)\,K]^2}{\pi k\sqrt{1-k^2}}$$

$$\vartheta = 2\sqrt{2k^2-1}\,K$$

Maldacena'98

At $k \to 1$:

$$\vartheta = -\ln\left(1-k^2\right)$$

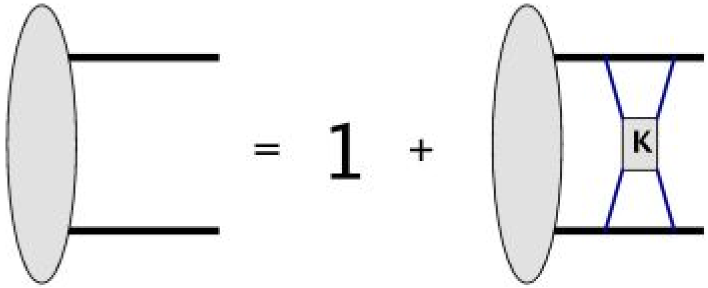
$$\alpha = \frac{2\sqrt{\lambda}}{\pi\sqrt{1-k^2}}$$

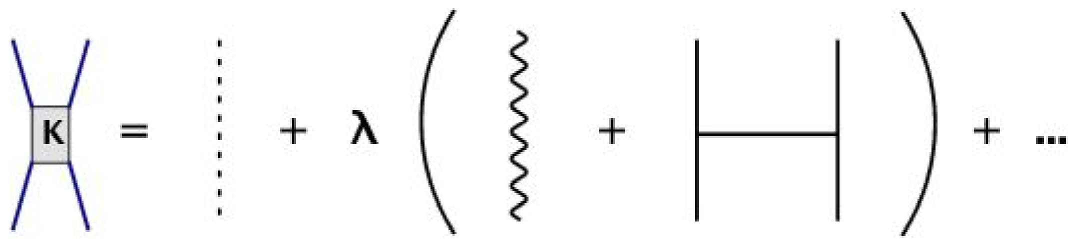
$$\Longrightarrow \quad \alpha = \frac{\sqrt{\lambda}}{2\pi}\,\mathrm{e}^{\frac{\vartheta}{2}}$$

Agrees with ladder calculation!

Correa,Henn,Maldacena,Sever'12

NLO ladders





$$V_{\rm eff}(x) = -\frac{\hat{\lambda}}{4\pi^2\,(x^2+1)} + \frac{\lambda\hat{\lambda}^2\ln\alpha}{2\pi^6\alpha^4\,(x^2+1)^3}$$

At strong coupling:

$$\alpha = \frac{\sqrt{\hat{\lambda}}}{\pi}\left(1 - \frac{\lambda \ln \hat{\lambda}}{2\hat{\lambda}}\right)$$ Bykov,Z.'12

In string theory:

$$\alpha = \frac{\sqrt{\lambda}}{2\pi}\, \mathrm{e}^{\frac{\vartheta}{2}} \left(1 - 2\vartheta\, \mathrm{e}^{-\vartheta}\right)$$

Agrees, since

$$\hat{\lambda} = \frac{\lambda}{4}\, \mathrm{e}^{\vartheta}$$

Questions

- String dual of N=2, $N_f=2N_c$ SCFT:
  - What can exact Wilson loop calculation tell about it?
  - Is dual string really tensionless?
- Wilson loops in other N=2 theories Russo'12
- Ladders from TBA?
- Why ladder calculation (weak coupling) agrees with string theory (strong coupling)?